Instituto de Ciências Matemáticas de São Carlos

ISSN - 0103-2577

THE HARTMAN-GROBMAN THEOREM FOR REVERSIBLE SYSTEMS ON BANACH SPACES

Hildebrando Munhoz Rodrigues
José Gaspar Ruas Filho

Nº 52

NOTAS DO ICMSC
Série Matemática

São Carlos
jul./97

THE HARTMAN-GROBMAN THEOREM FOR REVERSIBLE SYSTEMS ON BANACH SPACES

HILDEBRANDO MUNHOZ RODRIGUES† AND JOSÉ GASPAR RUAS-FILHO‡

ABSTRACT. In this note we give a version of the Hartman-Grobman Theorem for reversible systems defined on a Banach space. We prove that the homeomorphism that reduces the nonlinear system to the linearized one preserves the symmetry. Applications to coupled nonlinear second order ordinary differential equations and to coupled nonlinear wave equations are discussed.

1. INTRODUCTION

The object of this paper is to present a version of the Hartman-Grobman Theorem for reversible systems defined on a Banach space and applications to partial differential equations.

The proof of the Hartman-Grobman Theorem can be found for example in Hartman[10], in Pugh[14]. Our approach follows the one used by Pugh[14]. The proof presented by Pugh was based on some ideas of Moser[13] on the analysis of Anosov systems. A finite dimensional version of the Hartman-Grobman Theorem for reversible systems was presented in Rodrigues[15]. An important application was given in Rodrigues and Ruas-Filho[16], where hard estimates were obtained for generalized inverses related to Liapunov-Schmidt Method, to study bifurcation of subharmonic and homoclinic solutions of nonautonomous equations.

The idea of proving that symmetries on the equation imply symmetry on the solutions has been widely explored, as one can see in Fürkotter and Rodrigues[2,3], Galante and Rodrigues[4,5], Hale and Rodrigues[8,9], Rodrigues and Ruas-Filho[16], Rodrigues and Vanderbauwhede[18], Vanderbauwhede[19], etc.

2. REVERSIBILITY FOR SEMIGROUPS

Throughout this paper $\mathcal{B}$ will indicate a real Banach space. For the notations and basic results on reversible systems we suggest Hale[6] and Vanderbauwhede[18].

Definition 2.1. Let $\mathcal{B}$ be a real Banach space. Let $A : \mathcal{D} \subset \mathcal{B} \to \mathcal{B}$ be the infinitesimal generator of a C_0-semigroup $T(t)$. Let $S : \mathcal{B} \to \mathcal{B}$ be a real continuous linear isometry such that $S^2 = I$. We say that A is reversible with respect to S, if

(i) $\mathcal{D}$ is invariant under S.
(ii) $ASx = -SAx$, for every $x \in \mathcal{D}$.

1991 *Mathematics Subject Classification.* Primary 38C35, 34D10, 35F20.

Key words and phrases. The Hartman-Grobman Theorem, reversible systems, symmetry.

†Research partially supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) and Conselho Nacional de Desenvolvimento Científico e Tecnológico, (CNPq), Brazil, grant # 301994/85-4.

‡Research partially supported by Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq), Brazil, grant # 301890/85-4.

When dealing with Hilbert spaces the requirements on the symmetry operator S are that it be an involution, $S^2 = I$, and symmetric with respect to the inner product of the space, $\langle x, Sy\rangle = \langle Sx, y\rangle$, for all x and y in X. One can generalize this to semi-inner product spaces introduced by Lummer[12].

Let$X$ be a Banach space and X^* its dual. For each $x \in X$ there exists, by the Hahn-Banach theorem at least one (and we shall choose exactly one) functional $j(x) \in X^*$ such that $j(x)(x) = ||x||^2$. In the definition above one can easily show that if require that the operator S be an involution and satisfies

$$[Sx, y] = [x, Sy] \qquad \text{for all } x,\ y \in X$$

where the semi-inner product $[.,.]$ is defined by $[x, y] = j(y)(x)$, then S is a linear isometry in X.

Lemma 2.1. *Let $A : \mathcal{D} \subset \mathcal{B} \to \mathcal{B}$ be the infinitesimal generator of a C_0-semigroup $T(t)$. If A is reversible with respect to S, then the following holds:*

(i) *$T(t)$ is a group.*

(ii) *The spectrum of A, $\sigma(A)$ and its parts, namely, point, continuous and residual spectrum are invariant under the mapping:*

$$\lambda \in \mathbb{C} \mapsto -\lambda \in \mathbb{C}.$$

Moreover, if λ is an eigenvalue of A, associated to the eigenvector x_0, then $-\lambda$ is an eigenvalue of A, associated to the eigenvector Sx_0

Proof. For $t > 0$, we define $T(-t) := ST(t)S$. It is easy to check that $T(t)$ is a group. The second part follows from the fact that,

$$-S(\lambda - A)Sx = [(-\lambda) - A]x, \ \forall x \in \mathcal{D}.$$

□

Lemma 2.2. *Let $A : \mathcal{D} \subset \mathcal{B} \to \mathcal{B}$ be the infinitesimal generator of a C_0-semigroup $T(t)$. Let A be reversible with respect to S. Let*

$$W_s := \{x \in \mathcal{B} :\ T(t)x \to 0,\ as\ t \to \infty\},\ W_u := \{x \in \mathcal{B} : T(t)x \to 0,\ as\ t \to -\infty\}.$$

Suppose $\mathcal{B} = W_s \oplus W_u$ and that there exists projection $P : \mathcal{B} \to W_s$ onto, such that $SP = (I - P)S$ and

$$||T(t)Px|| \leq Ke^{-\alpha t}||x||,\ \forall t \geq 0,\ \forall x \in \mathcal{B}$$

where $K \geq 1$ and $\alpha > 0$ are real numbers.

Then the following hold:

(i) $||T(t)(I - P)x|| \leq Ke^{\alpha t}||x||,\ \forall t \leq 0,\ \forall x \in \mathcal{B}$.

(ii) *$\sigma(T(1))$ does not intersect the unity circle.*

(iii) *$B := T(1)$ is an isomorphism, $S \circ B \circ S = B^{-1} = T(-1)$ and the spectrum of B is invariant under the mapping $\lambda \to \lambda^{-1}$.*

(iv) *$W_u = (I - P)\mathcal{B}$ and $SW_s = W_u$.*

(v) *If $B_s := B_{|W_s}$, $B_u := B_{|W_u}$, then $S \circ B_s \circ S = (B^{-1})_u$.*

(vi) *If we define:*

$$|||x||| := \int_0^\infty \|T(t)Px\|dt + \int_{-\infty}^0 \|T(t)(I-P)x\|dt,$$

then the norm $|||\cdot|||$ *is equivalent to* $\|\cdot\|$, $|||B_s||| < 1$ *and* $|||B_u^{-1}||| < 1$

Proof. The second statement follows from the fact that the spectral radius of B_s is less than 1. Similar idea can be used for B_u. To prove (vi) we first observe that

$$\begin{aligned}|||Px||| &= \int_0^\infty \|T(t)Px\|dt = \sum_{k=0}^\infty \int_k^{k+1} \|T(t)Px\|dt = \\ &\leq \sum_{k=0}^\infty \int_0^1 Me^{-\alpha k}\|T(t)Px\|dt = \\ &= \frac{K}{1-e^{-\alpha}} \int_0^1 \|T(t)Px\|dt.\end{aligned}$$

Then

$$\begin{aligned}|||B_s x||| &= \int_0^\infty \|T(t)T(1)Px\|dt = \\ &= \int_0^\infty \|T(t)Px\|dt - \int_0^1 \|T(t)Px\|dt \\ &\leq (1 - \frac{1-e^{-\alpha}}{K})|||Px||| \leq (1 - \frac{1-e^{-\alpha}}{K})|||x|||.\end{aligned}$$

Therefore $|||B||| \leq 1 - \frac{1-e^{-\alpha}}{K} < 1$.

The proof of the other items are immediate. □

3. The Symmetric Hartman-Grobman Theorem

Let C_*^0 be the space of the bounded and uniformly continuous functions from $\mathcal{B}$ to $\mathcal{B}$, with the usual sup norm. Let $\mathcal{L}_\mu$ be the subspace of the functions of C_*^0, which are uniformly Lipschitz continuous with Lipschitz constant μ.

Theorem 3.1. (The symmetric Hartman-Grobman Theorem for mappings) *Suppose the assumptions of Lemma 2.2 are satisfied and let* $B := T(1)$. *Then there exists* $\mu_0 > 0$ *such that, for any* $f \in \mathcal{L}_{\mu_0}$ *satisfying* $(B+f)\circ S = S\circ(B+f)^{-1}$, *there is a unique homeomorphism* $h = h(f)$ *with* $h(0) = I$, $h - I \in C_*^0$, *and such that* $h\circ(B+f) = B\circ h$. *Moreover* $h\circ S = S\circ h$ *and* $S\circ h_u = h_s\circ S$. *In particular the set* $\{x \in \mathcal{B} : Sx = x\}$ *is invariant under* h.

Proof. From Pugh[14], it follows that there exists $\mu_0 > 0$ such that, for any $f \in \mathcal{L}_{\mu_0}$ there is a unique homeomorphism $h = h(f) \in C_*^0$, such that $h\circ(B+f) = B\circ h$ and $h(0) = I$.

From our assumptions we obtain,

$$\begin{aligned} S\circ h\circ S\circ(B+f) &= S\circ h\circ(B+f)^{-1}\circ S \\ &= S\circ h\circ h^{-1}\circ B^{-1}\circ h\circ S \\ &= S\circ B^{-1}\circ h\circ S \\ &= B\circ S\circ h\circ S \end{aligned}$$

From the uniqueness it follows that $S\circ h\circ S = h$ and so $h\circ S = S\circ h$.

Let h_s and h_u be respectively the projections of h on W^s and W^u. From Pugh[14] it follows that for any $x_0\in\mathcal{B}$,

$$h_s(x_0) = B_s\circ h_s((B+f)^{-1}(x_0)) \tag{3.1}$$

$$h_u(x_0) = (B^{-1})_u\circ h_u((B+f)(x_0)) \tag{3.2}$$

If we let $x_0 = Sy_0$ in (2.1) and (2.2), we obtain

$$\begin{aligned} h_s(Sy_0) &= B_s\circ h_s((B+f)^{-1}(Sy_0)) = B_s\circ h_s(S(B+f)(y_0)) \\ h_u(Sy_0) &= (B^{-1})_u\circ h_u((B+f)(Sy_0)) = (B^{-1})_u\circ h_u(S(B+f)^{-1}(y_0)) \end{aligned}$$

and then

$$\begin{aligned} S\circ h_s(Sy_0) &= S\circ B_s\circ h_s(S(B+f)(y_0)) = (B^{-1})_u\circ S\circ h_s(S(B+f)(y_0)) \\ S\circ h_u(Sy_0) &= S\circ(B^{-1})_u\circ h_u(S(B+f)^{-1}(y_0)) = (B)_s\circ S\circ h_u(S(B+f)^{-1}(y_0)) \end{aligned}$$

Uniqueness implies that $S\circ h_s\circ S = h_u$ and then $h_s\circ S = S\circ h_u$. The above calculation also proves that (3.1) and (3.2), in the symmetric case, are equivalent. □

Let $g: V\subset\mathcal{B}\to\mathcal{B}$ be a Lypschitz function, where V is an open neighborhood of the origin, symmetric with respect to S, that is, $SV = V$.

Definition 3.1. Consider the differential equation: $\dot{x} = Ax + g(x)$, where A is reversible with respect to S. We say that this equation is reversible, with respect to S in V, if $g(Sx) = -Sg(x)$ for all $x\in V$. In this case we will also say that $g(x)$ is reversible.

If $x(t)$ is a solution of a reversible system then $Sx(-t)$ is also a solution. If $x(t,x_0)$ denotes the solution of the above equation with initial condition x_0 at $t=0$, then $x(t,Sx_0) = Sx(-t,x_0)$.

Consider the systems:

$$\dot{y} = Ay \tag{3.3}$$

$$\dot{x} = Ax + f(x) \tag{3.4}$$

Theorem 3.2. (The Symmetric Hartman-Grobman Theorem for flows) *Let $A:\mathcal{D}\subset\mathcal{B}\to\mathcal{B}$ be the infinitesimal generator of a C_0-semigroup $T(t)$. Let A and f be reversible with respect to S. Suppose $\mathcal{B} = W_s\oplus W_u$ and that there exists projection $P:\mathcal{B}\to W_s$ onto, such that $SP = (I-P)S$ and*

$$\|T(t)Px\| \le Ke^{-\alpha t}\|x\|,\ \forall t\ge 0,\ \forall x\in\mathcal{B}$$

where K and α are positive real numbers.

Then there exists $\mu_1 > 0$, such that if $f\in\mathcal{L}_{\mu_1}$, there exists a unique homeomorphism $h:\mathcal{B}\to\mathcal{B}$, with $h - I\in C^0_*$ such that $h\circ\psi_t = \phi_t\circ h$, where ϕ_t and ψ_t are the flows generated by (3.3) and (3.4) respectively. Moreover $h\circ S = S\circ h$ and $S\circ h_u = h_s\circ S$. In particular, the set $\{x\in\mathcal{B}: Sx = x\}$ is invariant under h.

Proof. Consider the time one map $\psi_1(x)$ associated to the system (3.4). By the variation of constants formula we have,

$$\psi_1(x_0) = x(1, x_0) = T(1)x_0 + \int_0^1 T(1-s)f(x(s, x_0))ds := Bx_0 + F(x_0),$$

where $F \in \mathcal{L}_{\mu_0}$ and $\mu_0 := \frac{K\mu_1}{\alpha}$ is sufficiently small.

From the reversibility condition we obtain,

$$(B + F)(Sx_0) = \psi_1(Sx_0) = x(1, Sx_0) = Sx(-1, x_0) = S\psi_1^{-1}(x_0) = S(B + F)^{-1}(x_0).$$

The result now follows from Theorem 3.1 using the uniqueness of the homeomorphism h as in Pugh[14]. □

Theorem 3.3. **(The Hartman-Grobman Theorem: The Local Version.)** *Let $A : \mathcal{D} \subset \mathcal{B} \to \mathcal{B}$ be the infinitesimal generator of a C_0-semigroup $T(t)$. Let A be reversible with respect to S. Suppose $\mathcal{B} = W_s \oplus W_u$ and that there exists projection $P : \mathcal{B} \to W_s$ onto, such that $SP = (I - P)S$ and*

$$\|T(t)Px\| \le Ke^{-\alpha t}\|x\|, \ \forall t \ge 0, \ \forall x \in \mathcal{B}$$

where K and α are positive real numbers.

Let V be a neighborhood of the origin, such that $SV = V$ and suppose that $f : V \to \mathcal{B}$ is continuous and reversible with respect to S on V. Let $V_\delta := \{x \in V : \|x\| \le \delta\}$, where $\delta > 0$. Assume that for each $\delta > 0$ there exists $\mu_\delta > 0$, such that $f|V_\delta$ has Lipschitz constant μ_δ and $\mu_\delta \to 0$, as $\delta \to 0$.

Then there exists $\delta > 0$ and a homeomorphism $h : V_\delta \to V_1$, where V_1 is a neighborhood of the origin, such that $h \circ \psi_t = \phi_t \circ h$, where ϕ_t and ψ_t are the flows generated by (3.3) *and* (3.4) *respectively. Moreover $h \circ S = S \circ h$ and $S \circ h_u = h_s \circ S$. In particular, $h(\{x \in V_\delta : Sx = x\}) \subset \{x \in V_1 : Sx = x\}$.*

Proof. Let $\delta > 0$, sufficiently small such that $2\mu_\delta < \mu_1$, where μ_1 is as in Theorem 3.2. We extend $f|_{V_\delta}$ to a function $F : \mathcal{B} \to \mathcal{B}$ in the following way:

$$F|_{V_\delta} := f|_{V_\delta}, \ F(x) := f|_{V_\delta}(\frac{\delta x}{\|x\|}), \text{ if } \|x\| > \delta.$$

One can prove that F is continuous and has Lipschitz constant $2\mu_\delta$ on $\mathcal{B}$. Moreover since S is an isometry, if $\|x\| > \delta$, we have

$$F(Sx) = f|_{V_\delta}(\frac{\delta Sx}{\|Sx\|}) = -Sf|_{V_\delta}(\frac{\delta x}{\|x\|}) = -SF(x)$$

The result follows from Theorem 3.2. □

4. Applications

In this section we make some applications to ordinary and partial differential equations.

Our first application gives us a result that was very important to prove uniform estimate of generalized inverses related to an application of Liapunov-Schimidt Method in Rodrigues and Ruas-Filho[16].

Example 4.1. Consider the system:

$$\begin{aligned} \dot{x} &= y + f(x, y) \\ \dot{y} &= \alpha x + g(x, y) \end{aligned}$$

where f, g are C^1 functions with sufficiently small derivatives, at the origin, such that $f(x, -y) = -f(x, y)$ and $g(x, y) = g(x, -y)$, for all x, y in $\mathbb{R}^n$, $g(0,0) = 0$ and $\alpha > 0$.

The above system is reversible with respect to the matrix:

$$S := \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.$$

In this case Theorem 3.3 says that there exists a homeomorphism h which takes the above nonlinear system into $\dot{x} = y$ and $\dot{y} = \alpha x$. If $h := \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$, in term of the components, the condition $h \circ S = S \circ h$ is equivalent to $h_1(x, -y) = h_1(x, y)$ and $h_2(x, -y) = -h_2(x, y)$, for every x, y in $\mathbb{R}^n$. For $y = 0$, we have $h_2(x, 0) = 0$ for every x. Therefore the x-axis is invariant under h.

An interesting case that belongs to this class of examples is the one which comes from second order equations $\ddot{x} = g(x)$, which is equivalent to the system: $\dot{x} = y$ and $\dot{y} = g(x)$.

Our second example deals with a reversible system of damped hyperbolic partial differential equations.

Example 4.2. Let us consider the system of partial differential equations:

$$\begin{cases} \frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - c\frac{\partial u}{\partial t} + f(x, u, u_x, v, v_x) \\ \frac{\partial^2 v}{\partial t^2} = \frac{\partial^2 v}{\partial x^2} + c\frac{\partial v}{\partial t} + g(x, u, u_x, v, v_x) \end{cases}$$

with the boundary conditions:

$$u(t, 0) = u(t, 1) = v(t, 0) = v(t, 1) = 0.$$

The above second order system is equivalent to the following first order system:

$$\begin{cases} \frac{\partial u_1}{\partial t} = u_2 \\ \frac{\partial u_2}{\partial t^2} = \frac{\partial^2 u_1}{\partial x^2} - cu_2 + f(x, u_1, \frac{\partial u_1}{\partial x}, v_1, \frac{\partial v_1}{\partial x}) \\ \frac{\partial v_1}{\partial t} = v_2 \\ \frac{\partial v_2}{\partial t} = \frac{\partial^2 v_1}{\partial x^2} + cv_2 + g(x, u_1, \frac{\partial u_1}{\partial x}, v_1, \frac{\partial v_1}{\partial x}) \end{cases} \tag{4.1}$$

We will use the notation of semigroup theory. Consider the space $X := (H_1^0 \times L_2)^2$. Let

$$U := \begin{pmatrix} u_1 \\ u_2 \\ v_1 \\ v_2 \end{pmatrix}, \quad AU := \begin{pmatrix} u_2 \\ \frac{\partial^2 u_1}{\partial x^2} - cu_2 \\ v_2 \\ \frac{\partial^2 v_1}{\partial x^2} + cv_2 \end{pmatrix}, \quad F(U) := \begin{pmatrix} 0 \\ f(\cdot, u_1, \frac{\partial u_1}{\partial x}, v_1, \frac{\partial v_1}{\partial x}) \\ 0 \\ g(\cdot, u_1, \frac{\partial u_1}{\partial x}, v_1, \frac{\partial v_1}{\partial x}) \end{pmatrix}.$$

Therefore, equation (4.1), with the above boundary conditions, is equivalent to the following equation defined on X,

$$\dot{U} = AU + F(U) \tag{4.2}$$

where $\mathcal{D}(A) := ((H^2 \cap H_0^1) \times H_0^1)^2$. One can easily show that $A : \mathcal{D}(A) \subset X \to X$ is the infinitesimal generator of a C_0-semigroup $T(t)$.

Consider the following continuous linear operator defined on X,

$$SU := \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} U$$

It is easy to verify that $S^2 = I$, and that A is reversible with respect to S.

Let

$$PU := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} U.$$

We have that $SP = (I - P)S$ and if we let

$$W^s = \{(u_1, u_2, 0, 0) \in X; u_1 \in H_0^1, u_2 \in L^2\}$$

then P is onto W^s and due to presence of the damping term $-c\frac{\partial u_1}{\partial x}$ one can show that the condition

$$\|T(t)x\| \leq Ke^{-\alpha t}\|x\|, \ \forall t \geq 0, \ \forall x \in W^s$$

on the behavior of $T(t)$ on W^s is satisfied for some constant $\alpha = \alpha(c)$. To show this one can use energy estimates or more direct methods. See, for example, Hale[7].

If we assume that $f(x, w_1, w_2, w_3, w_4) = g(x, w_3, -w_4, w_1, -w_2)$, for every $(x, w_1, w_1, w_3, w_4) \in [0, 1] \times \mathbb{R}^4$, then we have that $F(SU) = -SF(U)$, for every $U \in X$ and so system (4.2) is reversible with respect to S.

In addition to the above conditions, if we assume that f and g are Lipschitz continuous with constant $\mu > 0$, where μ is sufficiently small, from Theorem 3.3, we obtain that equation (4.2) is conjugate to equation,

$$\dot{U} = AU$$

under a homeomorphism $h : X \to X$. Moreover $h(SU) = Sh(U)$, for every $U \in X$. If $h := \begin{pmatrix} h_1, & h_2, & h_3, & h_4 \end{pmatrix}^T$, we have:

$$h_1(v_1, -v_2, u_1, -u_2) = h_3(u_1, u_2, v_1, v_2), \quad h_2(v_1, -v_2, u_1, -u_2) = -h_4(u_1, u_2, v_1, v_2),$$

for every $U := (u_1, u_2, v_1, v_2) \in X$.

The following subspace of X,

$$Y := \{\begin{pmatrix} u \\ v \\ u \\ -v \end{pmatrix}, \ u \in H_0^1, \ v \in L_2\}.$$

is the fixed subspace of S. Theorem 3.3 implies that Y is invariant under h, wich implies that, $h_1(u, v, u, -v) = h_3(u, v, u, -v), \quad h_2(u, v, u, -v) = -h_4(u, v, u, -v), \ \forall \ u \in H_0^1, \ v \in L_2$.

The results of this example can be generalized, in a natural way, to other types of partial differential equations, as for instance, the beam equation

$$(4.3)\qquad \begin{aligned} u_{tt}+\gamma u_{xxxx} + \Gamma u_{xx} + \delta u_t &= f(x,u,v,u_x,v_x) \\ v_{tt}+\gamma v_{xxxx} + \Gamma u_{xx} - \delta v_t &= g(x,u,v,u_x,v_x) \end{aligned}$$

with appropriate boundary conditions. To apply Theorem 3.3 to equation (4.3) one can use the estimates obtained by Rodrigues and Silveira[17] to the linear part of [4.3].

More generally one can think of hyperbolic equations or systems like

$$\begin{aligned} \ddot{u} + Cu + \delta\dot{u} &= F(u,v) \\ \ddot{v} + Cv - \delta\dot{v} &= G(u,v) \end{aligned}$$

where A is a positive self-adjoint operator in a Hilbert space. See, for example, Lopes and Ceron[11].

References

[1] S. N. Chow and J. K. Hale, *Methods of Bifurcation Theory*,Springer-Verlag (1982).

[2] M. Fürkotter and H. M. Rodrigues, *Periodic solutions of forced nonlinear second order equations: symmetry and bifurcations*, SIAM J. Math. Anal. 17, pp. 1319-1331, (1986).

[3] M. Fürkotter and H. M. Rodrigues, *On harmonic and subharmonic solutions: symmetry and bifurcation*, Apl. Anal. 37, 63-93, (1990).

[4] L.F. Galante and H.M. Rodrigues, *On bifurcation and symmetry of solutions of nonlinear D_m-equivariant equations.* Dynamic Systems and Applications. 2, 75-100, (1993).

[5] L.F. Galante and H.M. Rodrigues, *On bifurcation and symmetry of solutions of symmetric nonlinear equations with odd forcings*,J. Math. Anal. Appl. vol. 196, 526-553, (1995).

[6] J. K. Hale, *Ordinary Differential Equations*, Krieger, (1978)

[7] J. K. Hale, *Asymptotic Behavior of Dissipative Systems*, Mathematical Surveys ans Monographs, 25, American Mathematical Society, (1988).

[8] J. K. Hale and H. M. Rodrigues, *Bifurcation in the Duffing equations with independent parameters I*, Proc. Roy. Soc. 77 (A), pp. 57-75, (1977).

[9] J. K. Hale and H. M. Rodrigues, *Bifurcation in the Duffing equations with independent parameters II*, Proc. Roy. Soc. 77 (A), pp. 317-326, (1978).

[10] P. Hartman, *Ordinary Differential Equations*, Wiley, (1964)

[11] O. Lopes and S. Ceron, *Existence of forced periodic solutions of dissipative semilinear hyperbolic equations and systems*, Ann. Mat. Pura Appl.

[12] G. Lummer, *Semi-inner-product Spaces*, Trans. Amer. Math. Soc., vol. 100, 29-43 (1961).

[13] J. Moser, *On a theorem of Anosov*, Journal of Differential Equations vol. 5, pp. 411-440,(1969).

[14] C. C. Pugh, *On a theorem of P. Hartman*, Amer. J. Math. 91,363-367, (1969)

[15] H. M. Rodrigues, *The symmetric Hartman-Grobman Theorem, Technical Report, CDSNS, GeorgiaTech* CDSNS-112, (1993)

[16] H. M. Rodrigues and J. G. Ruas-Filho, *Homoclinics and subharmonics of nonlinear two dimensional systems. Uniform Boundedness of generalized inverses.* Dynamic Systems and Applications 3 (1994), pp. 379-394.

[17] H. M. Rodrigues and M. Silveira, *Properties of Bounded Solutions of Linear and Nonlinear Evolution Equations*, Journal of Differential Equations, 70, 403-440, (1987).

[18] H. M. Rodrigues and A. Vanderbauwhede, *Symmetric perturbations of nonlinear equations: symmetry of small solutions.* Nonlinear Anal. T. M. A., 2, pp. 27-46, (1978).

[19] A. Vanderbauwhede, *Local Bifurcation and Symmetry*, Pitman, (1982)

Instituto de Ciências Matemáticas de São Carlos, Universidade de São Paulo - Departamento de Matemática, 13560-970, São Carlos, SP, Brazil
E-mail address: hmr@icmsc.sc.usp.br

Instituto de Ciências Matemáticas de São Carlos, Universidade de São Paulo - Departamento de Matemática, 13560-970, São Carlos, SP, Brazil
E-mail address: jgrfilho@icmsc.sc.usp.br

NOTAS DO ICMSC

SÉRIE MATEMÁTICA

051/97 ARRIETA, J. M.; CARVALHO, A.N.; - Abstract parabolic problems with critical nonlinearities and applications to Navier-Stokes and heat equations

050/97 BRUCE, J.W.; GIBLIN, P.J.; TARI, F. - Familes of surfaces: focal sets, ridges and umbilics.

049/97 MARAR, W.L; BALLESTEROS, J.J.NUÑO - Semiregular surfaces with two tripe points and ten cross caps

048/97 MARAR, W.L; MONTALDI, J..A.; RUAS, M.A.S. - Multiplicities of zero-schemes in quasihomogeneous corank-1 singularities.

047/96 LEME, B.T.; KUSHNER, L. - Some remarks on ideals of the algebra of smooth function germs.

046/96 LEME, B. T.; KUSHNER, L. - Relative stability on algebraic sets.

045/96 KUSHNER, L. - Finite relative determination on bouquet of subspaces.

044/96 RIEGER, J.H. - Notes on the complexity of exact view graph algorithms for piecewise smooth algebraic surfaces.

043/96 BASTO-GONÇALVES, J. - Analytic linearizability of vector fields depending on parameters and of certain poisson structures.

042/96 CARVALHO, A.N.; RODRIGUEZ,H.M.; DLOTKO,T. - Upper semicontinuity of attractors and synchronization.