UNIVERSIDADE DE SÃO PAULO

Instituto de Ciências Matemáticas e de Computação

ISSN 0103-2577

POLAR MULTIPLICITIES AND EULER OBSTRUCTION FOR RULED SURFACES

NIVALDO G. GRULHA JR.
MARCELO E. HERNANDES
RODRIGO MARTINS

Nº 357

NOTAS DO ICMC

SÉRIE MATEMÁTICA

São Carlos – SP
Out./2011

POLAR MULTIPLICITIES AND EULER OBSTRUCTION FOR RULED SURFACES

NIVALDO G. GRULHA JR., MARCELO E. HERNANDES AND RODRIGO MARTINS.

ABSTRACT. Given two integers $n \geq m \geq 0$ we exhibited (ruled) surfaces with multiplicity n and Euler obstruction m. As a consequence we give a finitely determined condition for ruled surfaces in terms of Euler obstruction.

1. INTRODUCTION

The local Euler obstruction at a point p of an algebraic variety X, denoted by $\mathrm{Eu}_X(p)$, was defined by MacPherson. It is one of the main ingredients in his proof of Deligne-Grothendieck conjecture concerning existence and unicity of characteristic classes for complex algebraic varieties [7]. An equivalent definition was given in [3] by J.-P. Brasselet and M.-H. Schwartz, using stratified vector fields. The Euler obstruction was deeply investigated by many authors as Brasselet, Schwartz, Sebastiani, Lê, Teissier, Sabbah, Dubson, Kato and others. For an overview about the Euler obstruction see [1, 2]. The computation of local Euler obstruction is not so easy by using the definition. Various authors propose formulae which make the computation easier. For instance Lê D.T. and B. Teissier provide a formula in terms of polar multiplicities [11].

For 1-dimensional algebraic varieties, that is, algebraic curves, there only one polar multiplicity which is the multiplicity $m_0(C)$ of curve C and by a Lê-Teissier formula we can clearly see that the local Euler obstruction $Eu_C(0)$ coincides with $m_0(C)$. In this context for 1-dimensional algebraic varieties X it is easy to present examples with $Eu_X(0) = n$ for all non-negative integer n.

The purpose of this paper is to exhibit surface X with polar multiplicities prescribed $m_0(X) \geq m_1(X)$ and for convenient choice of $m_0(X)$ and $m_1(X)$ we can present family of (ruled) surfaces with $Eu_X(0) = n$ for all non-negative integer n.

Our result concern the Euler obstruction of germs of ruled surfaces, a interesting generalizations of the Euler obstruction is the Euler obstruction of function, this invariant was defined in [4] for functions defined on singular spaces, it would be interesting to try to compute this invariant for functions defined on ruled surfaces. When $f : (X, 0) \to \mathbb{C}$ has isolated singularity at the origin (in the stratified sense) this invariant is closely releted to Milnor number of a function, see for instance [4, 9, 6].

2. Preliminaries

In this section we introduce some definitions and notations related with polar multiplicities, Euler obstruction and ruled surfaces.

2.1. The local Euler obstruction. Let us first introduce some objects in order to define the Euler obstruction. Let $(X,0) \subset (\mathbb{C}^N,0)$ be an equidimensional reduced complex analytic germ of dimension d in an open set $U \subset \mathbb{C}^N$. We consider a complex analytic Whitney stratification $\{V_i\}$ of U adapted to X such that $\{0\}$ is a stratum. We choose a small representative of $(X,0)$ such that 0 belongs to the closure of all the strata. In this way we will write $X = \cup_{i=0}^{q} V_i$ where $V_0 = \{0\}$ and $V_q = X_{\text{reg}}$, the set of smooth points of X. Moreover, we will assume that the strata $V_0, \ldots, V_{q-1}$ are connected and that the analytic sets $\overline{V_0}, \ldots, \overline{V_{q-1}}$ are reduced and we will denote $d_i = \dim V_i$ for $i \in \{1, \ldots, q\}$ (note that $d_q = d$).

Let $G(d,N)$ denote the Grassmanian of complex d-planes in $\mathbb{C}^N$. On the regular part X_{reg} of X the Gauss map $\phi : X_{\text{reg}} \to U \times G(d,N)$ is well defined by $\phi(x) = (x, T_x(X_{\text{reg}}))$.

Definition 2.1. *The Nash transformation (or Nash blow up) $\widetilde{X}$ of X is the closure of the image* $\text{Im}(\phi)$ *in $U \times G(d,N)$. It is a (usually singular) complex analytic space endowed with an analytic projection map $\nu : \widetilde{X} \to X$ which is a biholomorphism away from* $\nu^{-1}(\text{Sing}(X))$.

The fiber of the tautological bundle $\mathcal{T}$ over $G(d,N)$, at the point $P \in G(d,N)$, is the set of the vectors v in the d-plane P. We still denote by $\mathcal{T}$ the corresponding trivial extension bundle over $U \times G(d,N)$. Let $\widetilde{T}$ be the restriction of $\mathcal{T}$ to $\widetilde{X}$, with projection map π. The bundle $\widetilde{T}$ on $\widetilde{X}$ is called *the Nash bundle* of X.

An element of $\widetilde{T}$ is written (x,P,v) where $x \in U$, P is a d-plane in $\mathbb{C}^N$ based at x and v is a vector in P. We have the following diagram:

$$\begin{array}{ccc} \widetilde{T} & \hookrightarrow & \mathcal{T} \\ \pi \downarrow & & \downarrow \\ \widetilde{X} & \hookrightarrow & U \times G(d,N) \\ \nu \downarrow & & \downarrow \\ X & \hookrightarrow & U \end{array}$$

Let us recall the original definition of the Euler obstruction, due to MacPherson [7]. Let $z = (z_1, \ldots, z_N)$ be local coordinates in $\mathbb{C}^N$ around $\{0\}$, such that $z_i(0) = 0$. We denote by B_ε and S_ε the ball and the sphere centered at $\{0\}$ and of radius ε in $\mathbb{C}^N$. Let us consider the norm $\|z\| = \sqrt{z_1\overline{z}_1 + \cdots + z_N\overline{z}_N}$. Then the differential form $\omega = d\|z\|^2$ defines a section of the real vector bundle $T(\mathbb{C}^N)^*$, cotangent bundle on $\mathbb{C}^N$. Its pull back restricted to $\widetilde{X}$ becomes a section denoted by $\widetilde{\omega}$ of the dual bundle $\widetilde{T}^*$. For

ε small enough, the section $\widetilde{\omega}$ is nonzero over $\nu^{-1}(z)$ for $0 < \|z\| \leq \varepsilon$. The obstruction to extend $\widetilde{\omega}$ as a nonzero section of $\widetilde{T}^*$ from $\nu^{-1}(S_\varepsilon)$ to $\nu^{-1}(B_\varepsilon)$, denoted by $Obs(\widetilde{T}^*, \widetilde{\omega})$ lies in $H^{2d}(\nu^{-1}(B_\varepsilon), \nu^{-1}(S_\varepsilon); \mathbb{Z})$. Let us denote by $\mathcal{O}_{\nu^{-1}(B_\varepsilon), \nu^{-1}(S_\varepsilon)}$ the orientation class in $H_{2d}(\nu^{-1}(B_\varepsilon), \nu^{-1}(S_\varepsilon); \mathbb{Z})$.

Definition 2.2. *The local Euler obstruction of X at 0 is the evaluation of $Obs(\widetilde{T}^*, \widetilde{\omega})$ on $\mathcal{O}_{\nu^{-1}(\mathbb{B}_\varepsilon), \nu^{-1}(\mathbb{S}_\varepsilon)}$, i.e:*

$$\mathrm{Eu}_X(0) = \langle Obs(\widetilde{T}^*, \widetilde{\omega}), \mathcal{O}_{\nu^{-1}(B_\varepsilon), \nu^{-1}(S_\varepsilon)} \rangle.$$

The local Euler obstruction is independent of all choices involved.

The following interpretation of the local Euler obstruction has been given by Brasselet-Schwartz [3].

Let us consider a stratified radial vector field $v(x)$ in a neighborhood of $\{0\}$ in X, *i.e.*, there is ε_0 such that for every $0 < \varepsilon \leq \varepsilon_0$, $v(x)$ is pointing outwards the ball $\mathbb{B}_\varepsilon$ over the boundary $\mathbb{S}_\varepsilon = \partial \mathbb{B}_\varepsilon$.

Definition 2.3. *Let v be a radial vector field on $X \cap \mathbb{S}_\varepsilon$ and $\widetilde{v}$ the lifting of v on $\nu^{-1}(X \cap \mathbb{S}_\varepsilon)$ to a section of the Nash bundle. The local Euler obstruction (or simply the Euler obstruction)* $\mathrm{Eu}_X(0)$ *is defined to be the obstruction to extending $\widetilde{v}$ as a nowhere zero section of $\widetilde{T}$ over $\nu^{-1}(X \cap \mathbb{B}_\varepsilon)$.*

More precisely, let $\mathcal{O}(\widetilde{v}) \in H^{2d}\big(\nu^{-1}(X \cap \mathbb{B}_\varepsilon), \nu^{-1}(X \cap \mathbb{S}_\varepsilon)\big)$ be the obstruction cocycle to extending $\widetilde{v}$ as a nowhere zero section of $\widetilde{T}$ inside $\nu^{-1}(X \cap \mathbb{B}_\varepsilon)$. The local Euler obstruction $\mathrm{Eu}_V(0)$ is defined as the evaluation of the cocycle $\mathcal{O}(\widetilde{v})$ on the fundamental class of the pair $\big(\nu^{-1}(X \cap \mathbb{B}_\varepsilon), \nu^{-1}(X \cap \mathbb{S}_\varepsilon)\big)$. The Euler obstruction is an integer.

The Euler obstruction, $\mathrm{Eu}_X(x)$, is a constructible function on X, in fact it is constant along the strata of a Whitney stratification.

Definition 2.4. *Let $\nu : \widetilde{X} \to X$ be the proper analytic Nash modification of X, here $\widetilde{X} \subset X \times G$, where G is the Grassmannian manifold of d planes in $\mathbb{C}^{N+1}$. Let $\mathfrak{D} = \{D_d \subset D_{d-1} \subset \ldots \subset D_0 \subset \mathbb{C}^{N+1}\}$ be one flag, we associate $\mathfrak{D}$ with the Schubert variety $c_k(\mathfrak{D}) = \{E \in G / \dim E \cap D_{d-k} \geq k\}$.*

Definition 2.5. *The morphism $\gamma : \widetilde{X} \to G$ induced by the second projection $X \times G \to G$ allow us to define the absolute polar varieties $P_k(\mathfrak{D}) = \overline{\nu(\gamma^{-1}(c_k(\mathfrak{D})))}$. If $\mathfrak{D}$ is generic we just denote the polar variety by $P_k(V, y)$.*

With the aid of Gonzales-Sprinberg's purely algebraic interpretation of the local Euler obstruction [5], Lê and Teissier in [11] showed that the local Euler obstruction is an alternate sum of the multiplicities of the local polar varieties.

Theorem 2.6. [11] *Let $X \subset \mathbb{C}^N$ be an analytic space of dimension d reduced at 0. Then*

$$Eu_X(0) = \Sigma_{i=0}^{d-1} (-1)^{d-i-1} m_i(X, 0),$$

where $Eu_X(0)$ denotes the Euler obstruction of X at 0 and $m_i(X, 0)$ is the polar multiplicity of the polar varieties $P_i(X, 0)$.

2.2. **Ruled surfaces.** Let us now take two complex curves $\alpha : D \to \mathbb{C}^N$ and $\beta : D \to \mathbb{C}^N$, where $D \subset \mathbb{C}$ is a disc centered at the origin, we can consider both curves together as a map $(\alpha, \beta) : D \to \mathbb{C}^N \times \mathbb{C}^N$. We say that $\alpha(t) = (\alpha_1(t), \dots, \alpha_N(t))$ and $\beta(t) = (\beta_1(t), \dots, \beta_N(t))$ is a *pair of primitive parametrization* for (α, β) if it cannot be reparametrized by a power of a new variable. In what follows we consider only primitive parametrizations.

Notice that in a pair of primitive parametrization for (α, β) we may have that each parametrized curve is not primitive. For instance, the space complex curves $\alpha(t) = (t^2, t^4, t^6)$ and $\beta(t) = (t^3, t^6, t^9)$ are not primitive, but $(\alpha, \beta)(t) = \left((t^2, t^4, t^6), (t^3, t^6, t^9)\right)$ is a pair of primitive parametrization.

A ruled surface in $\mathbb{C}^3$ is locally the image of a map $f : D \times \mathbb{C} \to \mathbb{C}^3$ given by

$$f(t, u) = \alpha(t) + u\beta(t),$$

where α and β are space complex curves with $\beta \not\equiv 0$. We call $\alpha : D \to \mathbb{C}^3$ the *base curve* and $\beta : D \to \mathbb{C}^3$ is the *director curve.* Moreover, the straight lines $u \mapsto \alpha(t) + u\beta(t)$ are called rulings of the ruled surface.

If $(t_0, u_0) \in D \times \mathbb{C}$ and $f(t_0, u_0) = p$, then we will call a germ of ruled surface the image of the map germ $f : (D \times \mathbb{C}, (t_0, u_0)) \to (\mathbb{C}^3, p)$.

Considering that all map germs are C^∞, by Lemma 2.5 of [8], given a germ of ruled surface we can choose affine coordinates in $\mathbb{C}^3$ such that it is parametrized by a map germ $f : (\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$ of the form

$$f(t, u) = (0, \alpha_1(t), \alpha_2(t)) + u(1, \beta_1(t), \beta_2(t)). \tag{1}$$

Given a pair of primitive parametrization (α, β) of plane complex curve we will denote by $f_{(\alpha,\beta)}$ the ruled surface associated to these curves as in (1) and from now on we only consider ruled surfaces given in this way.

For example, considering $\alpha(t) = (t^2, 0)$ and $\beta(t) = (0, t)$, the pair (α, β) define the ruled surface $f_{(\alpha,\beta)}(t, u) = (u, t^2, ut)$ that is a well know Cross Cap parametrization.

Definition 2.7. *Given a pair* $(\gamma^{(1)}, \gamma^{(2)}) : D \to \mathbb{C}^2 \times \mathbb{C}^2$, *the* (pair) multiplicity *at the origin* $0 \in D \subset \mathbb{C}$ *is* $(m(\gamma^{(1)}), m(\gamma^{(2)}))$ *with* $m(\gamma^{(j)}) = min\{ord_t \gamma_i^{(j)}(t); i = 1, 2\}$.

3. Euler Obstruction for Ruled Surfaces

First we remark that, as a image of a finite application, all ruled surfaces are analytic sets, therefor we can compute its Euler obstruction at a point.

Fixing two non-negative integers $n_0 \geq n_1 \geq 0$ we can exhibited a family of ruled surface germs $(X, 0)$ in the such way that the polar multiplicities of its is precisely $m_0(X, 0) = n_0$ and $m_1(X, 0) = n_0 - n_1$ and as a consequence of Lê-Teissier formula we have that Euler Obstruction of $(X, 0)$ is $Eu_X(0) = n_1$.

With notations introduced in the previous section, it is clear that $P_0(X, D_2) = V$ then $m_0(X, 0)$ is exactly the multiplicity of X at the origin and we can compute by a intersection with a generic complex line L through the origin.

Proposition 3.1. *Let us take a pair of primitive parametrization (α, β) of plane complex curves and pair multiplicity (n_0, n_1). If X is the ruled surface given by $f_{(\alpha,\beta)}$ as in (1) then $m_0(X, 0) = n_0$.*

Proof. Considering L given by $\{w(a, b, c);\ w \in \mathbb{C}\}$ where $(a : b : c) \in \mathbb{P}^2_{\mathbb{C}}$ is generic we have that $L \cap X$ is described as following:

$$\begin{aligned} aw &= u \\ bw &= \alpha_1(t) + u\beta_1(t) \\ cw &= \alpha_2(t) + u\beta_2(t). \end{aligned}$$

It is easy verify that $L \cap X = \left\{ \frac{c - a\beta_2(t)}{b - a\beta_1(t)} \alpha_2(t) = \alpha_1(t); t \in \mathbb{C} \right\}$ where $c - a\beta_1(t), b - a\beta_1(t) \in \mathbb{C}\{t\}$ are unit because $(a : b : c)$ is generic.

Hence, $m_0(X, 0) = min\{ord_t \alpha_i(t); i = 1, 2\} = n_0$. □

In order to present germs ruled surfaces $(X, 0)$ with polar multiplicity and Euler obstruction prescribed, we consider that $(X, 0)$ is given by $f_{(\alpha,\beta)}$ where (α, β) pair of primitive parametrization with pair multiplicity (n_0, n_1) and $n_0 \geq n_1 \geq 0$.

Proposition 3.2. *Given a pair of primitive parametrization (α, β) of plane complex curves with pair multiplicity (n_0, n_1) and $n_0 \geq n_1 \geq 0$, then the germ (X, p) of ruled surface given by $f_{(\alpha,\beta)}$ is such that $m_1(X, p) = n_0 - n_1$.*

Proof. We have that the polar multiplicity $m_1(X, p)$ is the multiplicity of the the variety $P_1(X, L)$ defined by

$$\overline{\{q \in X; dim_{\mathbb{C}} T_p X \cap L \geq 1\}} \tag{2}$$

where L is a generic line through the $p = f_{(\alpha,\beta)}(t, u) = (x, y, z)$ which can be consider given by $\{(x, y, z) + w(a, b, c);\ w \in \mathbb{C}\}$ where $(a : b : c) \in \mathbb{P}^2_{\mathbb{C}}$ is generic.

The condition (2) is equivalent to

$$v \in span(v_1, v_2) = T_p X \text{ where } v = (a, b, c).$$

Without loss of generality we can consider $v_1 = (0, \alpha_1'(t) + u\beta_1'(t))$ and $v_2 = (1, \beta_1(t), \beta_1(t))$ where $\alpha_i'(t)$ and $\beta_i'(t)$ denote the derivative of $\alpha_i(t)$ and $\beta_i(t)$ with respect to t.

In this way we have that $v.(v_1 \wedge v_2) = 0$ that is

$$\begin{aligned} u\left(a(\beta_2(t)\beta_1'(t) + \beta_2'(t)\beta_1(t)) + b\beta_2'(t) - c\beta_1'(t)\right) = \\ (a(\beta_1(t)\alpha_2'(t) - \beta_2(t)\alpha_1'(t)) - b\alpha_2'(t) + c\alpha_1'(t)). \end{aligned}$$

Using the generality of $(a : b : c) \in \mathbb{P}^2$ and that $min\{ord_t \beta_i(t); i = 1, 2\} = n_1 \leq n_0 = min\{ord_t \alpha_i(t); i = 1, 2\}$ we have that

$$u = \frac{P(t)}{Q(t)} \in \mathbb{C}\{t\}. \tag{3}$$

where

$$P(t) = a(\beta_2(t)\beta_1'(t) + \beta_2'(t)\beta_1(t)) + b\beta_2'(t) - c\beta_1'(t)$$

and

$$Q(t) = a(\beta_1(t)\alpha_2'(t) - \beta_2(t)\alpha_1'(t)) - b\alpha_2'(t) + c\alpha_1'(t).$$

Hence $P_1(X, L)$ is the curve $\Psi(t)$ in (X, p) such that $f_{(\alpha,\beta)}(t, u) = 0$ with u as (3), that is,

$$\Psi(t) = \frac{1}{Q(t)}(P(t), Q(t)\alpha_1(t) + P(t)\beta_1(t), Q(t)\alpha_2(t) + P(t)\beta_2(t)).$$

Analyzing the multiplicity of each component of Ψ and taking in count the generality of $(a : b : c)$, we have that

$$ord_t(P(t)) < min\{ord_t(P(t)\alpha_1(t)+P(t)\beta_1(t)), ord_t(Q(t)\alpha_2(t)+P(t)\beta_2(t))\}.$$

But

$$ord_t(P(t)) = min\{ord_t\alpha_1'(t), ord_t\alpha_2'(t)\} = n_0 - 1$$

and

$$ord_t(Q(t)) = min\{ord_t\beta_1'(t), ord_t\beta_2'(t)\} = n_1 - 1.$$

In this way, the multiplicity of $P_1(X, L)$ is $ord_t(P(t)) - ord_t(Q(t)) = n_0 - n_1$, hence $m_1(X, p) = n_0 - n_1$. □

Now the next theorem is immediate from the above propositions. Notice that this result show us that we can find examples of germs of ruled surfaces for any given positive Euler obstruction.

Theorem 3.3. *If $(X, 0)$ is a germ of ruled surface given by $f_{(\alpha,\beta)}$ as in (1) where (α, β) is a pair of primitive parametrization of plane complex curve with pair multiplicity (n_0, n_1) and $n_0 \geq n_1 \geq 0$, then $Eu_X(0) = n_1$.*

Proof. It is following from Propositions 3.1, 3.2 and Lê-Tessier formula that

$$Eu_X(0) = m_0(X, 0) - m_1(X, 0) = n_1.$$

□

Corollary 3.4. *Let $(X, 0) \subset (\mathbb{C}^3, 0)$ be a germ of a ruled surface, if the Euler obstruction $Eu_X(0)$ is bigger than 1 then $(X, 0)$ have non-isolated singularity at the origin.*

Proof. The polar multiplicities are related with the Milnor number of the hypersurfaces with isolated singularities by the equation $m_i(X) = \mu_i(X) + \mu_{i+1}(X)$ where μ_i is the Milnor number of the intersection of the hypersurface with a hyperplane of dimension i [10]. Therefore we have in this case

$$Eu_X(0) = m_0(X) - \mu_1(X) - \mu_2(X).$$

It is well known that $m_0(X, 0) - 1 = \mu_1(X)$, therefore we have $Eu_X(0) = 1 - \mu_2(X)$, since the Milnor number is an integer bigger or equal to zero, we can conclude by the last result that all germs of ruled surfaces with Euler obstruction bigger than 1 have non-isolated singularities. □

Acknowledgement
The first author and the second one were partially supported by CNPq. The second author and the third one were partially supported by CAPES/PROCAD.

References

[1] BRASSELET, J.P.: Local Euler obstruction, old and new, *XI Brazilian Topology Meeting (Rio Claro, 1998)*, 140-147, World Sci. Publishing, River Edge, NJ, 2000.

[2] BRASSELET, J.P. and GRULHA JR., N. G.: Local Euler obstruction, old and new II, *London Mathematical Society - Lectures Notes Series 380 - Real and Complex Singularities*, Cambridge University Press, 2010, 23-45.

[3] BRASSELET, J.P. and SCHWARTZ, M.H.: Sur les classes de Chern d'un ensemble analytique complexe, *Astérisque* **82-83** (1981) 93-147.

[4] BRASSELET, J.P., MASSEY, D. and PARAMESWARAN, A. and SEADE, J.: Euler obstruction and defects of functions on singular varieties, *Journal London Math. Soc (2)* **70** (2004), no.1, 59-76.

[5] GONZALEZ-SPRINBERG, G *L'obstruction locale d'Euler et le Théorème de MacPherson*, Astérisque 82-83 (1981), 7–32.

[6] GRULHA JR., N. G.: The Euler Obstruction and Bruce-Roberts' Milnor Number, *Quart. J. Math.* 60(3) (2009) 291–302.

[7] MAC-PHERSON, R. D.: Chern classes for singular algebraic varieties, *Ann. of Math.* **100** (1974), 423-432.

[8] NUNO-BALLESTEROS, J.J. and MARTINS, R.: *Finitely determined singularities of ruled surfaces in* $\mathbb{R}^3$, *Proceedings of the Cambridge Philosophical Society*, 147, 701-733, 2009.

[9] SEADE, J., TIBAR, M. and VERJOVSKY A.: Milnor Numbers and Euler obstruction, *Bull. Braz. Math. Soc. (N.S)* **36** (2005), no. 2, 275-283.

[10] TEISSIER, B.: *Variétés polaires. II. Multiplicités polaires, sections planes et conditions de Whitney*, Lect. Notes in Math. **961** Springer, Berlin, 1982.

[11] LÊ D. T. and TEISSIER, B.: Variétés polaires Locales et classes de Chern des variétés singulières, *Ann. of Math.* **114**, (1981), 457-491.

Grulha Jr., N. G.
njunior@icmc.usp.br
ICMC-USP
Av. Trabalhador São-Carlense 400
São Carlos-SP 13566-590
Brazil

Hernandes, M. E.
mehernandes@uem.br
DMA-UEM
Av. Colombo 5790
Maringá-PR 87020-900
Brazil

Martins, R.
rmartins@uem.br
DMA-UEM
Av. Colombo 5790
Maringá-PR 87020-900
Brazil

SÉRIE MATEMÁTICA

NOTAS DO ICMC

356/11 LIMA, P. H. A. A. – Castelnuovo-Mumford Regularity of the Fiber Cone for Good Filtrations.

355/11 OLIVEIRA, R.D.S. ; ZHAO, Y. – On the Structural Stability of Quasihomogeneous Polynomial Vector Fields.

354/11 TIBAR, M. ; YING, C. ; SANTOS, R. N. A. – Singular Open Boo

353/11 GAFFNEY, T. ; GRULHA JR, N. G. – On the Multiplicity Polar Theorem, Collections of 1-Forms and Chern Numbers.

352/11 BAPTISTELLI, P.H. ; MANOEL, M. – Invariants and Relative Invariants under compact Lie Groups.

351/11 SILVA, P. L. D. ; SILVA, E. R. - Solvability near the characteristic set for a special class of complex vector fields.

350/11 BONOTTO, E. M. ; DEMUNER, D. P. - Autonomous dissipative semidynamical sustems with impulses.

348/11 ARAGÃO-COSTA, E. R. ; CARABALLO, T. ; CARVALHO, A. N. ; LANGA, J. A. – Stability of gradient semigroups under perturbation.

347/11 DUTERTRE, N. ; GRULHA JR, N. – Lê-Greuel type formula for the Euler obstruction and applications.

346/11 RUAS, M. ; FERNANDES, A. – Regidity of bi-lipschitz equivalence of weighted homogeneous function-germs in the plane.

345/11 MATTOS, D. de; SANTOS, E. L. dos; COELHO, F. R. de C.. – On the existence of G-equivariant maps.