UNIVERSIDADE DE SÃO PAULO

Instituto de Ciências Matemáticas e de Computação

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Resumo

Neste artigo, apresentamos uma análise Bayesiana para o modelo de volatilidade estocástica (SV) e, também, para uma forma generalizada do modelo SV cujo objetivo é estimar a volatilidade de séries temporais financeiras no caso univariado. Considerando alguns casos especiais dos modelos SV, usamos algoritmos de Monte Carlo em Cadeias de Markov (MCMC) e o software WinBugs para obter sumários a posteriori de interesse para as diferentes formas de modelos SV aplicados a séries financeiras. Também introduzimos algumas técnicas Bayesianas de discriminação de modelos para a escolha do melhor modelo a ser usado para estimar as volatilidades e fazer previsões de séries financeiras. Um exemplo empírico de aplicação da metodologia proposta é introduzido com a série financeira do IBOVESPA.

Abstrac

The purpose of this paper is twofold. First, we introduce a Bayesian analysis for volatility stochastic models (SV) and then a generalization of SV models to estimate the volatility of financial time series in the univariate case. Considering some special cases of the SV models, we use Markov Chain Monte Carlo algorithms (MCMC) and the software WinBugs to find the posterior summaries of interest in the different forms of the SV model applied to financial series. We also introduce some Bayesian model discrimination techniques in order to choose for the best model to estimate the volatilities and to get predictions of financial series. An example is considered to illustrate the proposed methodology considering a IBOVESPA financial series.

Stochastic Volatility Model for Financial Time Series: An Application with Brazilian Stock Market IBOVESPA

Milton Barossi
Departamento de Economia
Faculdade de Economia, Administração e Contabilidade
Universidade de São Paulo,
Av. dos Bandeirantes, 3900 Monte Alegre
14040-900, Ribeirão Preto, SP., Brasil
e-mail: *mbarossi@usp.br*

Jorge A. Achcar
Departamento de Medicina Social
Faculdade de Medicina de Ribeirão Preto
Universidade de São Paulo
Av. Bandeirantes, 3900
14048-900, Ribeirão Preto, SP. Brasil
e-mail: *achcar@fmrp.usp.br*

Marinho G. Andrade
Departamento de Matemática Aplicada e Estatística
Instituto de Ciências Matemáticas e de Computação,
Universidade de São Paulo,
C.P. 668, 13560-970, São Carlos, SP., Brasil
e-mail: *marinho@icmc.usp.br*

Abstract

The purpose of this paper is twofold. First, we introduce a Bayesian analysis for volatility stochastic models (SV) and then a generalization of SV models to estimate the volatility of financial time series in the univariate case. Considering some special cases of the SV models, we use Markov Chain Monte Carlo algorithms (MCMC) and the software *WinBugs* to find the posterior summaries of interest in the different forms of the SV model applied to financial series. We also introduce some Bayesian model discrimination techniques in order to choose for the best model to estimate the volatilities and to get predictions of financial series. An example is considered to illustrate the proposed methodology considering a IBOVESPA financial series.

Key words: keyword Stochastic Volatility model, Financial time series, MCMC methods, Bayesian model selection, Brazilian stock market IBOVESPA.

1 Introduction

The ARCH type models (autoregressive conditional heteroscedastic) introduced by Engle (1982) and the generalized autoregressive conditional heteroscedastic (GARCH) introduced by Bollerslev (1986) are been widely used to model the volatility of financial time series (see Taylor (1982), Tauchen and Pitts (1983), Bollerslev (1990), Bollerslev et al. (1992), Engle (2001)). The use of stochastic volatility models (SV) has been a good alternative to analyze financial time series in comparison with the usual type GARCH models (see Kim et al., 1998).

The SV model gives more flexibility in the modelling of financial time series, since they assume two processes for the noise: a first process for the observations and a second process for the latent volatilities.

Comparative studies between the classes of SV models and ARCH type models are widely studied in the literature (see for example, Taylor (1994), Ghysels et al. (1996), Shephard (1996)).

Bayesian methods using Markov Chain Monte Carlo (MCMC) methods has been considered to analyze financial time series assuming SV models (see for example, Meyer and Yu (2000)) since there are great difficulties to use standard classical inference methods given the complexity of the likelihood function.

In this paper we introduce a Bayesian analysis for the standard SV model and for a modified form of SV models to estimate the volatilities of univariate time series.

The paper is organized as follows: in Section 2, we introduce usual volatility models and also a generalized form of SV models. In Section3, we introduce a Bayesian analysis for SV models. In Section 4, we introduce a numerical illustration considering a time series selected from the Brazilian stock market BOVESPA index (IBOVESPA) (BOVESPA, 2007). Finally, in Section 5, we present a discussion of the obtained results.

2 Stochastic volatility Models

Considering a financial time series P_t, let us assume a transformed serie or log-return given by $y_t = \log(P_t/P_{t-1}) \approx (P_t - P_{t-1})/P_{t-1}$. In the presence of the heteroscedasticity, let us assume the following model for the time series:

$$y_t = \sigma_t \epsilon_t \tag{1}$$

where ϵ_t is a noise assumed to be independent and identically distributed with a normal distribution $N(0, \sigma_\epsilon^2)$. Let us denote by $H^t = \{y_{t-1}, y_{t-2}, \ldots\}$ the past values of y_t. The volatility of the series y_t is the conditional variance $\sigma_t^2 = E\{y_t^2|H^t\}$.Also, let us assume that the standard deviation is given by,

$$\sigma_t = \exp\{h_t/2\} \tag{2}$$

where h_t is a latent variable defined by a autoregressive model AR(1) given by,

$$h_t = \mu + \phi(h_{t-1} - \mu) + \eta_t \tag{3}$$

for $t = 2, \ldots, N$. Let us assume that h_1 is a random variable with a known distribution $P_1(h_1)$ and η_t is a noise associated with the latent variable with a normal distribution $N(0, \sigma_\eta^2)$. If $|\phi| < 1$, the unconditional mean and variance of h_t are given, respectively, by $E\{h_t\} = \mu$ and $V(h_t) = \sigma_\eta^2/(1 + \phi\rho_1)$, where ρ_1 is the autocorrelation coefficient between h_t and h_{t-1}.

With the assumptions (1),(2) e (3), we have,

$$y_t \backsim N(0, \sigma_\epsilon^2 \exp\{h_t\}) \tag{4}$$

$$h_1 \backsim N(\mu, \sigma_\eta^2) \tag{5}$$

$$h_t|h_{t-1} \backsim N(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2) \tag{6}$$

for $t = 2, 3, \ldots, N$.

A possible generalization for the SV model could be considered defining a model from the expression (1) and (2) with latent variable given by

$$h_t = \mu + \sum_{j=1}^{p} \phi_j (h_{t-j} - \mu) + \eta_t \tag{7}$$

for $t = p+1, ..., N$, with the roots of the polynomial $\phi(B) = 1 - \sum_{j=1}^{p} \phi_j B^j$ outside the unit ratio circle (B is a backward operation given by $B^k h_t = h_{t-k}$). Let us denote the model defined by (1)-(2) and (7) as a generalized stochastic volatility model (GSV).

In this case, we have,

$$h_t|h_{t-1}, \ldots, h_{t-p} \backsim N\left(\mu + \sum_{j=1}^{p} \phi_j(h_{t-j} - \mu), \sigma_\eta^2\right) \tag{8}$$

for $t = 2, 3, \ldots, N$.

The likelihood function for the GSV model, assuming equation (1)-(2) is given by,

$$L = \left\{\prod_{t=1}^{N} p(y_t|h_t)\right\} \tag{9}$$

From equation (4), we have,:

$$L = \left\{\prod_{t=1}^{N} \frac{1}{\sqrt{2\pi\sigma_\epsilon^2 \exp\{h_t\}}} \exp\left\{-\frac{y_t^2}{2\sigma_\epsilon^2 \exp\{h_t\}}\right\}\right\} \tag{10}$$

3 A Bayesian analysis for GSV model

For a Bayesian analysis of the stochastic volatility model defined by (1) and (2), with latent variable defined by (7), let us assume the following prior distributions for the parameters μ, $\Phi = (\phi_1, \ldots, \phi_p)^T$, σ_ϵ^2 and σ_η^2:

$$\begin{array}{lcl} \phi_j & \backsim & \text{Beta}(a_j, b_j),\ j = 1, \ldots, p \\ \sigma_\epsilon^2 & \backsim & \text{IG}(c_1, d_1) \\ \sigma_\eta^2 & \backsim & \text{IG}(c_2, d_2) \\ \mu & \backsim & \text{N}(0, e^2) \end{array} \tag{11}$$

where $\text{Beta}(a_j, b_j)$ denotes a beta distribution with mean $a_j/(a_j + b_j)$ and variance $a_j b_j/[(a_j + b_j)^2(a_j + b_j + 1)$; $\text{IG}(c, d)$ denotes a inverse gamma distribution with mean $d/(c-1)$ and variance $d^2/[(c-1)^2(c-2)]$, $c > 2$ and $N(\mu, \sigma^2)$ denotes a normal distribution with mean μ e variance σ^2. Let us assume that the hyperparameters (a_j, b_j), $j = 1, \ldots, p$, (c_i, d_i), $i = 1, 2$ and e^2 are known.

With $h_l = 0$ for $l = 0, -1, \ldots, -p+1$ in equation (8), the conditional density for h_t given $h_{t-1}^{(p)} = \{h_{t-1}, \ldots, h_{t-p}\}$, for $t = 1, \ldots, N$, is given by,

$$p(h_t|h_{t-1}^{(p)}) = \frac{1}{\sqrt{2\pi\sigma_\eta^2}} \exp\left\{-\frac{1}{2\sigma_\eta^2}\left[h_t - \mu - \sum_{j=1}^{p} \phi_j(h_{t-j} - \mu)\right]^2\right\} \tag{12}$$

Denoting by $\Phi = (\phi_1, \ldots, \phi_p)^T$, $\theta = (\mu, \Phi, \sigma^2_\epsilon, \sigma^2_\eta)$, $\mathbf{h} = (h_1, \ldots h_N)^T$ and assuming prior independence among the parameters μ, Φ, σ^2_ϵ and σ^2_η,the joint posterior density for $\varphi = (\theta, \mathbf{h})$ is given by:

$$\pi(\varphi|\mathbf{y}) = \prod_{t=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2_\epsilon \exp\{h_t\}}} \exp\left\{-\frac{y_t^2}{2\sigma^2_\epsilon \exp\{h_t\}}\right\} \times \tag{13}$$

$$\frac{1}{\sqrt{2\pi\sigma^2_\eta}} \exp\left\{-\frac{1}{2\sigma^2_\eta}\left[h_t - \mu - \sum_{j=1}^{p}\phi_j(h_{t-j} - \mu)\right]^2\right\}\pi(\theta)$$

where $\mathbf{y} = (y_1, \ldots y_N)^T$ and $\pi(\theta)$ is given by,

$$\pi(\theta) \propto \left\{\prod_{j=1}^{p} \phi_j^{a_j - 1}(1-\phi_j)^{b_j-1}\right\}(\sigma^2_\eta)^{-(c_1+1)/2}\exp\{-\frac{d_1}{\sigma^2_\eta}\}\times \tag{14}$$

$$(\sigma^2_\epsilon)^{-(c_2+1)/2}\exp\{-\frac{d_2}{\sigma^2_\epsilon}\}\exp\{-\frac{\mu^2}{2e^2}\}$$

We can write the posterior density as,

$$\pi(\varphi|\mathbf{y}) \propto \left(\sigma^2_\epsilon\right)^{-N/2} \exp\left\{-\frac{1}{2}\sum_{t=1}^{N} h_t - \frac{1}{2\sigma^2_\epsilon}\sum_{t=1}^{N} y_t^2 \exp\{-h_t\}\right\} \times$$

$$(\sigma^2_\eta)^{-N/2}\exp\left\{-\frac{1}{2\sigma^2_\eta}\sum_{t=1}^{N}\left(h_t - \mu - \sum_{j=1}^{p}\phi_j(h_{t-j} - \mu)\right)^2\right\}\pi(\theta)$$

Considering the vector notation $\Phi = (\phi_1, \ldots, \phi_p)^T$, $\mathbf{Y} = (y_1^2, \ldots y_N^2)^T$, $\mathbf{h} = (h_1, \ldots h_N)^T$, $\mathbf{E}(\mathbf{h}) = (e^{-h_1}, \ldots e^{-h_N})^T$, and the $\mathbf{X}$ matrix given by,

$$\mathbf{X} = \begin{pmatrix} h_0 & h_{-1} & \cdots & h_{-p+1} \\ h_1 & h_0 & \cdots & h_{-p+2} \\ \vdots & \vdots & \cdots & \vdots \\ h_p & h_{p-1} & \cdots & h_1 \\ h_{p+1} & h_p & \cdots & h_2 \\ \vdots & \vdots & \cdots & \vdots \\ h_{N-1} & h_{N-2} & \cdots & h_{N-p} \end{pmatrix}_{N\times p}$$

we can write the posterior density in the form,

$$\pi(\varphi|\mathbf{y}) \propto \left(\sigma^2_\epsilon\right)^{-N/2}\exp\left\{-\frac{1}{2}\mathbf{1}^T\mathbf{h} - \frac{1}{2\sigma^2_\epsilon}\mathbf{Y}^T\mathbf{E}(\mathbf{h})\right\}\times \tag{15}$$

$$(\sigma^2_\eta)^{-N/2}\exp\left\{-\frac{1}{2\sigma^2_\eta}(\mathbf{h} - \mu - \mathbf{X}\Phi)^T(\mathbf{h} - \mu - \mathbf{X}\Phi)\right\}\pi(\theta)$$

where $\mathbf{1} = (1, \ldots, 1)^T$ is the $N \times 1$ vector of 1's and μ is a $N \times 1$ vector of μ's.

To generate samples of the joint posterior distribution for $\varphi = (\theta, \mathbf{h})$, we use MCMC methods with the Gibbs sampling algorithm (see for example Gelfan and Smith, 1990) or the Metropolis-Hastings algorithm (see for example Smith and Roberts, 1993). These samples are generated from the conditional distributions $\pi(\theta_j|\theta_{(j)}, \mathbf{y})$, where $\theta_{(j)}$ denotes the vector of all components except the the jth component.

A great simplification is obtained using the software *WinBugs* (Spiegelhalter et al., 1999), where we only need to specify the distribution for the data and the prior distribution for the parameters of the model.

Similar results are obtained considering special cases of the GSV model with $p = 1$, $\phi_j = 0$ for $j = 2, \ldots, p$ (usual SV) and $\sigma_\epsilon^2 = 1$ or $\sigma_\eta^2 = 1$.

4 Model selection methods

Many model selection methods to choose the best stochastic model to be fitted by financial time series can be adopted under the Bayesian approach (see for example Berg et al. (2004)); among these methods we consider the following:

4.1 Bayesian information Criterion (BIC)

The Bayesian Information Criterion (BIC) is a model selection criterion introduced by Schwarz (1978) and modified by Carlin and Louis (2000) (see also Raftery et al., 2006) to be used considering the posterior density for the parameter of the fitted models. This criterion weights between the maximized log-likelihood function and the member of parameters of the model. The best model is the one that gives a larger value of BIC, given by,

$$BIC = E\{\ln L(\theta)\} - \frac{1}{2} p \ln(N) \tag{16}$$

where $E\{\ln L(\theta)\}$ is the expected value of the log-likelihood function based on the posterior density for θ; p is the dimension of the parameter vector and N is the sample size.

4.2 Deviance Information Criterion (DIC)

The Deviance Information Criterion (DIC) is a generalization for the BIC. This criterion is specially useful for selection models under Bayesian approach where samples of the posterior distribution for the parameters of the models where obtained using Monte Carlo Markov Chain (MCMC) methods. Similarly to BIC criterion, this criterion is a asymptotical approximation for large sample size and is useful when the posterior distribution is well approximated by a multivariate normal distribution.

The deviance is defined by:

$$D(\theta) = -2 \log L(\theta) + C$$

where θ is a vector of unknown parameters of the model; $L(\theta)$ is the likelihood function and C is a constant not needed to be known in the comparison of two models. The DIC criterion introduced by Spiegelhalter et al. (2002), is given by

$$DIC = D(\widehat{\theta}) + 2p_D \tag{17}$$

where $D(\widehat{\theta})$ is the deviance evaluated at the posterior mean and p_D is the effective number of parameters of the model, given by $p_D = \overline{D} - D(\widehat{\theta})$, where $\overline{D} = E\{D(\theta)\}$ is the posterior mean deviance measuring the quality of the data fit for the model. Smaller values of DIC indicate better models; these values also could be negatives.

The DIC is used to discriminate different models assumed to analyze financial time series.

4.3 Bayes Factor (B_{ij})

This criterion compares two models M_1 and M_2 using marginal densities or marginal likelihood functions. The model M_1 is better than the model M_2 if $P(D|M_2) < P(D|M_1)$. The marginal likelihood function is given by:

$$P(D|M_l) = \int_{\theta_l} L(D|\theta_l, M_l)\pi_0(\theta_l|M_l)d\theta_l \tag{18}$$

where $L(D|\theta_l, M_l)$, $l = 1, 2$ is the likelihood of the data under the hypothesis of model M_l and $\pi_0(\theta_l|M_l)$ is a prior density for the vector of parameters θ_l. The Bayes factor is given by the ratio,

$$B_{ij} = \frac{P(D|M_i)}{P(D|M_j)} \tag{19}$$

A modification of the Bayes factor introduced by Aitkin (1991) replaces the prior density $\pi_0(\theta_l|M_l)$ in (18) by the posterior density for θ_l. In this way, we can obtain a Monte Carlo estimate of the integral (18) from the generated samples of the posterior distribution for θ_l using MCMC methods. The marginal likelihood can be estimated by its Monte Carlo estimator,

$$\widehat{P}(D|M_l) = \frac{1}{M}\sum_{j=1}^{M} L(\theta_l^{(j)}|M_l) \quad \text{for } l = 1, 2$$

Another discrimination criterion was introduced by Raftery et al. (1995), (see also Raftery et al., 2006) given by the harmonic mean of the likelihood function

$$\widehat{f}(D|M_l) = \left\{ \frac{1}{M}\sum_{j=1}^{M} \frac{1}{L(D|\theta_l^{(j)}, M_l)} \right\}^{-1} \quad \text{for } l = 1, 2$$

The model selection methods introduced in this section are used to select the best model to be fitted by the log-returns of a financial time series considered in the next section.

5 A Bayesian Analysis of a Brazilian Stock Market IBOVESPA Series

Let us consider the log-returns $y_t = \ln(I_{t+1}/I_t)$, $t = 1, \ldots, 679$ where I_t are weekly BOVESPA indexes corresponding to the period from July 04, 1994 to July, 09 2007, and centralized around their averages. The plots of this series is given by in Figure 1.

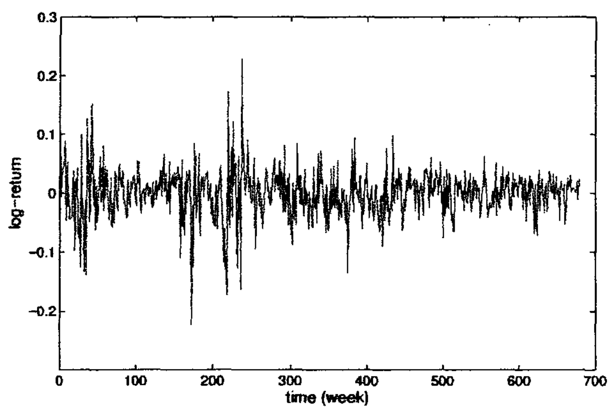

Figure 1. IBOVESPA centralized log-returns series

To analyze the data set, we first assume the stochastic volatility model (SV) defined by (1) to (6) considering $\sigma_\epsilon^2 = 1$. Let us assume the prior distribution (see 11), $\phi \backsim Beta(1,1)$; $\tau_\eta = 1/\sigma_\eta^2 \backsim gamma(1,1)$ and $\mu \backsim N(0,100)$, where $gamma(a,b)$ denotes a gamma distribution with mean a/b and variance a/b^2. The choice of a $gamma(1,1)$ distribution for τ_η corresponds to a inverse-gamma distribution for σ_η^2. Let us denote this model as "Model $1A$".

Observe that the normal $N(0,100)$ distribution from μ has a large variance, that is, we have an approximately non-informative prior for μ.

The posterior summaries for the parameters of the model (see Table 1) were obtained from the simulation of 1000 Gibbs Samples from the joint posterior distribution after a "burn-in samples", of size 5000 to eliminate the effect of the initial values for the parameters and selecting every $10th$ to get approximately uncorrelated samples. To simulate the Gibbs samples we used the software *WinBugs* Spiegelhalter et al. (1999).

Posterior summaries for the parameters of the SV model assuming unknown σ_ϵ^2 ("Model $1B$") also are given in Table 1. These posterior summaries were obtained in a similar way to the results obtained for the Model $1A$, assuming the same prior distribution for ϕ and μ; and a $gamma(8,1)$ prior for $\tau_\eta = 1/\sigma_\eta^2$ and a $gamma(100,100)$ prior distribution for $\tau_\epsilon = 1/(\sigma_\epsilon^2)$.

Similarly, we present in Table 1, the posterior summaries for the parameters of the generalized stochastic volatility (GSV) model defined by (1), (2) and (7) with $p = 2$, $\sigma_\epsilon^2 = 1$(Model $2A$) and also assuming σ_ϵ^2 unknown ("Model $2B$").

For "Model $2A$", we assume the priors $\phi_1 \backsim Beta(1,1)$, $\phi_2 \backsim Beta(1,5)$, $\tau_\eta = 1/(\sigma_\eta^2) \backsim gamma(1,1)$ and $\mu \backsim N(0,10)$.

For the "Model $2B$", we assume the prior distributions $\tau_\epsilon = 1/(\sigma_\epsilon^2) \backsim gamma(6,1)$, $\phi_1 \backsim Beta(5,10)$, $\phi_2 \backsim Beta(5,10)$, $\tau_\eta = 1/(\sigma_\eta^2) \backsim gamma(100, 100)$ and $\mu \backsim N(0,100)$.

To compare the SV model with usual existing type *ARCH* models, we assume model (1) with normal $N(0,1)$ independent identically distributed errors ϵ_t and σ_t^2 modeled by a $ARCH(2)$, given by:

$$y_t = \sigma_t \epsilon_t; \quad \text{with} \quad \epsilon_t \overset{i.i.d}{\backsim} N(0,1) \tag{20}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 y_{t-1}^2 + \alpha_2 y_{t-2}^2 \tag{21}$$

with $\alpha_0 > 0$, $\alpha_j \geq 0$, $j = 1,2$, where we should have $\alpha_1 + \alpha_2 < 1$. For this model we assume prior distributions $\alpha_0 \backsim gamma(1,1)$ and $\alpha_j \backsim gamma(0.1, 0.1)$, $j = 1,2$. Posterior summaries for α_0 and α_j, $j = 1,2$ were obtained from the simulation of 1000 Gibbs Samples, of the joint posterior distribution for α_0 and α_j, $j = 1,2$ after a "burn-in-sample" of size 5000 and selecting every $10th$ sample. These posterior summaries are given in Table 1.

Table 1. Posterior summaries for the assumed models

Model	θ	mean	s.d.	95%C.I.
$1A$	μ	-6.7700	0.1950	$(-7.139; -6.370)$
$(\sigma_\epsilon^2 = 1)$	ϕ	0.9218	0.0241	$(0.8689; 0.9642)$
	σ_η^2	8.4590	1.9970	$(5.214; 13.100)$
$1B$	σ_ϵ^2	0.9883	0.0789	$(0.838; 1.146)$
(σ_ϵ^2 unknown)	μ	-6.7060	0.2472	$(-7.140; -6.139)$
	ϕ	0.9447	0.0205	$(0.899; 0.981)$
	σ_η^2	12.1100	2.9260	$(7.172; 18.590)$
$2A$	μ	-6.7140	0.2436	$(-7.125; -6.162)$
$(\sigma_\epsilon^2 = 1)$	ϕ_1	0.5220	0.1568	$(0.252; 0.877)$
	ϕ_2	0.3686	0.1569	$(0.0582; 0.6866)$
	τ_η	6.2400	1.6410	$(3.859; 10.270)$
$2B$	σ_ϵ^2	1.0270	0.0977	$(0.8524; 1.2320)$
(σ_ϵ^2 unknown)	μ	-6.7260	0.2902	$(-7.219; -6.074)$
	ϕ_1	0.4780	0.0782	$(0.324; 0.647)$
	ϕ_2	0.4624	0.0792	$(0.305; 0.617)$
	σ_η^2	8,.4010	2.2190	$(5.064; 13.263)$
ARCH	α_0	0.00105	0.000089	$(0.00089; 0.00124)$
$(\sigma_\epsilon^2 = 1)$	α_1	0.2844	0.0663	$(0.169; 0.428)$
	α_2	0.0830	0.0449	$(0.00718; 0.189)$

C.I.: Credible interval

In Table 2 we have posterior Monte Carlo estimates for the model selection measures introduced in section 4 considering the 5 models fitted for the data set.

By observing the Monte Carlo estimates for DIC assuming model $1A$ ($DIC = -2620.34$) and the model $2A$ ($DIC = -2620.94$), and the model $1B$ ($DIC = -2626.01$) and the model $2B$ ($DIC = -2623.79$), we conclude that the model $1B$ is better fitted by the data when compared to models $1A$ and $2A$. Also observe that the model $1B$ (SV with σ_ϵ^2 unknown) is the best model when compared to model $2B$.

The Monte Carlo estimates for DIC assuming model $ARCH(2)$ is given by $DIC = -2495.98$. Therefore, we conclude that SV models are better fitted by the financial time series, that is, we have better volatility estimates considering SV models (smaller values for DIC).

From the BIC criterion (see Table 2), assuming Carlin and Louis (2000) modification, and using Monte Carlo estimates based on the generated Gibbs samples, we have the same conclusion as considering the DIC criterion. That is, "Model $1B$" is preferable to "Model $1A$" and "Model $2B$" is preferable to "Model $2A$". Both criterion shows that SV models are better fitted by the data when compared to ARCH-type models.

The same conclusion is obtained from the posterior Bayes Factor (see Table 3).

Table 2. Selection Criterion for the models

Models	$E\{\ln L(\theta)\}$	DIC	BIC	$\widehat{f}(D\|M_l)$
$1A$	1299.2	−2620.3	1296.0	1298.6
$1B$	1323.3	−2626.0	1320.1	1323.0
$2A$	1290.5	−2620.9	1284.0	1289.6
$2B$	1304.5	−2623.8	1298.0	1303.9
$ARCH$	1249.6	−2496.0	1243.1	1249.6

Table 3. Posterior Bayes Factor for models M_i and M_j

Models	$B_{ij} = P(D\|M_i)/P(D\|M_j),\ i \neq j$			$ARCH(j{=}5)$
$1A(i{=}1)$	$B_{12} = e^{-20.1}$	$B_{13} = e^{8.7}$	$B_{14} = e^{-5.3}$	$B_{15} = e^{49.6}$
$1B(i{=}2)$	$B_{21} = e^{24.1}$	$B_{23} = e^{32.8}$	$B_{24} = e^{18.8}$	$B_{25} = e^{73.7}$
$2A(i{=}3)$	$B_{31} = e^{-8.7}$	$B_{32} = e^{-32.8}$	$B_{34} = e^{-14.0}$	$B_{35} = e^{40.9}$
$2B(i{=}4)$	$B_{41} = e^{5.3}$	$B_{42} = e^{-18.8}$	$B_{43} = e^{14.0}$	$B_{45} = e^{54.9}$

In Figure 2 we have Monte Carlo estimates for the volatilities of the financial time series assuming the 4 models and in Figure 3 we have Monte Carlo estimates for the volatilities assuming the $ARCH$ models given by (20)-(21).

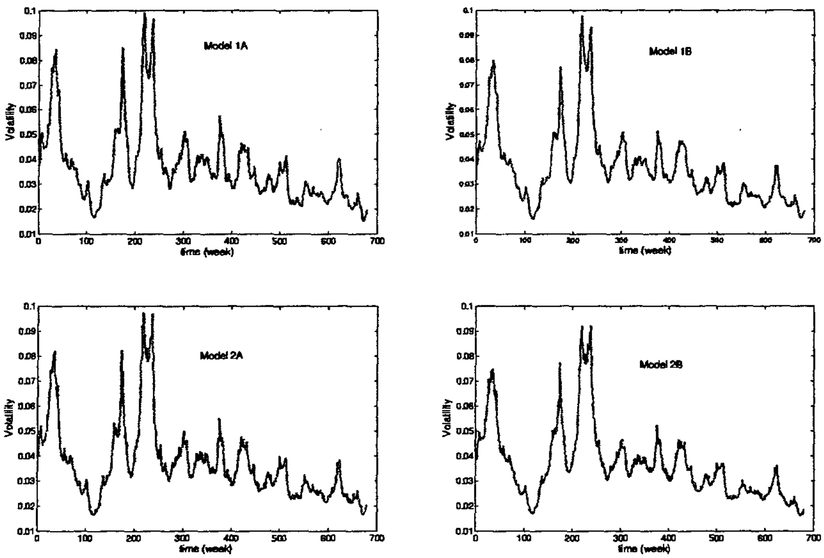

Figure 2. Monte Carlo estimates for the volatilities with assumed models $1A$, $2A$ ($\sigma_\eta^2 = 1$) e $1B$, $2B$ (σ_η^2 unknown)

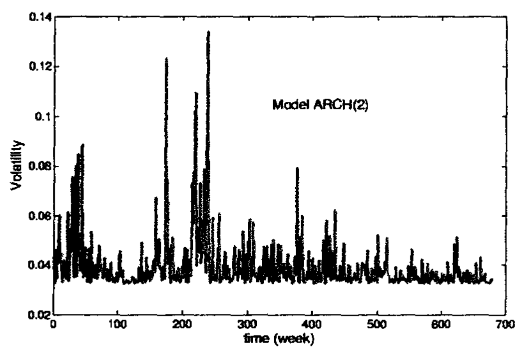

Figure 3. Monte Carlo estimates for the volatilities assuming ARCH(2) model

6 Concluding Remarks

From the obtained results of this paper, we observe that SV class of volatility model gives better inferential results as compared to the usual existing ARCH type models in the analysis of financial time series as it was observed in the analysis of the Brazilian BOVESPA index.

The superiority of SV models can be observed from the obtained statistical results and from the use of standard discrimination procedures to chose the best model to be fitted by financial data. In this way, we observed that model 1B (a generalized form of SV model assuming σ_ϵ^2 unknown) introduced in this paper, gives better for the IBOVESPA data set.

We also observe from the plots of figure 2, that the Monte Carlo estimates based on the generated Gibbs sampler for the volatilities assuming the different SV models considered in the analysis of the IBOVESPA give similar plots for weekly volatilities. These obtained estimates for the volatilities are in past agreement with standard economy theory and also present some surprising results that are peculiar to the Brazilian economy.

Some of these peculiar results are observed in some special years with international and Brazilian economy crisis, as it was observed in the years of 1997, 1998 and 1999. Different results also it was observed in the presidential election year of 2002, where the candidate of the left wing political Workers Party was promising radical transformations of the Brazilian economy. Later on, this fact was not observed (after the year of 2002).

These different standards for the volatilities of the IBOVESPA was well capture by the SV models (see figure 2), where we observe different volatility ranges in these special years. This fact was not captured using standard ARCH type models (see figure 3).

Another important point captured by the Bayesian analysis of the financial time series (IBOVESPA), was the presence of a seasonal trend for the Brazilian stock market IBOVESPA, where we observe an approximately cycle of 100 weeks to have a period of large ranges for the volatility of the IBOVESPA.

In this way, we see that using standard or generalized from of SV models, we can obtain accurate analysis for financial time series.

It is also important to point out that the Bayesian analysis of SV models can be obtained in a simple way using the software *WinBugs* (Spiegelhalter et al., 1999), and considering different prior distributions for the parameters of the model, since in many applications we have prior information of financial experts working in the stock market.

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