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Bayesian analysis of paired data in mixture of Normal distributions

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SUMMARY. Considering a pretest/posttest repeated measurement data we discuss the use of a mixture of two Normal distributions in the random effects model using Bayesian methods. The analysis of such a model is illustrated with a real data set.

KEY WORDS: Mixture of Normal distribution; Gibbs sampler; Bayesian inference; pretest/posttest data.

1 Introduction

The estimation of the parameters which results from the mixing of two Normal densities has been studied for a long time, dating back to Pearson (1894) and Charlier (1906). Since then, many works concerning the use of methods of moments estimation and maximum likelihood estimation has been developed. Recently, the problem of estimation of the parameters of finite mixture distributions using Bayesian methodology has been studied by many authors (see for example, Richardson and Green (1997), Diebolt and Robert (1994)). In this paper, we discuss the application of the mixture of two Normal distributions to a real data set from pretest/posttest experiment, where 26 preschoolers were evaluated with respect to a dental plaque index before and after toothbrushing either with a regular or with an experimental toothbrush. The aim of this study was to verify the eficacy of the hugger toothbrush with respect to the conventional one. As there are children with predispositions to form more dental plaque index than others we might take it into account.

In Section 2 we model the pretest/posttest data using mixture of two Normal distributions considering Bayesian methodology. In Section 3 we describe the computational aspects and finally in Section 4 we apply the model to the data presented in Singer and Andrade (1997).

2 The model

Let

$$r_{ij} = x_{ij} - y_{ij}, \tag{1}$$

with y_{ij} and x_{ij}, $i = 1, 2, j = 1, \ldots, n$ denoting respectively the observed values of the response (dental plaque index after the use of ith toothbrush by the jth individual) and explanatory (dental plaque index before the use of ith toothbrush by the jth individual) variables. In this context, r_{ij} represents the reduction of dental plaque index with the use of the ith toothbrush. The main goal of the study was to compare the efectiveness of the two toothbrushes under the study, which means to verify if the reduction of the plaque index is the same with the use of the hugger and conventional toothbrushes.

We define the reduction of dental plaque index as

$$r_{ij} = a_{ij} + e_{ij}, \tag{2}$$

with the error term e_{ij} independently distributed as $N(0, \sigma_{ei}^2)$, for $i = 1, 2, j = 1, \ldots, n$, and we suppose that

$$a_{ij} = \mu_{ti} + c_j, \tag{3}$$

with μ_{ti} denoting the effect of the ith toothbrush and c_j denoting the effect of the jth subject. To incorporate the possible predisposition of a child to form more plaque index than an other child, we assume c_j with a mixture of two normal distribution, that is,

$$c_j \sim p_1 N(\mu_1, \sigma_1^2) + p_2 N(\mu_2, \sigma_2^2), \tag{4}$$

with $p_1 + p_2 = 1$, c_j independently distributed for $j = 1, \ldots, n$ and c_j independent of e_{ij} for $i = 1, 2, j = 1, \ldots, n$.

Considering the model defined by (1), (2), (3) and (4), and defining

$$\mathbf{z}_j = \begin{bmatrix} r_{1j} \\ r_{2j} \end{bmatrix},$$

then

$$\mathbf{z}_j \sim p_1 N_2(\mu_{z_1}, \Sigma_{z_1}) + p_2 N_2(\mu_{z_2}, \Sigma_{z_2}), \tag{5}$$

with, $\mu_{z_i} = \begin{bmatrix} \mu_i + \mu_{t_1} \\ \mu_i + \mu_{t_2} \end{bmatrix}$ and $\Sigma_{z_i} = \begin{bmatrix} \sigma_i^2 + \sigma_{e_1}^2 & \sigma_i^2 \\ \sigma_i^2 & \sigma_i^2 + \sigma_{e_2}^2 \end{bmatrix}$.

As can be seen, the likelihood function given in (5) is not straightforward to analyse. In this way, we consider to augment the data (Tanner and Wong, 1987) by defining the vectors, $\mathbf{w_j} = \begin{bmatrix} \mathbf{z_j} \\ c_j \end{bmatrix}$ and $\mathbf{b_j} = \begin{bmatrix} b_{1j} \\ b_{2j} \end{bmatrix}$, with $b_{1j} + b_{2j} = 1$, for $j = 1, \ldots, n$ and $b_{1j} \sim Bernoulli(h_{1j})$, where

$$h_{1j} = \frac{p_1 N_3(\mu_{w_1}, \Sigma_{w_1})}{\sum_{i=1}^{2} p_i N_3(\mu_{\mathbf{w_i}}, \Sigma_{w_i})}, \text{ with } \mu_{w_i} = \begin{bmatrix} \mu_i + \mu_{t_1} \\ \mu_i + \mu_{t_2} \\ \mu_i \end{bmatrix} \text{ and } \Sigma_{w_i} = \begin{bmatrix} \sigma_i^2 + \sigma_{e_1}^2 & \sigma_i^2 & \sigma_i^2 \\ \sigma_i^2 & \sigma_i^2 + \sigma_{e_2}^2 & \sigma_i^2 \\ \sigma_i^2 & \sigma_i^2 & \sigma_i^2 \end{bmatrix}.$$

Then, the likelihood function for $\mathbf{b_1}, \ldots, \mathbf{b_n}$ is given by,

$$L(\mathbf{b_1}, \ldots, \mathbf{b_n}) = \prod_{j=1}^{n} \frac{(p_1 N_3(\mu_{w_1}, \Sigma_{w_1}))^{b_{1j}} \, (p_2 N_3(\mu_{w_2}, \Sigma_{w_2}))^{b_{2j}}}{\left(\sum_{i=1}^{2} p_i N_3(\mu_{\mathbf{w_i}}, \Sigma_{w_i})\right)}$$

and

$$L(\mathbf{w},\mathbf{b}/\theta) = \prod_{j=1}^{n}\left(p_1 N_3(\mu_{w_1},\Sigma_{w_1})\right)^{b_{1j}}\left(p_2 N_3(\mu_{w_2},\Sigma_{w_2})\right)^{b_{2j}} \tag{6}$$

$$= (2\pi)^{-\frac{3n}{2}}(\sigma^2_{e_1})^{-\frac{n}{2}}(\sigma^2_{e_2})^{-\frac{n}{2}}(\sigma^2_1)^{-\frac{\sum_{j=1}^{n} b_{1j}}{2}}(\sigma^2_2)^{-\frac{\sum_{j=1}^{n} b_{2j}}{2}} p_1^{\sum_{j=1}^{n} b_{1j}} p2^{\sum_{j=1}^{n} b_{2j}}$$

$$exp\left\{-\frac{1}{2}\left[\sum_{j=1}^{n}\frac{(c_j - r_{1j} + \mu_{t_1})^2}{\sigma^2_{e_1}} + \sum_{j=1}^{n}\frac{(c_j - r_{2j} + \mu_{t_2})^2}{\sigma^2_{e_2}} + \right.\right.$$

$$\left.\left.\sum_{j=1}^{n}\frac{b_{1j}(c_j-\mu_1)^2}{\sigma^2_1} + \sum_{j=1}^{n}\frac{b_{2j}(c_j-\mu_2)^2}{\sigma^2_2}\right]\right\},$$

where $\theta = (\mu_{t_1},\ \mu_{t_2},\ \mu_1,\ \mu_2,\ \sigma^2_1,\ \sigma^2_2,\ \sigma^2_{e_1},\ \sigma^2_{e_2},\ p_1)^T$.

We are going to specify the prior distribution of θ as follows. The joint prior density for the unknowns can be expressed as $\pi(\theta)=\pi(\mu_{t1})\ \pi(\mu_{t_2})\ \pi(\mu_1)\ \pi(\mu_2)\ \pi(\sigma^2_1)\ \pi(\sigma^2_2)$ $\pi\left(\sigma^2_{e_1}\right)\ \pi\left(\sigma^2_{e_2}\right)\ \pi(p_1)$, where $\mu_{t1} \sim N(a_1, b_1^2)$, $\mu_{t2} \sim N(a_2, b_2^2)$, $\mu_1 \sim N(d_1, e_1^2)$, $\mu_2 \sim N(d_2, e_2^2)$, $\sigma^2_1 \sim IG(g_1, h_1)$, $\sigma^2_2 \sim IG(g_2, h_2)$, $\sigma^2_{e_1} \sim IG(r_1, s_1)$, $\sigma^2_{e_2} \sim IG(r_2, s_2)$ and $p_1 \sim Beta(t, w)$, where $Beta(a, b)$ denotes the Beta distribution with parameters $a > 0$, $b > 0$, $N(a, b^2)$ denotes the Normal distribution with location parameter a and scale parameter $b > 0$, $IG(a, b)$ denotes the Inverse Gamma distribution with shape parameter $a > 0$ and scale parameter $b > 0$ and $a_1, b_1, a_2, b_2, d_1, e_1, d_2, e_2, g_1, h_1, g_2, h_2, r_1, s_1, r_2, s_2, t$ and w are specified prior parameters.

For the purpose of comparisons we considered the model defined by (1), (2), (3) and (4), but instead of using mixture of two Normal distributions for the effect of the subject, we considered

$$c_j \sim N(\mu, \sigma^2), \tag{7}$$

yielding the following expression for the likelihood function

$$L(\mathbf{w}/\theta)) = (2\pi)^{-\frac{3n}{2}}(\sigma^2_{e_1})^{-\frac{n}{2}}(\sigma^2_{e_2})^{-\frac{n}{2}}(\sigma^2)^{-\frac{n}{2}} exp\left\{-\frac{1}{2}\left[\sum_{j=1}^{n}\frac{(c_j - r_{1j} + \mu_{t_1})^2}{\sigma^2_{e_1}} + \right.\right. \tag{8}$$

$$\left.\left.\sum_{j=1}^{n}\frac{(c_j - r_{2j} + \mu_{t_2})^2}{\sigma^2_{e_2}} + \sum_{j=1}^{n}\frac{(c_j-\mu)^2}{\sigma^2}\right]\right\},$$

with $\theta = (\mu_{t_1},\ \mu_{t_2},\ \mu,\ \sigma^2,\ \sigma^2_{e_1},\ \sigma^2_{e_2},\ p_1)^T$. Using the Gibbs sampler algorithm described in the next section, we considered the models defined by the equations (1), (2), (3) and (4) and by the equations (1), (2), (3) and (7) for the real data presented in Singer and Andrade (1997) and we compared the results (section 4). We also studied the case when $\sigma^2_{e_1} = \sigma^2_{e_2} = \sigma^2_e$, considering mixture of two Normal distributions, as well as the simple Normal distribution, which is described in section 4.

3 Posterior computations

Considering the model defined by the equations (1), (2), (3) and (4), we computed the posterior distributions by using the Gibbs sampler algorithm (Smith and Roberts, 1993). Denoting $\theta_{-j} = (\theta_1, \ldots, \theta_{j-1}, \theta_{j+1}, \ldots, \theta_9)$, with $\theta = (\theta_1, \ldots, \theta_9) = (\mu_{t1}, \mu_{t2}, \mu_1, \mu_2,\ \sigma^2_1,\ \sigma^2_2,\ \sigma^2_{e_1},\ \sigma^2_{e_2},\ p_1)$, the full conditional distributions of the unknowns are given as follows:

$$\pi\left(\mu_{t_1}/\theta_{-1}\right) \propto N\left(m_{t_1}, v^2_{t_1}\right), \quad \text{with} \quad m_{t_1} = \frac{b_1^2\left(\sum_{j=1}^n r_{1j} - \sum_{j=1}^n c_j\right) + \sigma^2_{e_1} a_1}{n b_1^2 + \sigma^2_{e_1}}$$

$$\text{and} \quad v^2_{t_1} = \frac{b_1^2 \sigma^2_{e_1}}{n b_1^2 + \sigma^2_{e_1}};$$

$$\pi\left(\mu_{t_2}/\theta_{-2}\right) \propto N\left(m_{t_2}, v^2_{t_2}\right), \quad \text{with} \quad m_{t_2} = \frac{b_2^2\left(\sum_{j=1}^n r_{2j} - \sum_{j=1}^n c_j\right) + \sigma^2_{e_2} a_2}{n b_2^2 + \sigma^2_{e_2}}$$

$$\text{and} \quad v^2_{t_2} = \frac{b_2^2 \sigma^2_{e_2}}{n b_2^2 + \sigma^2_{e_2}};$$

$$\pi\left(\mu_1/\theta_{-3}\right) \propto N\left(m_1, v_1^2\right), \quad \text{with} \quad m_1 = \frac{e_1^2 \sum_{j=1}^n c_j b_{1j} + \sigma_1^2 d_1}{e_1^2 \sum_{j=1}^n b_{1j} + \sigma_1^2} \quad \text{and}$$

$$v_1^2 = \frac{e_1^2 \sigma_1^2}{e_1^2 \sum_{j=1}^n b_{1j} + \sigma_1^2};$$

$$\pi\left(\mu_2/\theta_{-4}\right) \propto N\left(m_2, v_2^2\right), \quad \text{with} \quad m_2 = \frac{e_2^2 \sum_{j=1}^n c_j b_{2j} + \sigma_2^2 d_2}{e_2^2 \sum_{j=1}^n b_{2j} + \sigma_2^2} \quad \text{and}$$

$$v_2^2 = \frac{e_2^2 \sigma_2^2}{e_2^2 \sum_{j=1}^n b_{2j} + \sigma_2^2};$$

$$\pi\left(\sigma_1^2/\theta_{-5}\right) \propto IG\left(\nu_1, \alpha_1\right), \quad \text{with} \quad \nu_1 = g_1 + \frac{\sum_{j=1}^n b_{1j}}{2} \quad \text{and}$$
$$\alpha_1 = h_1 + \sum_{j=1}^n \frac{b_{1j}\left(c_j - \mu_1\right)^2}{2};$$

$$\pi\left(\sigma_2^2/\theta_{-6}\right) \propto IG\left(\nu_2, \alpha_2\right), \quad \text{with} \quad \nu_2 = g_2 + \frac{\sum_{j=1}^n b_{2j}}{2} \quad \text{and}$$
$$\alpha_2 = h_2 + \sum_{j=1}^n \frac{b_{2j}\left(c_j - \mu_2\right)^2}{2};$$

$$\pi\left(\sigma_{e_1}^2/\theta_{-7}\right) \propto IG\left(\nu_{e_1}, \alpha_{e_1}\right), \quad \text{with} \quad \nu_{e_1} = r_1 + \frac{n}{2} \quad \text{and}$$
$$\alpha_{e_1} = s_1 + \sum_{j=1}^n \frac{\left(c_j - r_{1j} + \mu_{t_1}\right)^2}{2};$$

$$\pi\left(\sigma_{e_2}^2/\theta_{-8}\right) \propto IG\left(\nu_{e_2}, \alpha_{e_2}\right), \quad \text{with} \quad \nu_{e_2} = r_2 + \frac{n}{2} \quad \text{and}$$
$$\alpha_{e_2} = s_2 + \sum_{j=1}^n \frac{\left(c_j - r_{2j} + \mu_{t_2}\right)^2}{2};$$

$$\pi\left(p_1/\theta_{-9}\right) \propto Beta\left(q, p\right), \quad \text{with} \quad q = t + \sum_{j=1}^n b_{1j} \quad \text{and}$$
$$p = w + \sum_{j=1}^n b_{2j};$$

The posterior distributions for the model defined by the equations (1), (2), (3) and (7), as well as the posterior distributions of the models defined by (1), (2), (3) and (4)(or (7)), considering $\sigma_{e_1}^2 = \sigma_{e_2}^2 = \sigma^2$ can be obtained in the same way.

4 An application

Considering the pretest/posttest data we first applied the models defined by the equations (1), (2), (3) and (4), and by the equations (1), (2), (3) and (7). Next, considering different initial values, we generated five parallel independent runs of the chain with size 14000, disregarding the first 12000. As the succesive realizations of each chain are correlated, we have considered a spacing of size 10, obtaining a sample of size 200 from each chain. To monitor the convergence of the chain, graphical plots (plot of the sequence of the samples generated, histogram, autocorrelation plot), as well as the between sequence information and within sequence information applying the approach developed in Gelman and Rubin (1992) were obtained. The potential scale reduction, $\hat{R}$, were less than 1.02 for all of the parameters (in both models) indicating the convergence of the chain.

In Table 1, we present the mean and 95 % credibility region of the parameters for the model defined by the equations (1), (2), (3) and (4) (Model I) and for the model defined by the equations (1), (2), (3) and (7) (Model II). To test the hypothesis of interest, we can either consider the function $\mu_{t_1} - \mu_{t_2}$, verifying if the value zero belongs to the credibility region or μ_{t_1}/μ_{t_2} and check if the value one belongs to the credibility region (see for example, Box and Tiao, 1973).

Table 1: Posterior mean and 95% credibility region

Model I					
μ_{t_1}	μ_{t_2}	μ_1	μ_2	σ_1^2	σ_2^2
0.756	0.383	0.644	0.722	0.006	0.018
(0.659,0.858)	(0.293,0.483)	(0.475,0.806)	(0.471,1.075)	(0.002,0,012)	(0.006,0.053)
Model II					
μ_{t_1}	μ_{t_2}	μ		σ^2	
0.765	0.390	0.592		0.027	
(0.661,0.872)	(0.289,0.489)	(0.223,0.969)		(0.008,0.080)	

Model I				
$\sigma^2_{e_1}$	$\sigma^2_{e_2}$	p_1	$\mu_{t_1}-\mu_{t_2}$	μ_{t_1}/μ_{t_2}
0.540	0.513	0.746	0.373	2.007
(0.318,0.865)	(0.311,0.841)	(0.361,0.958)	(0.240,0.505)	(1.505,2.723)
Model II				
$\sigma^2_{e_1}$	$\sigma^2_{e_2}$		$\mu_{t_1}-\mu_{t_2}$	μ_{t_1}/μ_{t_2}
0.592	0.578		0.375	1.997
(0.338,0.996)	(0.336,1.012)		(0.236,0.515)	(1.506,2.688)

If we compare the estimatives of the parameters using either Model I or Model II, these values are very close to each other and cleary in both cases we reject the hypothesis that the effect of the reduction of the plaque index using the hugger and the conventional toothbrushes are the same, which are in accordance with the results obtained in Singer and Andrade (1997). To compare model adequacy we considered the predictive function of each model and obtained

$$C_j = \hat{f}(\mathbf{w}_j/\mathbf{w}) = \frac{1}{S}\sum_{s=1}^{S} f(\mathbf{w}_j/\theta^{(s)}), j = 1, \ldots, n$$

where $\mathbf{w} = (\mathbf{w}_1, \ldots, \mathbf{w}_n)^T$ (see for example, Gelfand, 1996). These values are displayed in Figure 1. Considering

$$C = \prod_{j=1}^{n} C_j,$$

we obtain $C = 1.110 \times 10^{-25}$ for the Model I and $C = 9.151 \times 10^{-26}$ for the Model II. It means that the fit of the model using mixture of Normal distributions was better than the model where we did not consider mixture of Normal distributions. Another point, is the fact that the estimatives of the parameters $\sigma^2_{e_1}$ and $\sigma^2_{e_2}$ are very close. If we construct the 95 % credibility region for the diference of these parameters, i.e. $\sigma^2_{e_1} - \sigma^2_{e_2}$ and for $\sigma^2_{e_1}/\sigma^2_{e_2}$, we obtain $(-0.371, 0.411)$ and $(0.511, 2.094)$, respectiverly for the Model I. If we construct the same 95% credibility region for the Model II, we obtain $(-0.469, 0.515)$ and $(0.485, 2.248)$, what means that in both models (Model I and Model II), we do not reject the hypothesis that $\sigma^2_{e_1} = \sigma^2_{e_2}$. From these results, we adjusted the Model I and Model II, with $\sigma^2_{e_1} = \sigma^2_{e_2} = \sigma^2_e$, which is refered to as Model III and Model IV, respectiverly. The same analysis were performed and the values of the potencial scale reduction in Model III, were less than 1.040 for all cases and for Model IV these values were less than 1.017, indicating the convergence of the chain. The estimated values of the parameters, were very close to the ones obtained by using Model I and Model II. The conclusion about the hypothesis of interest, efficiency in reducting dental plaque index using hugger and conventional toothbrushes, was the same. The values of C_j is plotted in Figure 1 and the value of C was 1.365×10^{-25} for Model III and 1.112×10^{-25} for Model IV.

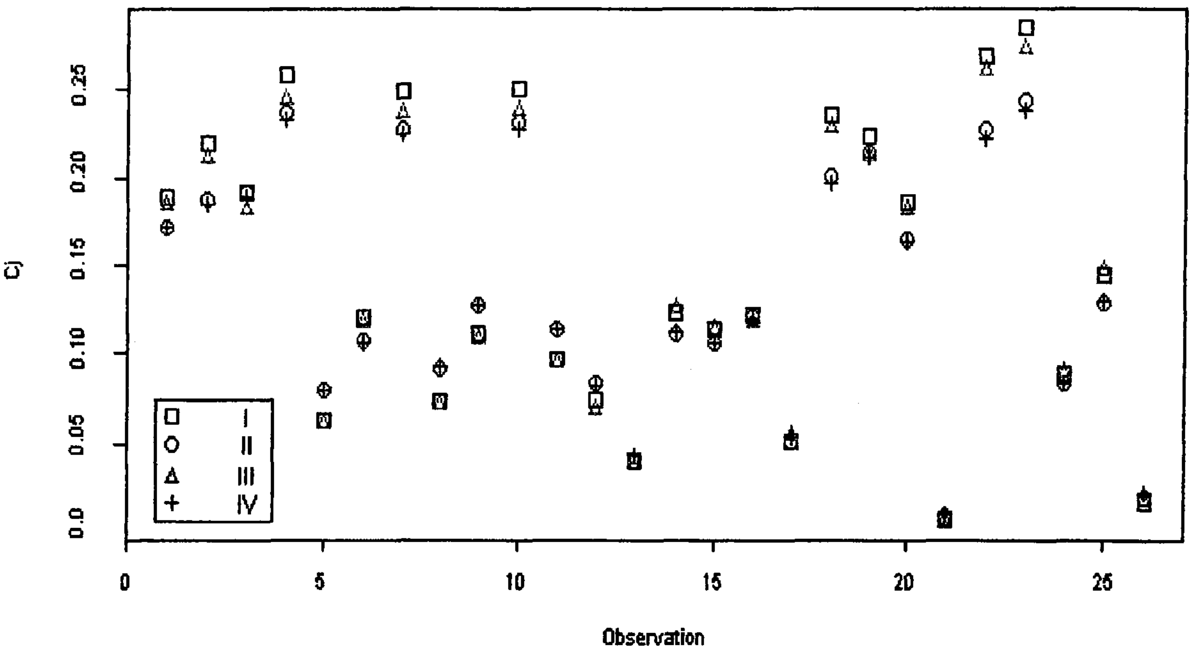

Figure 1: Plot of the values of C_j for the Models I, II, III and IV.

5 Conclusion

The use of the mixture of Normal distribution to model the effect of the subject in reducing the plaque index, as there are individuals with tendency to have more plaque index than others, showed to adjust better than the model where we did not consider this fact. The Model III, where we have used the mixture of Normal distributions for the effect of the individuals, with $\sigma^2_{e_1} = \sigma^2_{e_2} = \sigma^2_e$ was the one with the largest value of C, which means that this model adjusted better than the others for this data set.

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RESUMO. Considerando dados de medidas repetidas do tipo pré teste/pós teste discutimos o uso de misturas de duas distribuições Normais em um modelo de efeitos aleatórios com o uso de métodos Bayesianos. As análises de tais modelos são ilustradas através de um conjunto de dados reais.

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