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Instituto de Ciências Matemáticas e de Computação

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Josmar Mazuchelli
Francisco Louzada-Neto
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Josmar Mazucheli
DEs, Universidade Estadual de Maringá
Maringá, P.R., Brazil

Francisco Louzada-Neto
DEs, Universidade Federal de São Carlos
São Carlos, S.P., Brazil

Jorge A. Achcar
Departamento de Ciências de Computação e Estatística
Instituto de Ciências Matemáticas e de Computação
Universidade de São Paulo — Campus de São Carlos
Caixa Postal 668, CEP 13560-970, São Carlos, S.P., Brazil

Resumo

Neste trabalho é proposto um modelo onde o evento de interesse, morte ou falha, pode ocorrer devido a falhas precoces (*infant mortality*) ou devido ao desgaste (*wearout mortality*) do equipamento, porém a exata causa de falha é completamente desconhecida. Estimação dos parâmetros do modelo proposto é conduzida via máxima verossimilhança. Alguns casos particulares do modelo proposto são estudados via simulação Monte Carlo.

Keywords: Modelos de risco, Modelos de mistura, Simulação Monte Carlo.

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SUMMARY

In this paper we propose a model for handling lifetime data where an early lifetime can be related to the infant-mortality failure or to the wearout processes but we do not know which risk is responsible for the failure. Maximum likelihood estimation and a sampling-based approach for inference are described. Some particular cases of the proposed model are studied via Monte Carlo for size and power of hypothesis tests.

Keywords: Hazard models, Infant mortality failure, Mixture models, Monte Carlo study, Weibull model, Bootstrap.

1 Introduction

Studies where units with manufacturing defects usually lead to an early lifetime failure, hereafter called infant-mortality failure, while nondefective units will eventually failure from wearout are commom in reliability. In general, such situations can be modeled by a mixture of survival distributions (Lawless, 1982; Nelson, 1990). However, an early lifetime can be related to the infant-mortality failure or to the wearout processes but we do not know which risk is responsible for the failure (Meeker and Escobar, 1998). In this situation we only observe the minimum lifetime between the lifetimes related to the hypothetical unknown risks.

In this paper we propose a model for handling such special situation. The model is defined in Section 2 where we also discuss the estimation procedure and a sampling-based approach for inference. The results of a Monte Carlo study are presented in Section 3 where we study the size and power of hypothesis tests for comparing the proposed model with some of its particular cases and a mixture model. A brief discussion in Section 4 concludes the paper.

2 Model Formulation

Suppose that X_1 and X_2 denote the independent hypothetical failure times due to the two unknown competing risks. Consider that only a proportion p of m components are subjected to this $k = 2$ unknown failure causes and a proportion $(1-p)$ of these components failure due to only one of the unknown causes, say X_2. Then, the observed random variable $T = \min(X_1, X_2) \hat{} X_2$ is said to have a infant mortality mixture distribution if its density function is given by

$$f(t) = p\left[h_1(t) + h_2(t)\right] S_1(t)\, S_2(t) + (1-p)\; h_2(t)\, S_2(t)\,, \qquad (1)$$

where $S_j(t)$ and $h_j(t)$ are the survivals and hazards functions due to the cause j, for $j = 1, 2$.

Equivalently, the infant mortality hazard function is given by,

$$h(t) = \frac{p h_1(t)\, S_1(t)}{p S_1(t) + (1-p)} + h_2(t)\,. \qquad (2)$$

The model described in (1) and (2) represents a situation where, despite of the existence of two causes of failure at the initial ages, from an unknown age only the second one become active as the age progress. As an example, following Chan and Meeker (2000), we can consider a situation where units with manufacturing defects will usually lead to an infant-mortality failure early in their lifetimes, whereas the nondefective units will eventually failure from wearout. However, an early lifetime can also be related to the wearout process but we do not know which cause is responsible for the failure. When $p = 1$ model (1) is a bi-hazard competing risk model with two unkown causes of failure (Louzada-Neto, 1999). An advantage of model (2) is to accomodate several non-monotone hazard curves. Figure 1 presents some hazard curves when the $j-$th component follows a Weibull model with $h_j(t) = \beta_j/\mu_j(t/\mu_j)^{\beta_j - 1}$, where μ_j and β_j are the scale and shape parameters, respectively, for $j = 1, 2$. More general curve behaviours can be obtained if other lifetime distributions, such as log-normal, log-logistic and gamma, or even two distributions of different families, is considered for the $j-$th component.

[Figure 1 about here.]

Considering a sample of independent random variables $T_1, \ldots, T_n$ denoting the lifetimes with T_i having an associated indicator variable defined by $\delta_i = 1$ if $T_i = t_i$ is an observed failure time and $\delta_i = 0$ if it is a right-censored observation, the log-likelihood function for the parameters of any set of survival data subject to uninformative censoring can be written, up to an additive constant, as $\log L = \sum_{i=1}^{n}\{\delta_i \log h(t_i) - H(t_i)\}$, where $H(t_i) = \int_0^{t_i} h(x_i)\,dx_i$. The maximum likelihood estimates (MLE) can be obtained by direct maximization of the $\log L$ or by solving the system of nonlinear equations given by the partial derivatives of $\log L$ with respect to the parameters.

Large-sample inference for the parameters can be based on the MLEs and their estimated standard errors or on their profile likelihoods. However, a basic difficulty with the present setting is that the tests can be non regular (Davies, 1977; Cheng and Traylor, 1995). Particularly, if we are interested in testing $p = 0$ (or equivalently $p = 1$) and $\mu_j = 0$ (or equivalently $1/\mu_j = \infty$) the parameters will lie on the bound of the parameter space. An alternative direct approach is to bootstraping the LRS for testing in order to obtain its empirical distribution (Davison and Hinkley, 1997). This can be done parametrically letting $w = 2(l_2 - l_1)$ be the LRS for testing two alternative models, denoted by (1) and 2, where l_1 and l_2 are the log-likelihoods for each model. Large positive values of w give favourable evidence to model 2. The parametric bootstrap technique consists of generating B datasets from the model under the null hypothesis (model 1) with the parameters substituted by their MLEs, record $w_1^* < \cdots < w_B^*$, and use $w^*_{(B+1)(1-\alpha)}$ as the critical point to test the null hypothesis with size α. For power calculation purposes it is interesting to estimate the p-value for the alternative hypothesis, in which case we adopt the reverse procedure above (hereafter called reverse simulation), generating datasets from the model under the alternative (model 2) with the fitted parameter values. This is particularly interesting here once through the significance test we can find evidence against either both models. For interval estimation we can obtain bootstrap percentile confidence intervals for the parameters straightforwardly. For interval estimation, let θ be the parameter of interest. At each resample we calculate the MLE of θ and record $\hat{\theta}_1^* < \cdots < \hat{\theta}_B^*$, and use $\hat{\theta}^*_{(B+1)(\alpha/2)}$ and $\hat{\theta}^*_{(B+1)(1-\alpha/2)}$ as the lower and upper limits of the $100(1-\alpha)\%$ confidence interval for θ.

The adequacy of the bootstrap procedures described above is verified via simulation in Section 3, where we also make a comparison between the infant mixture distribution and the mixture model.

3 Simulation Study

For an assessment of the performance of the bootstrap test procedure described in Section 2 we study its size and power for sample sizes fixed at $n = 50, 70, 90, 110, 130, 150, 200$ with the number of bootstrap replications B fixed at 500. For a Monte Carlo evidence the bootstrap procedure was repeated $R = 1000$ times.

For testing a single-Weibull against model (1), for the empirical sizes we generate samples from single-Weibull model with parameters fixed at $\mu_1 = 1$ and $\beta_1 = 0.4, 0.8, 2.0, 6.0$. For the power calculations the samples were generated from model (1) with parameters fixed at $\mu_1 = \mu_2 = 1$ and $(\beta_1, \beta_2) = (6.0, 0.6)$, $(\beta_1, \beta_2) = (6.0, 2.0)$, $(\beta_1, \beta_2) = (2.0, 0.6)$, $(\beta_1, \beta_2) = (0.8, 0.4)$ and $p = 0.50, 0.75$. We have considered the constraint $\mu_1 = \mu_2 = 1$ since, in principle, it does not lead to the non-identifiability problem described in Section 2. Also, the β's values were choose such that we have strong bathtub, increasing, weak bathtub and decreasing hazard shapes, respectively, as in Figure 1.

From the empirical size calculations we do not reject the null hypothesis at a nominal significance of $\alpha = 0.05$ in all cases considered here and the empirical probabilities are not shown. The power of the tests for $\alpha = 0.05$ were summarized in Table 1 for p equals to 0.50 and 0.75 cases. The empirical powers increase with the ratio β_1/β_2 and are bigger for large n. For ratios moving towards one however the powers are rather moderate. A phenomenon related to the non-identifiability problem that arises in such situations.

[Table 1 about here.]

For testing a bi-Weibull (model (1) with $p = 1$) against model (1), for the empirical sizes we generate samples from bi-Weibull models with parameters fixed at $\mu_1 = \mu_2 = 1$ and $(\beta_1, \beta_2) = (6.0, 0.6)$, $(\beta_1, \beta_2) = (6.0, 2.0)$, $(\beta_1, \beta_2) = (2.0, 0.6)$ and $(\beta_1, \beta_2) = (0.8, 0.4)$. And, as above, for the power calculations the samples were generated from model (1) with parameters fixed at $\mu_1 = \mu_2 = 1$ and $(\beta_1, \beta_2) = (6.0, 0.6)$, $(\beta_1, \beta_2) = (6.0, 2.0)$, $(\beta_1, \beta_2) = (2.0, 0.6)$, $(\beta_1, \beta_2) = (0.8, 0.4)$ and and $p = 0.50, 0.75$. For all cases considered we do not reject the null hypothesis at a nominal significance level of $\alpha = 0.05$ and the empirical probabilities are not shown. The powers of the tests for $\alpha = 0.05$ were summarized in Table 2 for p equals to 0.75 and 0.50 cases. The empirical powers behave in similar manner as the empirical powers for testing a single-Weibull against model (1).

[Table 2 about here.]

Table 3 shows the bootstrap estimates and their standard deviations for generated datasets from model (1) with parameters fixed at $\mu_1 = \mu_2 = 1$, $\beta_1 = 2.0$, $\beta_2 = 0.6$ and p equals to 0.50 and 0.75 with sample sizes equal to $n = 50, 70, 90, 110, 130, 150, 200$. The bootstrap estimates are based on $R = 5000$ simulations and are very close to the true parameter values.

[Table 3 about here.]

It is interesting to note that model (1) is an alternative to a mixture model (Titterington et al., 1985) given by

$$f(t) = ph_1(t)S_1(t) + (1-p)\, h_2(t)\, S_2(t), \tag{3}$$

for fitting lifetime data. In this context, it is desirable to know whether it is possible to discriminate between the model (1)and the mixture model (3) for a particular set of lifetime data. To investigate this we considered pairs of simulated datasets with n observations each, for $n = 50, 70, 90, 110, 130, 150, 200$. The first dataset was generated from model (1) and the second one was generated from model (3). The parameters of both models were fixed at $\mu_1 = \mu_2 = 1$ e $(\beta_1, \beta_2) = (6.0, 0.6)$, $(\beta_1, \beta_2) = (6.0, 2.0)$, $(\beta_1, \beta_2) = (2.0, 0.6)$, $(\beta_1, \beta_2) = (0.8, 0.4)$ and $p = 0.50, 0.75$. The parametric simulation scheme adopted was analogous to the scheme adopted for comparing model (1) with its particular cases. The powers of the tests for $\alpha = 0.05$ were summarized in Table 4 p equals to 0.50 and 0.75 cases. The empirical powers results suggest that it is possible to discriminate between models (1)and (3).

[Table 4 about here.]

4 Concluding Remarks

The model (1) discussed in the paper can be effectively used for analysing lifetime data and it is a general framework. It can be used in several practical situations where we have infant-mortality and wearout type of failures, but the information about the cause which is responsible for the failure is completely unknown. Besides, model (1) can be considered straithforwardlly if the cause are partial masked. Our simulation study reveal that the bootstrap scheme adapted for testing is satisfactory according to their size and power. Model (1) is an alternative to the mixture formulation (3). Our comparation study of models (1) and (3) revels that it is possible to distinguish between them based on statistical criteria. Censoring is very common in reliability and survival studies. The parametric simulation approach considered in this paper should be investigated further in the censoring data context when the infant-hazard model is adopted.

Acknowledgements

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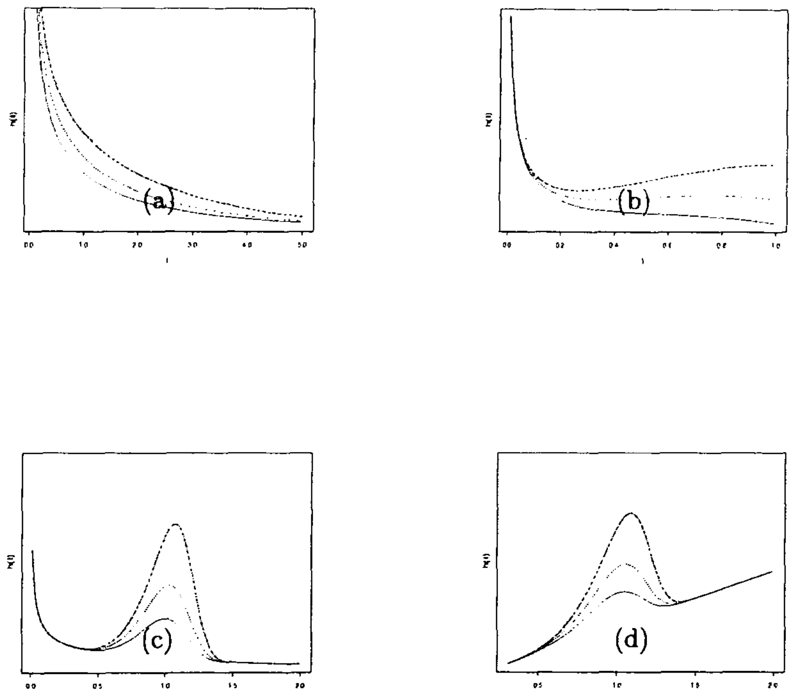

Figure 1: Typical hazard shapes that can be accommodated by (2). (—) $p = 0.4$, $(\cdots)$ $p = 0.6$ e $(-\cdot-)$ $p = 0.8$. (a): $\beta_1 = 0.8$, $\beta_2 = 0.4$; (b): $\beta_1 = 2.0$, $\beta_2 = 0.6$; (c): $\beta_1 = 6.0$, $\beta_2 = 0.6$; (d): $\beta_1 = 6.0$, $\beta_2 = 2.0$.

List of Tables

Table 1: The power (standard deviation) of the tests for $\alpha = 0.05$. Model (1) versus single-Weibull. $p = 0.75$ and $p = 0.50$

	n						
p	50	70	90	110	130	150	200
				Case 1: $\beta_1 = 6.0, \beta_2 = 0.6$			
0.75	0.9990 (0.0044)	1.0000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)
0.50	0.8730 (0.0551)	0.9640 (0.0159)	0.989 (0.0067)	0.9980 (0.0040)	0.9990 (0.0030)	0.9990 (0.0030)	1.000 (0.000)
				Case 2: $\beta_1 = 6.0,\ \beta_2 = 2.0$			
0.75	0.8025 (0.0650)	0.9665 (0.0267)	0.9955 (0.0096)	0.9985 (0.0058)	0.9995 (0.0022)	1.0000 (0.0000)	1.0000 (0.0000)
0.50	0.4615 (0.1097)	0.5915 (0.0873)	0.7040 (0.0879)	0.7415 (0.0683)	0.8800 (0.0510)	0.9110 (0.0538)	0.9750 (0.0201)
				Case 3: $\beta_1 = 2.0,\ \beta_2 = 0.6$			
0.75	0.9030 (0.0347)	0.9565 (0.0220)	0.9935 (0.0079)	0.9995 (0.0022)	1.0000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)
0.50	0.5870 (0.0949)	0.6825 (0.0902)	0.7820 (0.0954)	0.9075 (0.0416)	0.9085 (0.0321)	0.9690 (0.0141)	0.9770 (0.0149)
				Case 4: $\beta_1 = 0.8,\ \beta_2 = 0.4$			
0.75	0.7975 (0.0694)	0.9165 (0.0402)	0.9765 (0.0188)	0.9910 (0.0118)	0.9985 (0.0036)	0.9995 (0.0022)	1.0000 (0.0000)
0.50	0.4460 (0.1070)	0.5640 (0.1194)	0.7490 (0.0602)	0.7840 (0.0781)	0.8880 (0.0452)	0.9055 0.0373)	0.9530 (0.0300)

Table 2: The power (standard deviation)of the tests for $\alpha = 0.05$. Model (1) versus bi-Weibull. $p = 0.75$ and $p = 0.50$

p	n = 50	70	90	110	130	150	200
			Case 1: $\beta_1 = 6.0, \beta_2 = 0.6$				
0.75	0.8260 (0.0937)	0.9565 (0.0445)	0.9945 (0.0218)	0.9955 (0.0116)	1.0000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)
0.50	0.6585 (0.0964)	0.9155 (0.1156)	0.9720 (0.0277)	0.9905 (0.0186)	0.9990 (0.0030)	1.0000 (0.0000)	1.0000 (0.0000)
			Case 2: $\beta_1 = 6.0$, $\beta_2 = 2.0$				
0.75	0.4240 (0.1491)	0.4940 (0.1271)	0.6875 (0.1066)	0.7385 (0.1228)	0.7680 (0.1360)	0.9315 (0.0492)	0.9540 (0.0588)
0.50	0.6585 (0.0964)	0.9155 (0.1156)	0.9720 (0.0277)	0.9905 (0.0186)	0.9990 (0.0030)	1.0000 (0.0000)	1.0000 (0.0000)
			Case 3: $\beta_1 = 2.0$, $\beta_2 = 0.6$				
0.75	0.4395 (0.1070)	0.5460 (0.1188)	0.5710 (0.1529)	0.8235 (0.1113)	0.8755 (0.0862)	0.9575 (0.0559)	0.9945 (0.0092)
0.50	0.6495 (0.1608)	0.8310 (0.0871)	0.9760 (0.0256)	0.9980 (0.0051)	0.9995 (0.0022)	1.0000 (0.0000)	1.0000 (0.0000)
			Case 4: $\beta_1 = 0.8$, $\beta_2 = 0.4$				
0.75	0.3840 (0.1055)	0.4530 (0.1094)	0.5125 (0.1523)	0.5810 (0.1462)	0.6605 (0.1417)	0.8140 (0.0995)	0.9215 (0.0611)
0.50	0.6240 (0.1445)	0.8570 (0.0589)	0.9620 (0.0262)	0.9895 (0.0264)	0.9995 (0.0022)	1.000 (0.0000)	1.0000 (0.0000)

Table 3: Bootrstrap estimates and standard deviation.

	n						
	50	70	90	110	130	150	200
True values: $\beta_1 = 2.0$, $\beta_2 = 0.6$ and $p = 0.5$.							
β_1	2.4450 (1.1936)	2.4139 (1.0612)	2.3452 (0.9790)	2.2430 (0.8965)	2.2394 (0.8359)	2.1904 (0.7945)	2.1890 (0.6710)
β_2	0.6111 (0.0874)	0.6077 (0.0731)	0.6083 (0.0639)	0.6094 (0.0578)	0.6079 (0.0529)	0.6069 (0.0488)	0.6046 (0.0419)
p	0.5418 (0.1904)	0.5408 (0.1624)	0.5263 (0.1469)	0.5129 (0.1356)	0.5090 (0.1255)	0.5078 (0.1181)	0.5071 (0.1025)
True values $\beta_1 = 2.0$, $\beta_2 = 0.6$ and $p = 0.75$.							
β_1	2.4052 (0.9264)	2.2210 (0.7597)	2.1741 (0.6550)	2.1621 (0.5998)	2.0907 (0.5415)	2.0835 (0.4956)	2.0667 (0.4308)
β_2	0.6239 (0.0971)	0.6188 (0.0814)	0.6124 (0.0712)	0.6091 (0.0637)	0.6105 (0.0584)	0.6060 (0.0537)	0.6047 (0.0463)
p	0.7264 (0.1553)	0.7427 (0.1312)	0.7519 (0.1147)	0.7478 (0.1042)	0.7507 (0.0968)	0.7510 (0.0898	0.7508 (0.0783)

Table 4: The power (standard deviation) of the tests for $\alpha = 0.05$. Model (1) versus mixture-Weibull. $p = 0.75$ and $p = 0.50$.

	n						
p	50	70	90	110	130	150	200
			Case 1: $\beta_1 = 6.0$, $\beta_2 = 0.6$				
0.75	0.4395 (0.1056)	0.6210 (0.1115)	0.6925 (0.0932)	0.8195 (0.0881)	0.9160 (0.0454)	0.9480 (0.0304)	0.9855 (0.0163)
0.50	0.8925 (0.0843)	0.9825 (0.0155)	0.9960 (0.0073)	0.9995 (0.0022)	1.000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)
			Case 2: $\beta_1 = 6.0$, $\beta_2 = 2.0$				
0.75	0.5575 (0.0795)	0.7080 (0.0779)	0.7970 (0.1106)	0.8180 0.0941)	0.8945 (0.0676)	0.8985 (0.0419)	0.9705 (0.0163)
0.50	0.8055 (0.0901)	0.9705 (0.0248)	0.9925 (0.0113)	0.9985 (0.0048)	1.0000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)
			Case 3: $\beta_1 = 2.0$, $\beta_2 = 0.6$				
0.75	0.6035 (0.0973)	0.6570 (0.0826)	0.7325 (0.0893)	0.8305 (0.0825)	0.9365 (0.0302)	0.9525 (0.0295)	0.9840 (0.0136)
0.50	0.8215 (0.0755)	0.9630 (0.0276)	0.9765 (0.0165)	0.9975 (0.0043)	0.9975 (0.0043)	1.0000 (0.0000)	1.0000 0.0000)
			Case 4: $\beta_1 = 0.8$, $\beta_2 = 0.4$				
0.75	0.5445 (0.1052)	0.6085 (0.1099)	0.6280 (0.1252)	0.7580 (0.0852)	0.7795 (0.0919)	0.8960 (0.0547)	0.9920 (0.0154)
0.50	0.7495 (0.0754)	0.8995 (0.0440)	0.9790 (0.0145)	0.9935 (0.0079)	1.0000 (0.0000)	1.0000 (0.0000)	1.0000 (0.0000)

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