UNIVERSIDADE DE SÃO PAULO

USE OF MARKOV CHAIN MONTE CARLO METHODS FOR A BAYESIAN ANALYSIS OF SOFTWARE RELIABILITY MODELS

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RESUMO

Neste artigo,consideramos inferencia Bayesiana para alguns modelos de confiabilidade de software com dados de sobrevivencia entre falhas.Algoritmos Metropolis-Hastings com passos de Gibbs sao propostos para desenvolver inferencia Bayesiana para dois modelos especiais:o modelo de Goel e Okumoto (1978) e o modelo de Schick e Wolverton (1978).

Tambem exploramos alguns criterios de selecao de modelos Bayesianos considerando um conjunto de dados de confiabilidade de software introduzido por Jelinski e Moranda (1972).

Use of Markov Chain Monte Carlo methods for a Bayesian analysis of software reliability models

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Abstract

In this paper, we consider Bayesian inference for some software reliability models considering interfailure time data. Metropolis-Hastings algorithms along with Gibbs steps are proposed to perform the Bayesian inference for two special models: the Goel and Okumoto (1978) model and the Schick and Wolverton (1978) model.

We also explore some Bayesian model selection criteria considering a software reliability data set introduced by Jelinski e Moranda (1972).

Keywords: software reliability models, interfailure time data, Gibbs sampling, Metropolis algorithm.

1 Introduction

Software reliability is the probability of a computer program to be free of error in operation during a specified period of time. The software failures are related to errors in syntax or logic (see for example, Singpurwalla and Wilson, 1994). The literature in statistics and especially in software engineering presents many different models for software reliability. Among these stochastic models we have two strategies: (i) modelling times between successive failures or, (ii) modelling the number of failures of the software up to a given time.

For modelling interfailure time data, Jelinski and Moranda (1972) suppose that the total number of bugs in the program is N, and they assume that each time the software fails, one bug is corrected. The failure rate of the ith time between failures T_i is assumed to be a constant proportional to N-i+1, which is the number of bugs remaining in the program.

Thus, the failure rate for T_i is given by

$$\lambda_i = \lambda_{JM}(N - i + 1) \qquad (1)$$

where i=1,2,3,...

That is, T_i has an exponential density,

$$f(t_i \mid \lambda_i) = \lambda_i e^{-\lambda_i t_i} \qquad (2)$$

where $T_i \geq 0$ and λ_i is given by (1).

Some modifications of the JM model (1) are introduced in the literature. Moranda (1975), assume that the fixing of bugs that cause early failures in the system reduces the failure rate more than the fixing of bugs that occur later. Therefore, Moranda (1975) assume that the failure rate should remain constant for each T_i , but it decreases geometricaly in i after each failure, that is, for constants θ e K,

$$\lambda_i = \theta K^{i-1} \qquad (3)$$

where $\theta > 0$ and $0 < K < 1$.

Goel and Okumoto (1978) propose a model similar to JM model (1), but assuming that there is a probability p, $0 \le p \le 1$, of fixing a bug when it is encountered. Thus, the failure rate of T_i is given by

$$\lambda_i = \lambda_{GO}[N - p(i-1)] \tag{4}$$

When p=1, we have the JM model (1).

Schick and Wolverton (1978) assume that the failure rate is proportional to the number of bugs remaining in the system and the time elapsed since the last failure. Thus,

$$\lambda_i = \lambda_{SW}(N - i + 1)t \tag{5}$$

Other model for software interfailure time data is introduced by Littlewood e Verral (1973) to relax the assumption of perfect repair in the JM model (1). They assume that the ith time between failures has an exponential density (2) with failure rate λ_i, and that instead of λ_i be considered decreasing with certainty, as it is assumed in the JM model (1), they assume the sequence of λ's to be stochastically decreasing, that is,

$P(\lambda_{i+1} < \lambda) \ge P(\lambda_i < \lambda)$, for $i = 1,2,\ldots$ and $\lambda \ge 0$.

In this way, they consider a gamma distribution for λ_i with shape parameter α and scale $\psi(i)$, where $\psi(i)$ is a monotonically increasing function of i,

$$\pi(\lambda_i \mid \alpha, \psi(i)) = \frac{[\psi(i)]^\alpha}{\Gamma(\alpha)} \lambda_i^{\alpha-1} e^{-\psi(i)\lambda_i} \tag{6}$$

The function $\psi(i)$ is supposed to describe the quality of the programmer and the programming task. Mazzuchi and Soyer (1988) assume $\psi(i) = \beta_o + \beta_1 i$.

For modelling the number of failures of the software up to a given time, the literature presents many different models assuming homogeneous or non-homogeneous Poisson process (see for example, Goel and Okumoto, 1979; Goel, 1983; Ohba and Yamada, 1982; Musa and Okumoto, 1984).

The use of Gibbs sampling with Metropolis-Hastings algorithms (see for example, Gelfand and Smith, 1990) has been used by many authors to get Bayesian inference for software reliability models (see for example, Kuo and Yang, 1996; Yang, 1994; Kuo, Lee, Choi and Yang, 1996; Achcar, Dey and Niverthi, 1996).

Yang (1994) considers the use of Metropolis-within-Gibbs algorithms to obtain Bayesian inferences on the Jelinski e Moranda (1972) model and on the Littlewood and Verral (1973) model, considering the introduction of latent variable (data augmentation
technique) to simplify the conditional posterior densities required in the Gibbs algorithm.

In this paper, we present Bayesian inference for two special software reliability models using Metropolis-within-Gibbs algorithms: the Goel and Okumoto (1978) model and the Schick and Wolverton (1978) model. We also explore some Bayesian model selection criteria (see Kuo and Yang, 1996) considering a software reliability data set introduced by Jelinski e Moranda (1972).

2 Bayesian Inference for the Goel and Okumoto Model

Assuming the GO model (4), the likelihood function for λ_{GO}, p and N (with a failure truncated model) is given by

$$L(\lambda_{GO}, N, p) = \lambda_{GO}^{n} A(N,p) \exp\{-\lambda_{GO} B(N,p)\} \tag{7}$$

where $A(N,p)=\prod_{i=1}^{n}[N-p(i-1)]$ and $B(N,p)=\sum_{i=1}^{n}[N-p(i-1)]t_i$.

Since $t_i = x_i - x_{i-1}$, where x_i denotes the ordered epochs of failure time, we have $B(N,p)=p\sum_{i=1}^{n} x_i + (N-np)x_n$.

Assuming prior independence, consider the following prior densities for λ_{GO}, p and N:

$$\begin{aligned} &\text{(i)} && \lambda_{GO} \sim \Gamma[a_1, b_1], \\ &\text{(ii)} && N \sim P(\theta_1), \\ &\text{(iii)} && p \sim B[a_2, b_2], \end{aligned} \tag{8}$$

where a_1, b_1, a_2, b_2 and θ_1 are known constants.

The joint posterior density for λ_{GO}, p and N is given by

$$\pi(\lambda_{GO}, N, p \mid D_n) \propto \frac{\lambda_{GO}^{n+a_1-1} A(N,p)\theta_1^N}{N!} \times$$
$$p^{a_2-1}(1-p)^{b_2-1} \exp\left\{-\left[b_1 + p\Sigma_{i=1}^n x_i + (N-np)x_n\right]\lambda_{GO}\right\}, \quad (9)$$

where $\lambda_{GO} > 0$; N=n,n+1,...; and $0 \le p \le 1$.

In this case, the conditional densities for the Gibbs algorithm are given by

(i) $$\lambda_{GO} \mid N, p, D_n \sim \Gamma[n + a_1, b_1 + p\Sigma_{i=1}^n x_i + (N-np)x_n],$$

(ii) $$\pi(N \mid \lambda_{GO}, p, D_n) \propto \frac{e^{-\theta_1}\theta_1^N}{N!} \psi_1(N, p, \lambda_{GO}),$$

where

$$\psi_1(N, p, \lambda_{GO}) = \exp\{\ln A(N,p) - (N-np)x_n\lambda_{GO}\} \quad (10)$$

and

(iii) $$\pi(p \mid N, \lambda_{GO}, D_n) \propto p^{a_2-1}(1-p)^{b_2-1} \psi_2(N, p, \lambda_{GO}),$$

where

$$\psi_2(N, p, \lambda_{GO}) = \exp\{\ln A(N,p) - p\lambda_{GO}\Sigma_{i=1}^n x_i - (N-np)x_n\lambda_{GO}\}.$$

Observe that, the variables N and p should be generated using the Metropolis-Hastings algorithm (see for example, Chib and Greenberg, 1995).

3 Bayesian Inference for the Schick and Wolverton Model

Assuming the SW model (5), observe that the density function for the interfailure time T_i is given by,

$$f_i(t) = \lambda_{SW} t(N - i + 1)\exp\{-\lambda_{SW}(N - i + 1)t^2/2\} \quad (11)$$

Also assuming a failure truncated model, the likelihood function for λ_{SW} and N is given by

$$L(\lambda_{SW}, N) = \lambda_{SW}^{n} A_1(N) \exp\left\{-\frac{\lambda_{SW}}{2} B_1(N)\right\} \tag{12}$$

where $= A_1(N) = \prod_{i=1}^{n} t_i [N - i + 1]$ and $B_1(N) = \sum_{i=1}^{n} [N - i + 1] t_i^2$.

Assuming prior independence, consider the following prior densities for λ_{SW} and N:

$$\begin{aligned} &\text{(i)} \qquad \lambda_{SW} \sim \Gamma[a_3, b_3], \\ &\text{(ii)} \qquad N \sim P(\theta_2), \end{aligned} \tag{13}$$

where a_3, b_3 and θ_2 are known constants.

The joint posterior density for λ_{SW} and N is given by

$$\begin{aligned} \pi(\lambda_{SW}, N \mid D_n) &\propto \frac{\lambda_{SW}^{n+a_3-1} A_1(N) \theta_2^N}{N!} \times \\ &\times \exp\left\{-\lambda_{SW}\left[b_3 + \frac{N}{2}\sum_{i=1}^{n} t_i^2 - \frac{1}{2}\sum_{i=1}^{n}(i-1)t_i^2\right]\right\}, \end{aligned} \tag{14}$$

where $\lambda_{SW} > 0$ and N=n,n+1,....

The conditional posterior densities for the Gibbs algorithm are given by

$$\begin{aligned} &\text{(i)} \qquad \lambda_{SW} \mid N, D_n \sim \Gamma\left[n + a_3, b_3 + \frac{N}{2}\sum_{i=1}^{n} t_i^2 - \frac{1}{2}\sum_{i=1}^{n}(i-1)t_i^2\right], \\ &\text{(ii)} \qquad \pi(N \mid \lambda_{SW}, D_n) \propto \frac{e^{-\theta_2}\theta_2^N}{N!} A_1(N) \exp\left\{-\frac{\lambda_{SW}}{2} N \sum_{i=1}^{n} t_i^2\right\}, \end{aligned} \tag{15}$$

Considering the transformation $N' = N - n$, we observe that $\frac{A_1(N)}{N!} = \frac{\prod_{i=1}^{n} t_i}{N'!}$
Therefore, we get,

(i) $$\lambda_{SW} \mid N', D_n \sim \Gamma\left[n + a_3, b_3 + \frac{N'}{2}\sum_{i=1}^{n} t_i^2 + \frac{n}{2}\sum_{i=1}^{n} t_i^2 - \frac{1}{2}\sum_{i=1}^{n}(i-1)t_i^2\right],$$

and (16)

(ii)
$$N'|\lambda_{SW}, D_n \sim P\left(\theta_2 e^{-\lambda_{sw}\sum_{i=1}^{n} t_i^2/2}\right).$$

4 Some Considerations on Model Selection

For model selection, we could use the predictive density for t_i given $D_{(i)}$, $i=1,2,\ldots,n$ where $D_{(i)}$ denotes the set of interfailure times not including t_i, that is, $D_{(i)}=\{t_1,t_2,\ldots,t_{i-1},t_{i+1},\ldots,t_n\}$.

The predictive density for t_i given $D_{(i)}$, is given by

$$c_i = f(t_i|D_{(i)}) = \int f(t_i|\underset{\sim}{\theta})\,\pi(\underset{\sim}{\theta}|D_{(i)})\,d\underset{\sim}{\theta} \qquad (17)$$

where $\pi(\underset{\sim}{\theta}\,|D_{(i)})$ is the posterior density for a vector of parameters $\underset{\sim}{\theta}$, given the data set $D_{(i)}$.

Using the Gibbs samples, (17) can be approximated by its Monte Carlo estimate,

$$\hat{f}(t_i|D_{(i)}) = \frac{2}{RS}\sum_{r=1}^{R}\sum_{s=\frac{S}{2}+1}^{S} f(t_i|\underset{\sim}{\theta}^{(r,s)}) \qquad (18)$$

where $\underset{\sim}{\theta}^{(r,s)}$ are generated for S iterations in each of R chains considering different initial values for $\underset{\sim}{\theta}$.

We can use $c_i = f(t_i\,|D_{(i)})$ in model selection. In this way, we consider plots of c_i versus i ($i=1,2,\ldots,n$) for different models; large values of c_i (in average) indicates the better model. We also could choose the model such that $c(l) = \prod_{i=1}^{n} c_i(l)$ is maximum (l indexes models).

5 A Numerical illustration

In table 1, we have a software reliability data set introduced by Jelinski and Moranda (1972). The data consists of the number of days between the 26 failures that occurred during the production phase of a software (NTDS data - Naval Tactical Data System).

From the data of table 1, we have n=26 and $x_n = x_{26} = 250$.

i	t_i	x_i	i	t_i	x_i	i	t_i	x_i
1	9	9	10	7	70	19	6	104
2	12	21	11	1	71	20	1	105
3	11	32	12	6	77	21	11	116
4	4	36	13	1	78	22	33	149
5	7	43	14	9	87	23	7	156
6	2	45	15	4	91	24	91	247
7	5	50	16	1	92	25	2	249
8	8	58	17	3	95	26	1	250
9	5	63	18	3	98			

Table 1 - NTDS data ($t_i = x_i - x_{i-1}$)

Assuming the GO model (4), we consider (from (8)) the prior densities $N \sim P(30)$, $\lambda_{GO} \sim \Gamma[0.2,20]$ and $p \sim B[2.5,2.5]$. From (10), the conditional distributions for the Gibbs-within-Metropolis algorithm are given by $\lambda_{GO} | N, p, D_n \sim \Gamma[26.2, 20 + p\sum_{i=1}^{26} x_i + 250\ (N\text{-}26p)]$, $\pi(N | \lambda_{GO}, p, D_n) \propto (e^{-30} 30^N / N!)\, \psi_1(N, p, \lambda_{GO})$ where $\psi_1(N, p, \lambda_{GO}) = \exp\{\ln A(N,p) - 250(N - 26p)\lambda_{GO}\}$, and $\pi(p | \lambda_{GO}, N, D_n) \propto p^{2.5-1}(1-p)^{2.5-1} \psi_2(N, p, \lambda_{GO})$, where $\psi_2(N, p, \lambda_{GO}) = \exp\{\ln A(N,p) - p\lambda_{GO} \sum_{i=1}^{26} x_i - 250(N - 26p)\lambda_{GO}\}$. Generating 5 separate Gibbs-within-Metropolis chains each of which with 1000 iterations, we selected for each parameter, the 205^{th}, 210^{th}, 215^{th}, ... , 995^{th}, 1000^{th} iterations, which for 5 chains yields a sample of size 1000. In table 2, we have the obtained posterior summaries for the parameters λ_{GO}, p and N, and in figure 1 we have the approximate marginal posterior densities considering the S=1000 Gibbs Samples.

	Mean	Median	S.D.	95% Credible interval
λ_{GO}	0.00634	0.00582	0.00257	0.002980;0.0126824
N	28.272	28	5.091	19;39
p	0.62189	0.64403	0.17523	0.241834;0.905575

Table 2: Posterior summaries for the GO model.

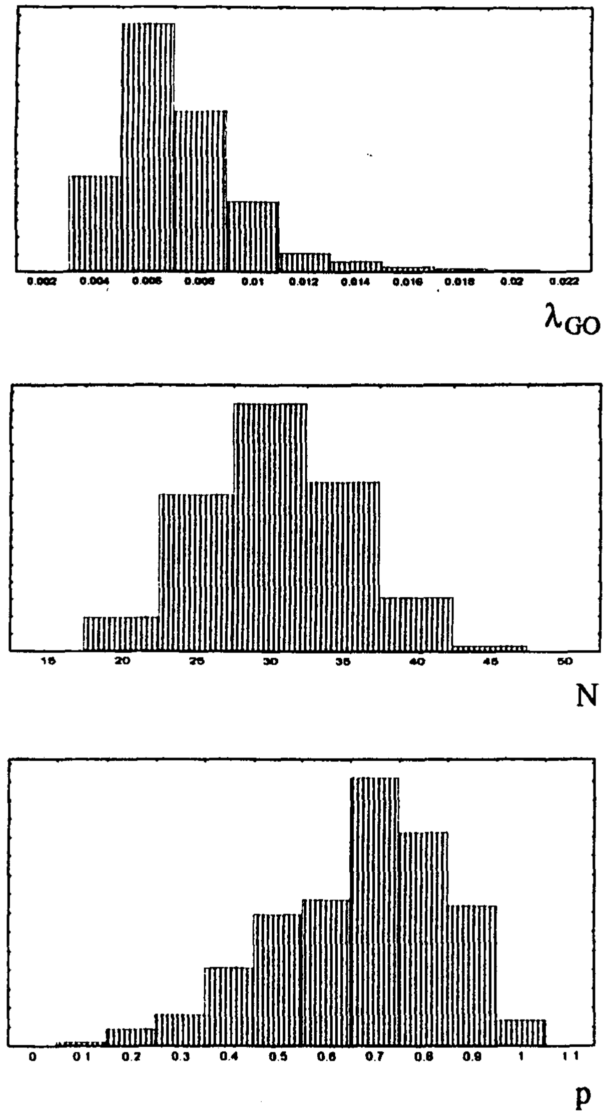

Figure 1 - Marginal posterior densities for λ_{GO}, N and p (GO model)

Assuming the SW model (5), consider (from (13)) the prior density $\lambda_{SW} \sim \Gamma[6.25,625]$ and $N \sim P(30)$. From the conditional distributions (16), $\lambda_{SW} \mid N', D_n \sim \Gamma[32.25, 23950 + 5157N']$, and $N' \mid \lambda_{SW}, D_n \sim P(30e^{-5157\lambda_{SW}})$, where N'=N-26, we generated 5 separate Gibbs chains each of which ran for 1000 iterations. For each parameter, we selected the last 200 iterations, which for 5 chains yields a sample of size 1000. In table 3, we have the obtained posterior summaries for the parameters λ_{SW} and N', and figure 2, we have the marginal posterior densities considering the S=1000 Gibbs Samples.

	Mean	Median	S.D.	95% Credible interval
λ_{SW}	0.00133	0.00131	0.000243	0.0008919;0.001842
N	0.056667	0.06	0.253266	0,1

Table 3: Posterior Summaries for the SW model.

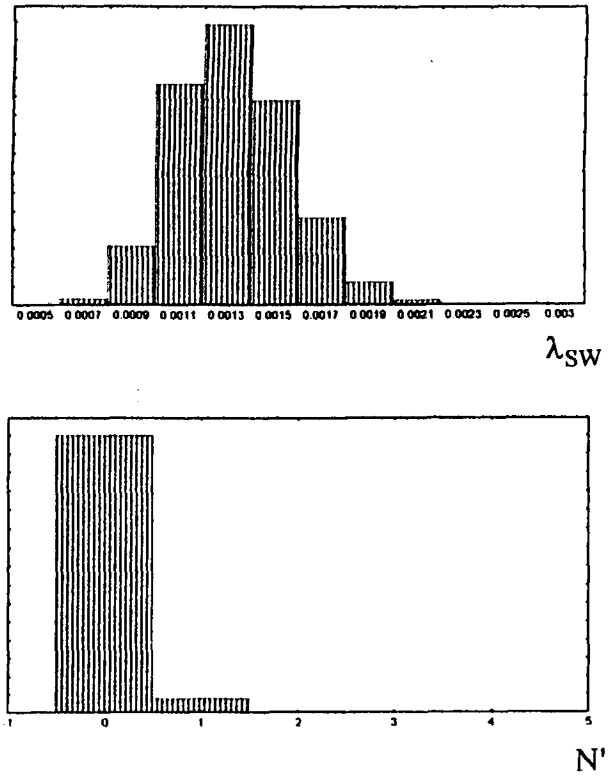

Figure 2: Marginal Posterior densities for λ_{SW} and N' (SW model)

In table 4, we have the values of the predictive densities $c_i = f(t_i \mid D_{(i)})$ (see section 4) evaluated at the observed values t_i (see table 1) and approximated by its Monte Carlo estimates based on S=1000 Gibbs Samples for each considered model. In figure 3, we have the plot of c_i against i for both models. We also have in table 4, the values for $c(l) = \prod_{i=1}^{26} c_i$, considering the different models.

From table 4 and figure 3, observe that c(l) has the largest value considering the GO model (4), which indicates, in general, better fit for the software reliability data of table 1.

i	SW model	GO model
1	0.0755903	0.036218
2	0.0367796	0.022907
3	0.0506388	0.027207
4	0.0953618	0.081413
5	0.0986254	0.050981
6	0.0529042	0.108578
7	0.0947465	0.068415
8	0.0887319	0.044542
9	0.0882486	0.067466
10	0.0899458	0.051205
11	0.0211354	0.112529
12	0.0830276	0.057838
13	0.0185271	0.106627
14	0.07625	0.040008
15	0.056227	0.070967
16	0.0146012	0.097535
17	0.0377261	0.076013
18	0.0341827	0.074076
19	0.0527312	0.053429
20	0.0093412	0.084955
21	0.0538004	0.032668
22	0.0062214	0.005317
23	0.00329447	0.045058
24	0.0000004	0.000167
25	0.0054276	0.063214
26	0.0013967	0.065062
c(l)	2.0×10^{-43}	2.8×10^{-36}

Table 4 - Value for $c_i = f(t_i \mid D_{(i)})$, i=1,2,...,26.

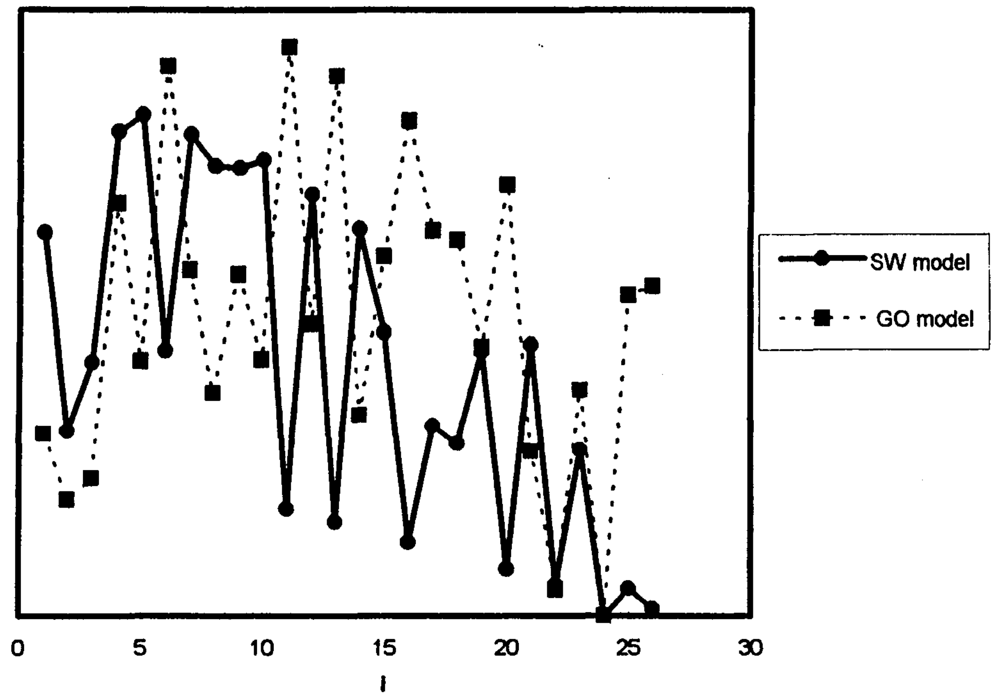

Figure 3: Plots of c_i versus i for SW model (5) and GO model (4).

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