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RESUMO

Neste artigo,consideramos uma analise Bayesiana para uma forma generalizada do modelo de confiabilidade de software introduzido por Moranda (1975) considerando densidades a priori nao-informativas para os parametros. Tambem apresentamos duas ilustracoes numericas, onde observamos melhor ajuste da forma generalizada do modelo de Moranda para os dados de confiabilidade de software,em comparacao com o modelo usual de Moranda.

A Generalized Moranda Software Reliability Model: A Bayesian Approach

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Abstract

In this paper, we consider a Bayesian analysis for a generalized form of the software reliability model introduced by Moranda (1975) considering noninformative prior densities for the parameters. We also present two numerical illustrations, where we observe better fit of the generalized form of Moranda's model for the software reliability data, in comparison with the usual Moranda's model.

Keywords: software reliability data, generalized Moranda's model, Bayesian analysis.

1 Introduction

When the data on software failures are given in terms of the number of failures that occur in fixed time periods, Moranda (1975) proposed a Poisson process to describe the number of failures in each successive time period. With this de-eutrophication model, it is assumed that the intensity function (see for example, Singpurwalla and Wilson, 1994) should be constant in a particular period, but form a decreasing geometric sequence over them. Thus the number of failures in the ith time period is a homogeneous Poisson process with intensity function,

$$\lambda_1 = \lambda k_1^{i-1} \tag{1}$$

where $0 < k_1 < 1$ and $i = 1, 2, 3, \ldots$

By scaling, so that the time periods are of length 1, the distribution of the number of failures m_i in the ith time period is given by

$$P(X = m_i) = \frac{e^{-\lambda_i}\lambda_i^{m_i}}{m_i!} \quad (2)$$

where $m_i = 0, 1, 2, \ldots$

A generalization of Moranda's model could be given by the introduction of an additional parameter k_2 in the intensity function (1), that is,

$$\lambda_1 = \lambda_a k_1^{i^{k_2}} \quad (3)$$

Observe that with $\lambda_a = \lambda / k_1$ and $k_2 = 1$, we have the Moranda's model (1).

Also scaling, so that the time periods are of length 1, we obtain maximum likelihood estimators for the parameters λ_a, k_1 and k_2. In this way, considering $m_1, m_2, \ldots, m_n$ the observed number of failures during the first n time periods, the likelihood function for λ_a, k_1 and k_2 is given by

$$L_1(\lambda_a, k_1, k_2) \propto \lambda_a^{d_1} k_1^{d_2(k_2)} exp\{-\lambda_a A(k_1, k_2)\} \quad (4)$$

where $d_1 = \sum_{i=1}^{n} m_i, d_2(k_2) = \sum_{i=1}^{n} i^{k_2} m_i$ and $A(k_1, k_2) = \sum_{i=1}^{n} k_1^{i^{k_2}}$.

The maximum likelihood estimator for λ_a is given by $\widehat{\lambda}_a = d_1 / A(\widehat{k}_1, \widehat{k}_2)$ and the maximum likelihood estimators $\widehat{k}_1$ and $\widehat{k}_2$ could be obtained by maximizing $-d_1 ln A(k_1, k_2) + d_2(k_2) ln k_1$.

Since we are assuming a homogeneous Poisson process with intensity function (3), the times between failures $T_i, i = 1, 2, \ldots, n$, have independent exponential distributions. Thus, when the times between successive failures T_i are available and assuming the generalized form of Moranda's model (3) for the hazard functions of the exponential distributions with λ'_a, k'_1 and k'_2 in place of λ_a, k_1 and k_2, respectively, the likelihood function for λ'_a, k'_1 and k'_2 is given by

$$L_2(\lambda'_a, k'_1, k'_2) \propto \lambda'^{n}_a k'^{b_1(k'_2)}_1 exp\{-\lambda'_a B(k'_1, k'_2)\} \quad (5)$$

where $b_1(k_2') = \sum_{i=1}^{n} i^{k_2'}$ and $B(k_1', k_2') = \sum_{i=1}^{n} t_i k_1'^{i^{k_2'}}$.

In this case, the maximum likelihood estimator for λ_a' is given by $\widehat{\lambda}_a' = n/B(k_1', k_2')$ and the maximum likelihood estimators $\widehat{k}_1'$ and $\widehat{k}_2'$ could be obtained by maximizing $-nlnB(k_1', k_2') + b_1(k_2')lnk_1'$.

Approximate confidence intervals for λ_a', k_1' and k_2' could be obtained by using the usual asymptotic normality for the maximum likelihood estimators $\widehat{\lambda}_a', \widehat{k}_1'$ and $\widehat{k}_2'$ (see for example, Lawless, 1982).

2 A Bayesian Analysis Considering the Number of Failures in Fixed Time Periods

Assuming prior independence for the parameters of model (3), let us consider a noninformative prior density (see for example, Box and Tiao, 1973) for λ_a, k_1 and k_2 given by

$$\pi(\lambda_a, k_1, k_2) \propto 1/\lambda_a \tag{6}$$

where $\lambda_a > 0, 0 < k_1 < 1$ and $-\infty < k_2 < \infty$.

Combining the prior density (6) with the likelihood function (4), the joint posterior density for λ_a, k_1 and k_2 is given by

$$\pi_1(\lambda_a, k_1, k_2 | m_1, \ldots, m_n) \propto \lambda_a^{d_1 - 1} k_1^{d_2(k_2)} exp\{-\lambda_a A(k_1, k_2)\} \tag{7}$$

where $d_1, d_2(k_2)$ and $A(k_1, k_2)$ are given in (4).

Integrating out λ_a in (7), we get the joint marginal posterior density for k_1 and k_2, given by

$$\pi_1(k_1, k_2 | m_1, \ldots, m_n) \propto \frac{k_1^{d_2(k_2)}}{\{A(k_1, k_2)\}^{d_1}} \tag{8}$$

Marginal posterior densities or posterior moments for k_1 and k_2 could be obtained by using numerical or approximation methods for integration (see for example, Naylor and Smith, 1982; or Tierney and Kadane, 1986).

2.1 Marginal Posterior Density for k_1 Assuming $k_2 = 1$ Known

Assuming $k_2 = 1$ (usual Moranda's model) and a joint noninformative prior density for λ_a and k_1 proportional to $\lambda_a^{-1}, \lambda_a > 0$ and $0 < k_1 < 1$, the marginal posterior density for k_1 is given by

$$\pi_1(k_1|m_1, \ldots, m_n) \propto \frac{k_1^{d_2}}{\{A(k_1)\}^{d_1}} \tag{9}$$

where $0 < k_1 < 1, d_1 = \sum_{i=1}^n m_i, d_2 = \sum_{i=1}^n i m_i$ and $A(k_1) = \sum_{i=1}^n k_1^i$.

The mode of (9) is the solution of the polynomial equation,

$$\frac{\sum_{i=1}^n i\tilde{k}_1^i}{\sum_{i=1}^n \tilde{k}_1^i} = \frac{d_2}{d_1} \tag{10}$$

2.2 Predictive Density for m_i Given $m_1, \ldots, m_{i-1}$

Assuming k_1 and k_2 known in (3), the predictive density for the number of failures m_i given $m_1, m_2, \ldots, m_{i-1}, i = 2, 3, \ldots$, is given by

$$f_1(m_i|m_1, \ldots, m_{i-1}) = \int_0^\infty f_1(m_i|\lambda_a)\, \pi_1(\lambda_a|m_1, \ldots, m_{i-1})\, d\lambda_a \tag{11}$$

where $f_1(m_i|\lambda_a) = e^{-\lambda_i}\lambda_i^{m_i}/m_i!, m_i = 0, 1, 2, \ldots; \lambda_i$ is given by (3) and $\pi_1(\lambda_a|m_1, \ldots, m_{i-1})$ is the posterior density for λ_a.

Also considering a noninformative prior density for λ_a proportional to λ_a^{-1}, we have from (11),

$$f_1(m_i|m_1,\ldots,m_{i-1}) \propto$$
$$\propto \frac{k_1^{m_i i^{k_2}}}{m_i!}\int_0^\infty \lambda_a^{d_1(i)+m_i-1} exp\left\{-\lambda_a\left[k_1^{i^{k_2}} + A_i(k_1,k_2)\right]\right\} d\lambda_a \tag{12}$$

where $d_1(i)=\sum_{j=1}^{i-1} m_j$ and $A_i(k_1,k_2)=\sum_{j=1}^{i-1} k_1^{j^{k_2}}$.

That is,

$$f_1(m_i|m_1,\ldots,m_{i-1}) = \frac{c_1\theta_i^{m_i}\Gamma\left[d_1(i)+m_i\right]}{m_i!} \tag{13}$$

where $\theta_i = k_1^{i^{k_2}}/[k_1^{i^{k_2}} + A_i(k_1,k_2)], \Gamma(x)$ is the gamma function and c_1 is the normalizing constant given by

$$c_1^{-1} = \sum_{m_i=0}^{\infty} \frac{\theta_i^{m_i}\Gamma\left[d_1(i)+m_i\right]}{m_i!}.$$

Observe (from $\Gamma(x+1)=x\Gamma(x)$) that the normalizing constant c_1 given in (13) could be written in the form,

$$c_1^{-1} = \Gamma\left[d_1(i)\right]\left\{1+\frac{\theta_i}{1!}d_1(i)+\frac{\theta_i^2}{2!}d_1(i)\left(d_1(i)+1\right)+\ldots\right\}$$

That is,

$$c_1^{-1} = \Gamma\left[d_1(i)\right]\sum_{j=0}^{\infty}\binom{d_1(i)+j-1}{j}\theta_i^j.$$

Since,

$$\sum_{j=0}^{\infty}\binom{d_1(i)+j-1}{j}\theta_i^j = \sum_{k=d_1(i)}^{\infty}\binom{k-1}{k-d_1(i)}\theta_i^{k-d_1(i)}$$

(see for example, Abramowitz and Stegun, 1972), we have $c_1^{-1}=\Gamma[d_1(i)](1-\theta_i)^{-d_1(i)}$.

Thus,

$$f_1(m_i|m_1,\ldots,m_{i-1}) = \frac{\theta_i^{m_i}(1-\theta_i)^{d_1(i)}\Gamma[d_1(i)+m_i]}{m_i!\Gamma[d_1(i)]} \tag{14}$$

where $m_i = 0, 1, 2, \ldots$

The prediction mean value for m_i given $m_1, \ldots, m_{i-1}$ is,

$$E(m_i|m_1,\ldots,m_{i-1}) = \frac{k_1^{ik_2}\sum_{j=1}^{i-1} m_j}{\sum_{j=1}^{i-1} k_1^{jk_2}}. \tag{15}$$

3 A Bayesian Analysis Considering the Times Between Failures

Considering the noninformative prior density (6) and the likelihood function (5) when we have the times between successive failures $t_i, i = 1, 2, \ldots, n$, the joint posterior density for λ'_a, k'_1 and k'_2 is given by

$$\pi_2(\lambda'_a, k'_1, k'_2|t_1,\ldots,t_n) \propto \lambda_a'^{n-1} k_1'^{b_1(k'_2)} exp\{-\lambda'_a B(k'_1, k'_2)\} \tag{16}$$

where $b_1(k'_2)$ and $B(k'_1, k'_2)$ are given in (5).

The joint marginal posterior density for k'_1 and k'_2 is given by

$$\pi_2(k'_1, k'_2|t_1,\ldots,t_n) \propto \frac{k_1'^{b_1(k'_2)}}{\{B(k'_1, k'_2)\}^n}. \tag{17}$$

Assuming $k'_2 = 1$ (the usual Moranda model (1)) and a noninformative prior density for λ'_a and k'_1 proportional to $\lambda_a'^{-1}, \lambda'_a > 0$ and $0 < k'_1 < 1$, the marginal posterior density for k'_1 is given by

$$\pi_2(k'_1|t_1,\ldots,t_n) \propto \frac{k_1'^{b_1}}{\{B(k'_1)\}^n}, \tag{18}$$

where $b_1 = \sum_{i=1}^{n} i$ and $B(k_1') = \sum_{i=1}^{n} t_i k_1'^i$.

3.1 Predictive Density for t_i Given $t_1, \ldots, t_{i-1}$

Assuming k_1' and k_2' known in (3), the predictive density for t_i given $t_1, \ldots, t_{i-1}$ is given by

$$f_2(t_i | t_1, \ldots, t_{i-1}) = \int_0^\infty f_2(t_i | \lambda_a') \pi_2(\lambda_a' | t_1, \ldots, t_{i-1}) \, d\lambda_a' \tag{19}$$

where $f_2(t_i | \lambda_a') = \lambda_a' k_1'^{i^{k_2'}} exp\{-\lambda_a' k_1'^{i^{k_2'}} t_i\}, t_i > 0$ and $\pi_2(\lambda_a' | t_1, \ldots, t_{i-1})$ is the posterior density for λ_a'.

With a noninformative prior density proportional to $\lambda_a'^{-1}$, the predictive density (19) for t_i is given by

$$f_2(t_i | t_1, \ldots, t_{i-1}) \propto \int_0^\infty \lambda_a'^{i-1} exp\left\{-\lambda_a' \left[k_1'^{i^{k_2'}} t_i + B_i(k_1', k_2')\right]\right\} d\lambda_a' \tag{20}$$

where $B_i(k_1', k_2') = \sum_{j=1}^{i-1} t_j k_1'^{j^{k_2'}}$.

That is,

$$f_2(t_1 | t_1, \ldots, t_{i-1}) = \frac{c_2}{\left\{k_1'^{i^{k_2'}} t_i + B_i(k_1', k_2')\right\}^i}, \tag{21}$$

where $t_i > 0$ and $c_2^{-1} = \int_0^\infty \{k_1'^{i^{k_2'}} t_i + B_i(k_1', k_2')\}^{-i} dt_i$.

4 Some Applications

4.1 A Data Set of Observed Number of Failures

In table 1, we have a data set introduced by Goel (1985) consisting of the observed number of failures per hour of a software tested for 25 hours.

CPU hour of testing	Number of failures per CPU hour	CPU hour of testing	Number of failures per CPU hour
1	27	14	5
2	16	15	5
3	11	16	6
4	10	17	0
5	11	18	5
6	7	19	1
7	2	20	1
8	5	21	2
9	3	22	1
10	1	23	2
11	4	24	1
12	7	25	1
13	2		

Table 1 - Data on software failures during system test

Considering Moranda's model with intensity function (1), the maximum likelihood estimators for k_1 and λ are given by $\widehat{k}_1 = 0.88285$ and $\widehat{\lambda} = 16.67257$.

Considering the generalized form of Moranda's model (3), the maximum likelihood estimators for λ_a, k_1 and k_2 are given by $\widehat{\lambda}_a = 252.5160, \widehat{k}_1 = 0.1000$ and $\widehat{k}_2 = 0.2480$.

In table 2, we have the maximum likelihood estimators for the intensity functions (1) and (3), respectively. We observe different inference results considering (1) and (3).

i	m_i	$\widehat{\lambda}_i$ From (1)	$\widehat{\lambda}_i$ From (3)	i	m_i	$\widehat{\lambda}_i$ From (1)	$\widehat{\lambda}_i$ From (3)
1	27	16.6726	25.5041	14	5	3.3000	2.9944
2	16	14.7193	16.5280	15	5	2.9134	2.7721
3	11	12.9949	12.3567	16	6	2.5721	2.5759
4	10	11.4725	9.8670	17	0	2.2708	2.4017
5	11	10.1285	8.1922	18	5	2.0048	2.2461
6	7	8.9419	6.9818	19	1	1.7699	2.1063
7	2	7.8944	6.0638	20	1	1.5625	1.9801
8	5	6.9695	5.3427	21	2	1.3795	1.8657
9	3	6.1530	4.7613	22	1	1.2179	1.7616
10	1	5.4322	4.2825	23	2	1.0752	1.6665
11	4	4.7958	3.8815	24	1	0.9492	1.5794
12	7	4.2340	3.5410	25	1	0.8380	1.4993
13	2	3.7379	3.2484				

Table 2 - Maximum likelihood estimators for $\lambda_i, i = 1, 2, \ldots, 25$.

Assuming the noninformative prior density (6) for the parameters λ_a, k_1 and k_2, we get Bayesian estimators for k_1 and k_2 from the joint marginal posterior density (8). The mode of (8) is given by $\tilde{k}_1 = 0.1000$ and $\tilde{k}_2 = 0.2480$.

Assuming $k_2 = 1$ (Moranda's model (1)), the mode of the marginal posterior density (9) for k_1 is given by $\tilde{k}_1 = 0.88284$ (see (10)).

In table 3, we have the mean prediction values for m_i (see (15)) considering the predictive density (14) with $k_1 = 0.88284$ and $k_2 = 1$ (Moranda's model) and $k_1 = 0.1000$ and $k_2 = 0.2480$ (generalized Moranda's model).

We also have in table3, in parenthesis, the sum of relative prediction errors, given by

$$SRPE = \sum_{i=2}^{25} \frac{|m_i - E(m_i/m_1, \ldots, m_{i-1})|}{E(m_i/m_1, \ldots, m_{i-1})} \tag{22}$$

We observe in table 3, better predictions considering the generalized form of Moranda's model with intensity function (3) for the software data set of table 1.

i	m_i	$E(m_i/m_1,\ldots,m_{i-1})$ Using (3) with $k_1 = 0.88284, k_2 = 1$	$E(m_i/m_1,\ldots,m_{i-1})$ Using (3) with $k_1 = 0.1000, k_2 = 0.2480$
2	16	23.8367	17.5309
3	11	17.8000	12.6761
4	10	13.9570	9.8310
5	11	11.6043	8.1941
6	7	10.1629	7.2630
7	2	8.6390	6.2944
8	5	7.0682	5.2814
9	3	6.0984	4.6960
10	1	5.2085	4.1509
11	4	4.3992	3.6420
12	7	3.8678	3.3374
13	2	3.5207	3.1758
14	5	3.0643	2.8975
15	5	2.7533	2.7360
16	6	2.4787	2.5947
17	0	2.2533	2.4903
18	5	1.9533	2.2838
19	1	1.7668	2.1898
20	1	1.5505	2.0409
21	2	1.3630	1.9087
22	1	1.2092	1.8045
23	2	1.0659	1.6977
24	1	0.9476	1.6134
25	1	0.8369	1.5254
SRPE		(13.051)	(11.542)

Table 3 - Mean prediction values for m_i considering models (1) and (3)

A simple procedure to compare models (1) and (3) (see for example, Box, 1980; or Mazzuchi and Soyer, 1988) for the software failure data is to consider the ratio

$$R = \prod_{i=2}^{n} \frac{f_1\left(m_i|m_1,\ldots,m_{i-1}; k_1 = 0.1000, k_2 = 0.2480\right)}{f_1\left(m_i|m_1,\ldots,m_{i-1}; k_1 = 0.88284, k_2 = 1\right)} \tag{23}$$

where $f_1(m_i|m_1,\ldots,m_{i-1}; k_1, k_2)$ is the predictive density (14).

With the software data of table 1, we get $R = 174.194$, which indicates that model (3) with $k_1 = 0.1000$ and $k_2 = 0.2480$ is preferable.

Equation (23) provides a global measure for comparing the two models. An alternative procedure is to use a local measure at each stage, given by

$$R_i = \frac{f_1(m_i|m_1,\ldots,m_{i-1}; k_1 = 0.1000, k_2 = 0.2480)}{f_1(m_i|m_1,\ldots,m_{i-1}; k_1 = 0.88284, k_2 = 1)} \quad (24)$$

In table 4, we observe in general, a better fit of the generalized Moranda's model (3) for the software failure data of table 1.

i	R_i	i	R_i
2	2.17827	14	0.89585
3	2.77265	15	0.98624
4	1.65583	16	1.16775
5	0.69353	17	0.79256
6	1.63097	18	1.55814
7	4.79791	19	0.81453
8	1.35867	20	0.80852
9	1.79111	21	1.13453
10	2.20416	22	0.82475
11	1.00149	23	1.34474
12	0.61620	24	0.87613
13	1.14384	25	0.91641

Table 4 - Ratio of predictive densities for models (3) and (1).

4.2 A Data Set of Times Between Failures

In table 5, we have a data set consisting of the observed times between failures of a software.

Considering the generalized form of Moranda's model with hazard function (3) for an exponential distribution for the software times between failures $t_i, i = 1, 2, \ldots, 30$, we get from (5) the maximum likelihood estimators for λ'_a, k'_1 and k'_2 given by $\widehat{\lambda}'a = 1.91326, \widehat{k}'_1 = 0.102$ and $\widehat{k}'_2 = 0.300$, respectively.

Considering the hazard function (3) with $k'_2 = 1$ (usual Moranda's model), the maximum likelihood estimators for λ'_a and k'_1 are given by $\widehat{\lambda}'_a = 0.07851$ and $\widehat{k}'_1 = 0.89$.

5, 10, 12, 17, 20, 26, 30, 40, 43, 50, 58, 65, 75, 80, 90,100,110,121,130,145,160,170,180,195,215, 230,242,265,280,298

Table 5 - Data on software failures (in days) during system test.

In table 6, we have the maximum likelihood estimators for the mean lifetimes $\theta_i = 1/\lambda_i, i = 1, 2, \ldots, 30$ considering models (1) and (3). We observe very different inference results considering models (1) and (3).

i	t_i	$\widehat{\theta}_i$ From (3)	$\widehat{\theta}_i$ From (1)	i	t_i	$\widehat{\theta}_i$ From (3)	$\widehat{\theta}_i$ From (1)
1	5	5.124	14.311	16	100	99.052	82.193
2	10	8.685	16.080	17	110	109.061	92.352
3	12	12.493	18.068	18	121	119.614	103.766
4	17	16.631	20.301	19	130	130.728	116.591
5	20	21.132	22.810	20	145	142.415	131.001
6	26	26.018	25.629	21	160	154.691	147.192
7	30	31.304	28.797	22	170	167.569	165.385
8	40	37.004	32.356	23	180	181.066	185.825
9	43	43.130	36.355	24	195	195.196	208.793
10	50	49.697	40.848	25	215	209.974	234.598
11	58	56.716	45.897	26	230	225.416	263.594
12	65	64.200	51.570	27	242	241.538	296.173
13	75	72.163	57.944	28	265	258.355	332.778
14	80	80.617	65.105	29	280	275.885	373.908
15	90	89.576	73.152	30	298	294.143	420.122

Table 6 - Maximum likelihood estimators for $\theta_i = 1/\lambda_i, i = 1, 2, \ldots, 30$

The mean of the predictive density (21) for t_i given $t_1, \ldots, t_{i-1}$, is,

$$E\left(t_i|t_1, \ldots, t_{i-1}\right) = \frac{\int_0^\infty t_i \left[k_1'^{i^{k_2'}} t_i + B_i\left(k_1', k_2'\right)\right]^{-i} dt_i}{\int_0^\infty \left[k_1'^{i^{k_2'}} t_i + B_i\left(k_1', k_2'\right)\right]^{-i} dt_i} \tag{25}$$

where $B_i(k_1', k_2')$ is defined in (20).

In table 7, we have numerically obtained values of (25) assuming k_1' and k_2' known. We also have in parenthesis in table 7, the sums of relative prediction errors with $E(t_i|t_1, \ldots, t_{i-1})$ in place of $E(m_i|m_1, \ldots, m_{i-1})$ in (22).

In table 7, we have numerically obtained values of (25) assuming k_1' and k_2' known. We also have in parenthesis in table 7, the sums of relative prediction errors with $E(t_i|t_1,\ldots,t_{i-1})$ in place of $E(m_i|m_1,\ldots,m_{i-1})$ in (22).

We observe better prediction values considering the generalized form of Moranda's model (3) with $k_1' = 0.102$ and $k_2' = 0.300$ assumed known, especially for large values of i.

i	t_i	$E(t_i\|t_1,\ldots,t_{i-1})$ Using (3) with $k_1' = 0.89, k_2' = 1$	$E(t_i\|t_1,\ldots,t_{i-1})$ Using (3) with $k_1' = 0.102, k_2' = 0.300$
3	12	17.550	26.574
4	17	16.607	25.674
5	20	18.802	28.950
6	26	21.462	32.890
7	30	25.134	37.913
8	40	29.152	43.258
9	43	34.497	49.898
10	50	39.953	56.481
11	58	46.147	63.637
12	65	53.182	71.396
13	75	60.961	79.597
14	80	69.812	88.489
15	90	79.319	97.603
16	100	89.982	107.325
17	110	101.853	117.637
18	121	115.011	128.496
19	130	129.624	139.947
20	145	145.677	151.867
21	160	163.636	164.555
22	170	183.657	178.007
23	180	205.622	191.926
24	195	229.727	206.342
25	215	256.418	221.425
26	230	286.175	237.410
27	242	319.034	254.089
28	265	355.125	271.266
29	280	395.271	289.431
30	298	439.473	308.232
SRPE		(4.7586)	(3.0460)

Table 7 - Mean prediction values for t_i considering models (1) and (3).

5 Concluding Remarks

The use of generalized Moranda's model (3) could be an useful alternative to analyse software reliability data, since in many applications, we could have difficulties to get good fit of software reliability data for the existing software reliability models (see for example, Singpurwalla and Wilson, 1994).

It is interesting to observe that we could consider any other choice of prior density, especially when we have expert opinion on the parameters of the model.

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