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N O T A S

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Abstract

The purpose of this paper is to consider the Bayesian estimation of the slope in a simple regression model with measurement errors via the orthogonal parametrization. The influence of the prior form and of the measurement errors on the posterior mean and variance is studied for the normal and Student-t priors, respectively.

1 Introduction

The classical simple regression model assumes that the independent variable is defined by

$$\begin{aligned} Y_i &= \alpha + \beta x_i + \epsilon_i. \\ i &= 1, \ldots, n \end{aligned} \tag{1}$$

where $(x_1, \ldots, x_n)$ is fixed in a repeated sampling and ϵ_i are independent $N(0, \sigma_\epsilon^2)$ random variables. It is assumed that x_i is measured without error. However, in practice, this assumption is often violated . There is a lot of work on the problem of parameter estimation when the x_i contains errors, see for example, Fuller (1987) for references.

In this paper, using the Bayesian approach with normal and Student-t priors, we try to solve the estimation problem by working with a conditional orthogonal likelihood model introduced by Rodrigues and Cordani (1990). As in Rodrigues and Cordani (1990), we shall studied models of type (1), with $\alpha = 0$, where instead of observing x_i one observes the sum

$$X_i = x_i + u_i. \tag{2}$$

The usual assumption is that

$$(x_i, u_i, e_i) \sim N[(0,0,0), \text{diag}(\sigma_x^2, \sigma_u^2, \sigma_e^2)]. \tag{3}$$

From (1)-(3), (X_i, Y_i) is distributed as a bivariate normal and the conditional distribution of Y_i, given X_i, is normal with mean θX_i and variance $\sigma_u^2 K_x \beta^2 + \sigma_e^2$, where

$$K_x = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2} \quad \text{and} \tag{4}$$

$$\theta = K_x \beta. \tag{5}$$

From (5), we see that the regression coefficient β was attenuated by the factor K_x. This factor, is called the reliability of X_i in many areas, see for example, Fuller (1987). Working with the orthogonal parametrization suggested by Rodrigues and Cordani (1990) and informative priors for β we obtain, when the reliability ratio K_x is known, the posterior distribution, the posterior mean and the posterior variance of β. Following a recent paper by Pericchi and Smith (1992), the influence of the normal and Student-t prior means and the measurement errors on the the posterior mean and variance is discussed in details. The results of this paper provide some aspects of robust Bayesian analysis for models with measurement errors. Related results, without measurement errors, have been discussed by Lindley (1968) and O'Hagan (1979) for the Student-t case, by Pericchi and Smith (1992) for double exponential and Student-t cases and by Pericchi and Walley (1991). Our robust Bayesian analysis is more general than the robust problem studied by Pericchi and Smith (1992) in the sense that we have a simple conditional regression model with unknown variance obtained by the orthogonal parametrization (Cox and Reid, 1987).

2 Bayesian Inference with normal prior and Known reliability ratio

In this section, our interest is to make a Bayesian inference about β under normal prior and the known reliability ratio. There are a number of situations, particulary in psychology and sociology, where the factor K_x is known. The Bayesian procedure is simplified by working with the orthogonal parametrization (Cox and Reid, 1987)

$$\sigma^2 = \sigma_u^2 K_x \beta^2 + \sigma_\epsilon^2. \tag{6}$$

If $(X_1, Y_1), \ldots, (X_n, Y_n)$ are independently and identically distributed according to model (1)-(3), the conditional likelihood function in the new parametrization is

$$L(\beta, \sigma^2) \propto \sigma^{-\frac{n}{2}} \exp\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(Y_i - \beta K_x X_i)^2\} \tag{7}$$

It is well known that the maximum likelihood estimators of β and σ^2 are

$$\begin{aligned} \hat{\beta} &= K_x^{-1}\frac{\sum_{i=1}^{n} X_i Y_i}{\sum_{i=1}^{n} X_i^2} \quad \text{and} \\ \hat{\sigma^2} &= \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \hat{\beta} X_i)^2. \end{aligned} \tag{8}$$

The same result was obtained by Fuller (1987) using a different approach and a more general model. Our problem is then to analyse the hierarchical Bayesian regression model given by

$$\begin{cases} Y \mid Z, \beta, \phi \sim NM[Z\beta, \phi^{-1} I] \\ \beta \mid \phi \sim N[\mu, \phi^{-1}] \\ n_0 \sigma_0^2 \phi \sim \chi^{(2)}(n_0) \end{cases} \tag{9}$$

where $MN(\quad , \quad)$ means the multivariate normal distribution, $Y' = (Y_1, \ldots, Y_n)$, $Z' = (Z_1, \ldots, Z_n) = K_x(X_1, \ldots, X_n)$, $\phi = \frac{1}{\sigma^2}$ and I is the identity matrix. The parameters μ, n_0 and σ_0^2 are supposed to be known. The Bayesian regression model (9) consists of two-stage prior with unknown variance and is a particular case of a general hierarchical model introduced by Lindley and

Smith (1971). From (9), the joint prior and posterior distributions of (β, ϕ) and the marginal distribution of β given the data are , respectively, given by

$$
\begin{aligned}
\pi(\beta,\phi) &\propto \phi^{\frac{n_0+1}{2}-1}\exp\{-\frac{\phi}{2}[n_0\sigma_0^2+(\beta-\mu)^2]\}, \\
\pi(\beta,\phi \mid \text{data}) &\propto \phi^{\frac{n_0+n+1}{2}-1}\exp\{-\frac{\phi a_\beta}{2}\} \quad \text{and} \\
\pi(\beta \mid \text{data}) &\propto [1+\frac{h}{\nu}(\beta-\hat{\beta}_1)^2]^{-\frac{\nu+1}{2}}
\end{aligned} \tag{10}
$$

where

$$
\begin{aligned}
a_\beta = n_0\sigma_0^2 &+ (n-1)\hat{\sigma}^2+(1+\sum_{i=1}^{n} Z_i^2)(\beta-\hat{\beta}_1)^2+\frac{\sum_{i=1}^n Z_i^2}{1+\sum_{i=1}^n Z_i^2}(\mu-\hat{\beta})^2, \\
h &= \frac{\nu(1+\sum_{i=1}^n Z_i^2)}{n_0\sigma_0^2+(n-1)\hat{\sigma}^2+\frac{\sum_{i=1}^n Z_i^2}{1+\sum_{i=1}^n Z_i^2}(\mu-\hat{\beta})^2}, \\
\hat{\beta}_1 &= \frac{1}{1+\sum_{i=1}^n Z_i^2}\mu+\frac{\sum_{i=1}^n Z_i^2}{1+\sum_{i=1}^n Z_i^2}\hat{\beta} \quad \text{and} \\
\nu &= n_0+n.
\end{aligned} \tag{11}
$$

We see from (10), that the marginal posterior of β, given the data, is the Student-t posterior with scale h, degrees of freedom ν , location given by

$$E[\beta \mid \text{data}] = \hat{\beta}_1 \tag{12}$$

and the posterior variance given by

$$\text{Var}[\beta \mid \text{data}] = \frac{1}{n_0+n-2}\cdot\frac{\nu}{h}. \tag{13}$$

Notice from (11) that (12) is a weighted average of the known prior mean, μ, and the maximun likelihood estimator of β in (8), with weight for $\hat{\beta}$ inversely proportional to the variance of $\hat{\beta}$ and weight for μ inversely proportional to the prior variance of μ. Form (12) can be written as

$$\hat{\beta}_1 = E[\beta \mid \text{data}] = \hat{\beta}+\frac{1}{1+K_x^2\sum_{i=1}^n X_i^2}(\mu-\hat{\beta}). \tag{14}$$

As suggested by Pericchi and Smith (1992), form (14) provides insight into the influence of the prior mean on the posterior mean. Since $\mu - \hat{\beta}$ is unbounded for all choices of μ, then $E[\beta \mid \text{data}] - \hat{\beta}$ is unbounded. The discrepancies between the prior mean and the observed m.l.e. $\hat{\beta}$ can lead to unbounded departures of $E[\beta \mid \text{data}]$ from $\hat{\beta}$. The technical attractive property of our normal prior in model (9) is only of secondary importance compared to the influence of the prior mean on the posterior mean and variance. From the modern subjetive Bayesian point of view this influence of the normal prior is often unfortunate. See Lindley (1968) for further discussion. We shall show that this lack of robustness is not shared with the Student-t prior for β. A similar result was obtained by Pericchi an Smith (1992) for normal location model with known variance without measurements errors.

3 Approximate Posterior Moments from a One-Stage Student-t prior

A Student-t prior for β with location μ, scale ϕ and degrees of freedom n, denoted by $t(\beta \mid \mu, \phi)$, is a prior of the form

$$\pi(\beta) \propto \phi^{\frac{1}{2}}[1 + \phi\frac{(\beta - \mu)^2}{n}]^{-\frac{n+1}{2}}. \tag{15}$$

With this notation we have the following two-stage conditional Bayesian regression model:

$$\begin{cases} Y \mid Z, \beta, \phi \sim MN[Z\beta, \phi^{-1}I] \\ \beta \mid \phi \sim t[\beta, n, \mu, \phi] \\ n_0\sigma_o^2\phi \sim \chi^{(2)}(n_0) \end{cases}, \tag{16}$$

Using the well-known mixture form

$$t[\beta \mid n, \mu, \phi] = \int_0^\infty n(\beta \mid \mu, \frac{1}{\phi\lambda})g(\lambda \mid \frac{n}{2}, \frac{2}{n})d\lambda. \tag{17}$$

where $n(|)$ and $g(|)$ denote normal and gamma probability functions, respectively (see, for example, Berger (1985)), the joint posterior of (β, ϕ), given the data, based on (16) is

$$\pi(\beta,\phi\mid \text{data}) \;\propto\; \phi^{\frac{n_0+n}{2}-1}\left(\int n(\beta\mid \mu,\frac{1}{\phi\lambda})g(\lambda\mid \frac{n}{2},\frac{2}{n})d\lambda\right)\exp\{-\frac{\phi}{2}[n_0\sigma_0^2+(n-1)\hat{\sigma}^2+ \\ +\sum_{i=1}^{n} Z_i^2(\beta-\hat{\beta})^2]\} \qquad (18)$$

Integrating (18) with respect to ϕ and interchanging the order of integration, we easily see that,

$$\pi(\beta\mid \text{data}) \propto \int_0^\infty g(\lambda\mid \frac{n}{2},\frac{2}{n})\lambda^{\frac{1}{2}}[d_\lambda+(\lambda+\sum_{i=1}^{n} Z_i^2)(\beta-\hat{\beta}_1(\lambda))^2]^{-\frac{n_0+n+1}{2}}d\lambda, \qquad (19)$$

where

$$d_\lambda = n_0\sigma_0^2+(n-1)\hat{\sigma}^2+\frac{\lambda\sum_{i=1}^{n} Z_i^2}{\lambda+\sum_{i=1}^{n} Z_i^2}(\mu-\hat{\beta})^2 \qquad \text{and}$$

$$\hat{\beta}_1(\lambda) = \frac{1}{\lambda+\sum_{i=1}^{n} Z_i^2}[\lambda\mu+\sum_{i=1}^{n} Z_i^2\hat{\beta}]. \qquad (20)$$

To get an approximation for the integral (19), we will make use of the Laplace method (see Tierney and Kadane, 1986)) and Taylor series approximations. These are resumed in the following results:

LEMMA 1

If

$$(\mu-\hat{\beta})^2 \leq \frac{(\lambda+\sum_{i=1}^{n} Z_i^2)}{\lambda}(n_0+n+1)a_n,$$

then

$$d_\lambda^{-\frac{n_0+n+1}{2}} \doteq [n_0\sigma_0^2+(n-1)\hat{\sigma}^2]^{-\frac{n_0+n+1}{2}}\exp\{-\frac{(\mu-\hat{\beta})^2}{2(\frac{b_n}{\lambda}+a_n)}\}, \qquad (21)$$

where

$$b_n = \frac{n_0\sigma_0^2+(n-1)\hat{\sigma}^2}{n_0+n+1} \qquad \text{and}$$

$$a_n = \frac{b_n}{\sum_{i=1}^{n} Z_i^2}.$$

Proof:

Using the fact that $ln(1+x) \doteq x$, for $|x| < 1$, we easily obtain the result (21).

It will be convenient for the next Lemma to introduce the notation

$$h(\frac{b_n}{\lambda} + z) = \exp\{-\frac{(\mu - \hat{\beta})^2}{2(\frac{b_n}{\lambda} + z)}\} \tag{22}$$

where $z = a_n$. Using series approximation we will get the next Lemma.

LEMMA 2

$$h(\frac{bn}{\lambda} + a_n) = h(\frac{b_n}{\lambda})[1 + \frac{\lambda^2(\mu - \hat{\beta})^2}{2b_n \sum_{i=1}^n Z_i^2} + O(n^{-2})]. \tag{23}$$

Proof:

Expanding $h(\frac{b_n}{\lambda} + z)$ in (22) around $z = 0$, we obtain the approximation

$$h(\frac{b_n}{\lambda} + a_n) = h(\frac{b_n}{\lambda}) + h'(\frac{b_n}{\lambda})a_n + O(n^{-2}). \tag{24}$$

From (24), the result (23) is easily obtained.

LEMMA 1 and 2 will be useful to obtain an approximation to integral (19). So, we return to integral (19) and using LEMMA 1 and 2 and retaining only the two first terms of (24) , we have

$$\pi(\beta \mid \text{data}) \propto \int_0^\infty g(\lambda \mid \frac{n}{2}, \frac{2}{n})\lambda^{\frac{1}{2}}\exp\{-\frac{\lambda}{2b_n}(\mu - \hat{\beta})^2\}[1 + \\ + \frac{(\lambda + \sum_{i=1}^n Z_i^2)(\beta - \hat{\beta}_1(\lambda))^2}{d_\lambda}]^{-\frac{n_0+n+1}{2}}[1 + \frac{\lambda^2(\mu - \hat{\beta})^2}{2b_n \sum_{i=1}^n Z_i^2}]d\lambda. \tag{25}$$

Now, the integral in (25) can be written as

$$\pi(\beta \mid \text{data}) \propto \int_0^\infty \exp\{-nh(\lambda)\}f(\lambda, \beta)d\lambda \tag{26}$$

where

$$\begin{aligned} -nh(\lambda) &= \frac{(n-1)}{2} ln(\lambda) - \frac{n\lambda}{2} - \frac{\lambda(\mu-\hat{\beta})^2}{2b_n} \qquad \text{and} \\ f(\lambda,\beta) &= [1+\frac{\lambda^2(\mu-\hat{\beta})^2}{2b_n\sum_{i=1}^n Z_i^2}][1+\frac{(\lambda+\sum_{i=1}^n Z_i^2)(\beta-\hat{\beta}_1(\lambda))^2}{d_\lambda}]^{-\frac{n_0+n+1}{2}} \qquad (27) \end{aligned}$$

The Laplace method states that (25) is given by

$$\pi(\beta \mid \text{data}) \propto [1+\frac{(\hat{\lambda}+\sum_{i=1}^n Z_i^2)}{d_{\hat{\lambda}}}(\beta-\hat{\beta}_1(\hat{\lambda}))^2]^{-\frac{n_0+n+1}{2}} \qquad (28)$$

where $\hat{\lambda}$ is the solution of

$$\frac{\partial(-nh(\lambda))}{\partial\lambda} = 0,$$

that is, from the above equations we have that

$$\hat{\lambda} = \frac{(n-1)b_n}{(\mu-\hat{\beta})^2 + nb_n}. \qquad (29)$$

We note from (20) and (28) that the marginal posterior of β is the Student-t

,

$t(\beta \mid \nu, \mu, h^*)$, where

$$\begin{aligned} \hat{\beta}_1(\hat{\lambda}) = E[\beta \mid \text{data}] &= \frac{1}{\hat{\lambda}+\sum_{i=1}^n Z_i^2}[\hat{\lambda}\mu + \sum_{i=1}^n Z_i^2\hat{\beta}], \qquad (30) \\ h^* &= \frac{\nu(\hat{\lambda}+\sum_{i=1}^n Z_i^2)}{d_{\hat{\lambda}}}, \qquad \nu = n_0 + n \qquad \text{and} \\ \text{Var}[\beta \mid \text{data}] &= \frac{\nu}{(\nu-2)h^*}. \end{aligned}$$

Combining equations (29) and (30), we obtain

$$E[\beta \mid \text{data}] - \hat{\beta} = \frac{(n-1)b_n(\mu-\hat{\beta})}{b_n[(n-1)+nK_x^2\sum_{i=1}^n X_i^2] + K_x^2\sum_{i=1}^n X_i^2(\mu-\hat{\beta})^2} \qquad (31)$$

and

$$\text{Var}\quad [\beta \mid \text{data}] = \frac{n_0\sigma_0^2 + (n-1)\hat{\sigma}^2 + \frac{\hat{\lambda}\sum_{i=1}^n Z_i^2}{\hat{\lambda}+\sum_{i=1}^n Z_i^2}(\mu - \hat{\beta})^2}{(\nu-2)(\hat{\lambda} + \sum_{i=1}^n Z_i^2)} \tag{32}$$

$$= \frac{(\nu+1)b_n[(\mu-\hat{\beta})^2 + nb_n] + \frac{(n-1)b_n \sum_{i=1}^n Z_i^2(\mu-\hat{\beta})^2[(\mu-\hat{\beta})^2+nb_n]}{(n-1)b_n + \sum_{i=1}^n Z_i^2[(\mu-\hat{\beta})^2+nb_n]}}{(\nu-2)\{b_n(n-1) + \sum_{i=1}^n Z_i^2[(\mu-\hat{\beta})^2 + nb_n]\}}.$$

Numerical investigations show that equation (30) is a good approximation to $E[\beta \mid \text{data}]$. Table 1 presents comparative values of approximation (30) and the numerically calculated $E[\beta \mid \text{data}]$.

Table 1: Values approximation (30) for
$n = n_0 = 10,\quad b_n = \sigma_0^2 = \hat{\sigma}^2 = 50,\quad \hat{\beta} = 0,\quad \sum_{i=1}^n X_i = 10$

μ	$K_x = 1$		$K_x = 0.8$		$K_x = 0.5$	
	$E[\beta \mid \text{data}]$	$\hat{\beta}_1$	$E[\beta \mid \text{data}]$	$\hat{\beta}_1$	$E[\beta \mid \text{data}]$	$\hat{\beta}_1$
0.5	0.1903	0.1887	0.1563	0.1544	0.1078	0.1054
5.0	0.1869	0.1864	0.1533	0.1524	0.1058	0.1038
10.0	0.1789	0.1809	0.1462	0.1477	0.1002	0.1000

If one is to use a Bayesian approach to a problem it is important to known the implications of a particular choice of prior distribution. If these implications are unacceptable then the prior we choose is unacceptable. Beale and Lindley, in the discussion of Lindley (1968), agree that the discrepancy between $E[\beta \mid \text{data}]$ and $\hat{\beta}$ proportional to the discrepancy between $\hat{\beta}$ and the prior mean μ is often a undesirable implication. This undesirable property of prior distribution arises because there is a feeling that a discrepancy between the prior mean and the data can somehow " discredit" the prior value μ. In this case, the Bayesian must find an appropriate prior such that the discrepancy between the posterior mean and the m.l.e. $\hat{\beta}$ is closed to zero. Lindley carried out an approximate analysis which indicated that this undesirable behaviour would be avoided if the prior distribution had the form of Student-t, and this is confirmed by the results which follow.

Figure 1 and 2 show the qualitative forms of dependence of the posterior mean and variance on discrepancy between the prior mean regression coefficient parameter and the observed m.l.e. $\hat{\beta}$, for $K_x = 1, 0.8, 0.5$; $b_n = 1$; $n = n_0 = 10$; $X_i = 1$, $1 = 1, \ldots, 10$ and $\nu = 20$. Figure 3 and 4

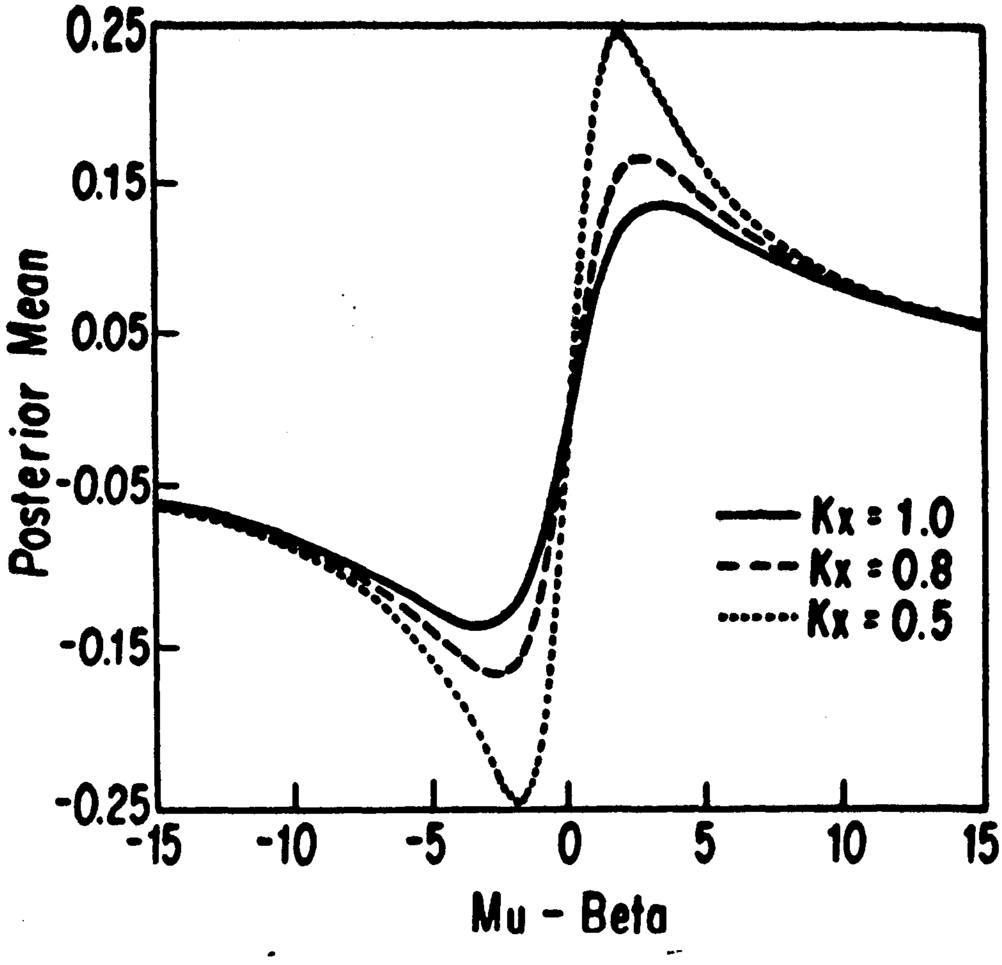

Figure 1: Student prior: $n = n_0 = 10$, $b_n = 1$

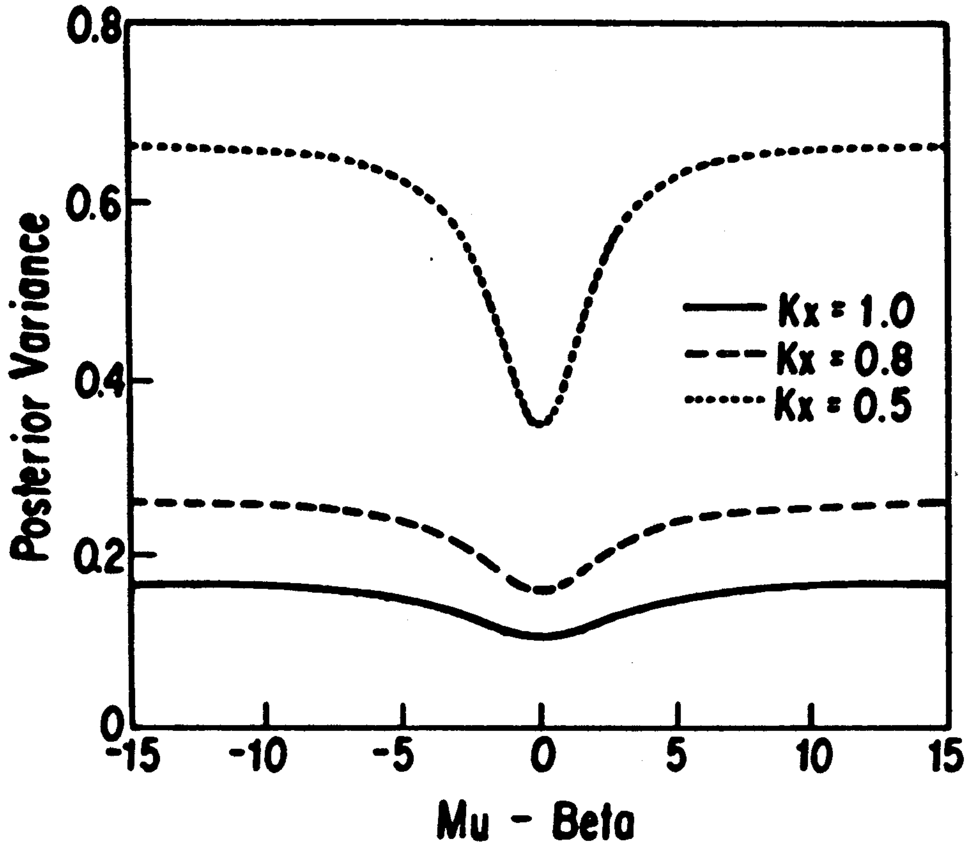

Figure 2: Student prior: $n = n_0 = 10, \quad b_n = 1$

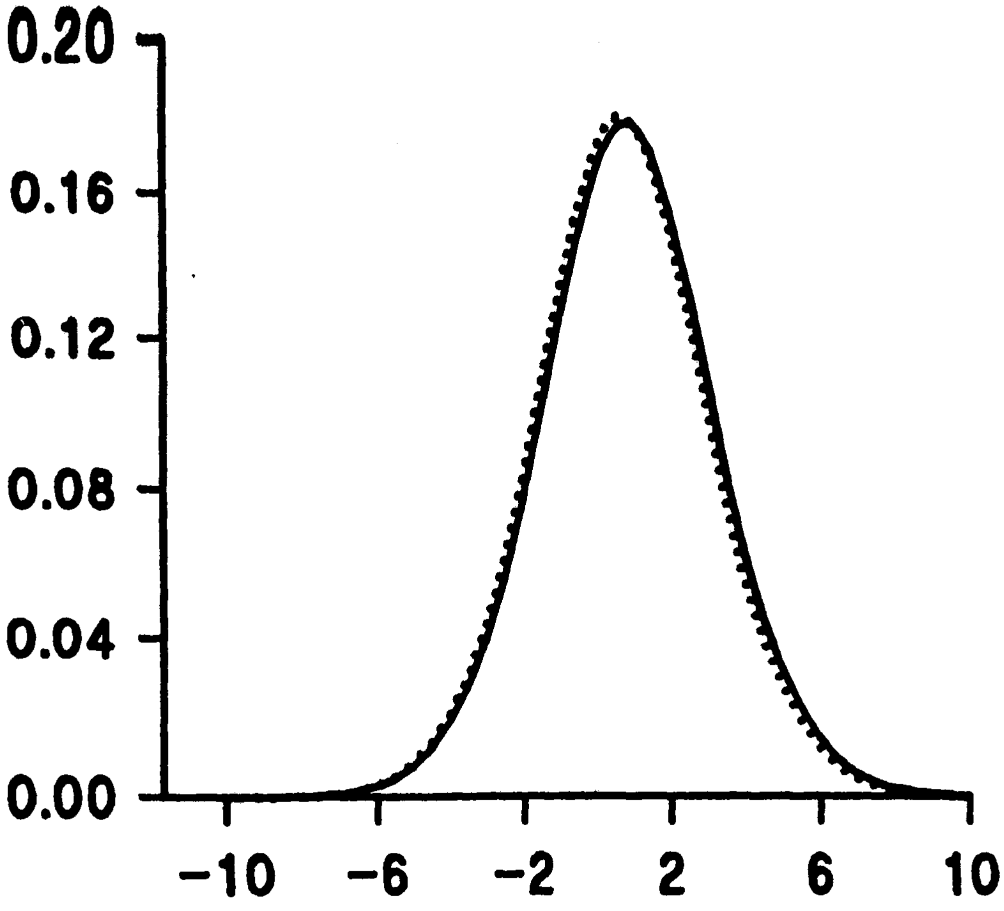

Figure 3: dashed line: Laplace approximation (28); solid line: exact integral (25) for $k_x = 1.0$

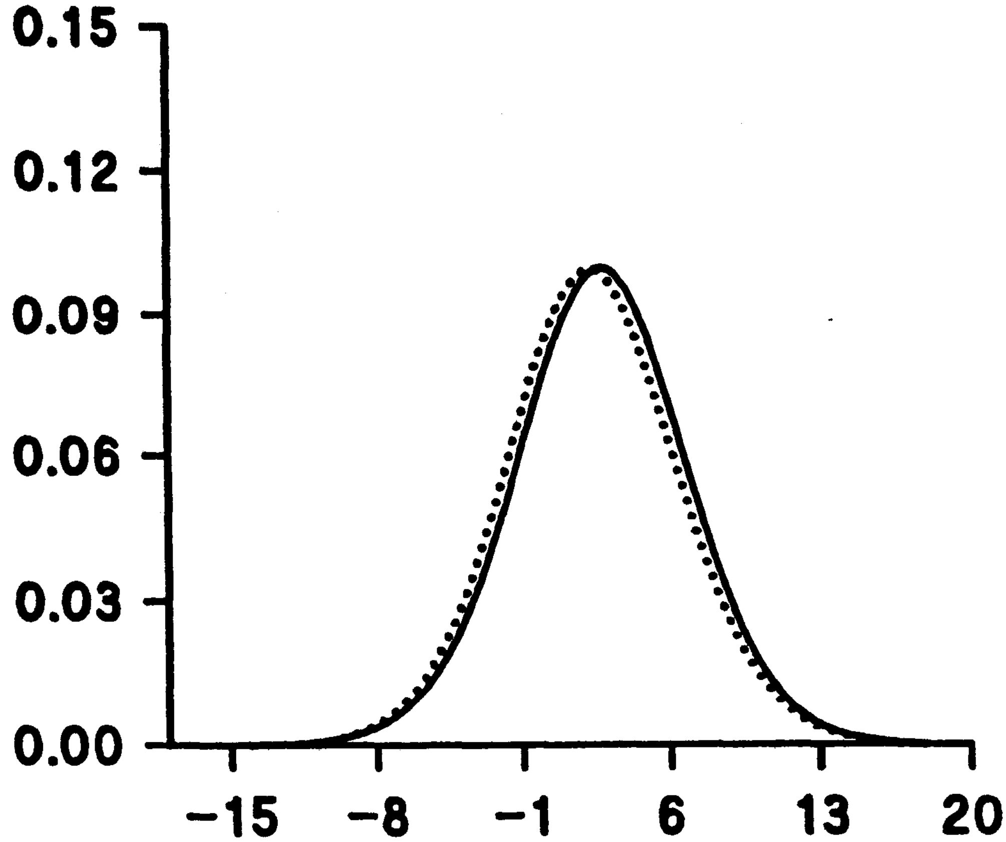

Figure 4: dashed line: Laplace approximation (28); solid line: exact integral (25) for $K_x = 0.5$

display the Laplace approximation (28) and the numerically calculated integral (25) for $n = n_0 = 10$, $b_n = 50$, $\sigma_0 = 50$, $\hat{\sigma}^2 = 50$, $\hat{\beta} = 0$, $\mu = 10$, $\sum_{i=1}^{n} X_i = 10$ and $K_x = 1.0$ and $K_x = 0.5$, respectively. The Laplace approximations are seen to be remarkably good.

The use of Student-t prior to provide a robust analysis for normal location parameter was suggested by many authors; see, for example Pericchi and Smith (1992), Dawid (1973), West (1981), Lindley (1968), O'Hagan (1979) and Smith (1983).

For $K_x = 1$, the dependence is similar to the dependence found by Pericchi and Smith (1992). However, the influence of the prior mean on the posterior mean, close to zero, is increasing as K_x decreases. Figure 1 and 2 show that is important to have in mind the measurement errors when studying Bayesian robustness analysis. It is interesting to observe the form of Figure 1, that is, when $| \mu - \hat{\beta} |$ is increasing the Student-t prior provides a kind of increasing " discounting" or trimming. The posterior variance (Figure 2) is monotonic in $| \mu - \hat{\beta} |$. These qualitatives aspects of the posterior mean and variance can provide some guidance in Bayesian analysis.

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