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Extreme Value Regression Models: An Useful Reparametrization For The Survival Function

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Abstract

In this paper, we consider some aspects of accurate approximate inferences for the survival function at a specified time t_0 considering extreme value regression models using a modified form of reparametrization proposed by Guerrero and Johnson (1982) and exploring a nonnormality measure for likelihood functions and posterior densities introduced by Kass and Slate (1982). We illustrate the proposed methodology considering a lifetime data set with two treatments introduced by Lee (1980).

Keywords: extreme value distribution, regression models, survival function, reparametrization.

1 Introduction

In the medical lifetime data analysis or in the reliability studies in engineering, usually the researchers have interest to get inferences on the survival function of a patient at a specified time t_0. In these situations, it is common the presence of one or more factors which could affect the lifetimes of the unities, and it is required the use of a regression model.

In situations where we have enough information to fit an appropriate parametrical model for the survival time T, an useful distribution used in many applications is given by the Weibull distribution (see for example, Lawless, 1982) with density,

$$f(t;\alpha,\delta) = \frac{\delta}{\alpha}\left(\frac{t}{\alpha}\right)^{\delta-1} exp\left\{-\left(\frac{t}{\alpha}\right)^{\delta}\right\} \tag{1}$$

where $t > 0; \delta > 0$, and $\alpha > 0$ are shape and scale parameters, respectively.

Assuming that the vector of covariates $\underset{\sim}{x} = (x_1, x_2, \ldots, x_p)'$ affect only the scale parameter α, we have a proportional hazards model for the survival data (see for example, Kalbfleisch and Prentice, 1980).

Considering a logarithm transformation for the survival time T, we have from (1), an extreme value distribution for $Y = log(T)$ with density,

$$f\left(y|\mu\left(\underset{\sim}{x}\right),\sigma\right) = \frac{1}{\sigma}exp\left\{\frac{y-\mu\left(\underset{\sim}{x}\right)}{\sigma} - exp\left(\frac{y-\mu\left(\underset{\sim}{x}\right)}{\sigma}\right)\right\} \tag{2}$$

where $\mu(\underset{\sim}{x}) = log\alpha(\underset{\sim}{x})$ and $\sigma = 1/\delta$.

From (2), we can write the location - scale model,

$$y = \mu\left(\underset{\sim}{x}\right) + \sigma z \tag{3}$$

where the random variable Z has a standard extreme value distribution with density $exp\{z - e^z\}, -\infty < z < \infty$.

A very useful form for $\mu(\underset{\sim}{x})$ in (3) is given by

$$\mu\left(\underset{\sim}{x}\right) = \underset{\sim}{x}'\underset{\sim}{\beta} \tag{4}$$

where $\underset{\sim}{x} = (x_1, x_2, \ldots, x_p)'$ is a vector of p covariates and $\underset{\sim}{\beta} = (\beta_1, \beta_2, \ldots, \beta_p)'$ is a vector of regression parameters.

Assuming a random sample of size n of patients for whom the logarithms of lifetimes have density (2) with $\mu(\underset{\sim}{x})$) given by (4), the logarithm of the likelihood function for $\underset{\sim}{\beta}$ and σ is given by

$$l(\underset{\sim}{\beta}, \sigma) = -nlog\sigma + \sum_{i=1}^{n}\left(y_i - \underset{\sim}{x}_i'\underset{\sim}{\beta}\right)\Big/\sigma - \sum_{i=1}^{n} exp\left\{\frac{y_i - \underset{\sim}{x}_i'\underset{\sim}{\beta}}{\sigma}\right\} \tag{5}$$

In the special case of only one covariate, we could consider (from (3) and (4)) the model,

$$y = \beta_0 + \beta_1 x + \sigma z \tag{6}$$

where the random variable Z has a standard extreme value distribution.

The Fisher information matrix for β_0, β_1 and σ considering model (6) is (see for example, Lawless, 1982) given by

$$I = \frac{1}{\sigma^2}\begin{pmatrix} n & \sum_{i=1}^{n} x_i & n(1-\gamma) \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & (1-\gamma)\sum_{i=1}^{n} x_i \\ n(1-\gamma) & (1-\gamma)\sum_{i=1}^{n} x_i & n\left[1 + \left(\frac{\pi^2}{6} + \gamma^2 - 2\gamma\right)\right] \end{pmatrix} \tag{7}$$

where $\gamma = 0.5772\ldots$ is the Euler constant.

For large values of n, the maximum likelihood estimators $\hat{\beta}_0, \hat{\beta}_1$ and $\hat{\sigma}$ have an asymptotic normal distribution $N\{(\beta_0, \beta_1, \sigma); I^{-1}\}$.

2 Inferences for the Survival Function at a Fixed Time t_0

Assuming the log-linear model (6) with Z having a standard extreme value density $exp(z - e^z)$, the survival function at a fixed time $y_0 = log(t_0)$ with $x = x_0$, is given by

$$S(y_0) = P\{Y > y_0\} = exp\left\{-exp\left(\frac{y_0 - \beta_0 - \beta_1 x_0}{\sigma}\right)\right\} \tag{8}$$

With the transformation $log(-log\ S(y_0)) = (y_0 - \beta_0 - \beta_1 x_0)/\sigma$, the logarithm of the likelihood function for $S(y_0), \beta_0$ and σ is (from (5)) given by

$$\begin{aligned} l(S, \beta_0, \sigma) &= -nlog\sigma + n\bar{y}/\sigma - ny_0\bar{x}/(\sigma x_0) - \\ &-n\beta_0/\sigma + n\beta_0\bar{x}/(\sigma x_0) + (n\bar{x}/x_0)\,log(-logS) - \\ &-e^{\beta_0/\sigma}\textstyle\sum_{i=1}^{n} e^{A_i(S)}\ , \end{aligned} \tag{9}$$

where $A_i(S) = \frac{1}{\sigma}\left(y_i - \frac{y_0 x_i}{x_0}\right) + \frac{\beta_0 x_i}{\sigma x_0} + \frac{x_i}{x_0} log(-logS)$.

Assuming σ and β_0 known, the logarithm of the likelihood function for $S(y_0)$ is given by

$$l(S) \propto (n\bar{x}/x_0)\,log(-logS) - e^{-\beta_0/\sigma}\sum_{i=1}^{n} e^{A_i(S)}. \tag{10}$$

In this case, the maximum likelihood estimator for $S(y_0)$ satisfies the equation,

$$\hat{S}\left(-log\hat{S}\right)\sum_{i=1}^{n}A_i'\left(\hat{S}\right)e^{A_i(\hat{S})}=-n\bar{x}e^{\beta_0/\sigma}/x_0 \tag{11}$$

where $A_i'(S)=-x_i/(x_0S(-logS))$.

Usually, inferences on $S(y_0)$ are based on the asymptotical normality for the maximum likelihood estimator of $S(y_0)$. The accuracy of these asymptotical results could be very poor, expecially, for small or moderate sample sizes.

To check the accuracy of the obtained asymptotical inferences, we could consider a nonnormality measure for likelihood functions or posterior densities of interest based on the standardized third derivative of the logarithm of the likelihood function (see for example, Sprott, 1973,1980; or Kass and Slate, 1992) given (see appendix) by

$$STD(\hat{S})=\left|\frac{\sum_{i=1}^{n}x_i^3e^{A_i(\hat{S})}-3x_0\left(log\hat{S}+1\right)\sum_{i=1}^{n}x_i^2e^{A_i(\hat{S})}}{e^{-\beta_0/2\sigma}\left(\sum_{i=1}^{n}x_i^2e^{A_i(\hat{S})}\right)^{3/2}}\right| \tag{12}$$

The normality of the likelihood function is appropriate if $STD(\hat{S})\cong 0$.

If, based on the obtained value for $STD(\hat{S})$, we conclude that the normality of the likelihood function is not appropriate, we could explore different parametrizations or transformations of $S(y_0)$ to improve the normality of the likelihood function. When σ and β_0 are unknown, we could consider the profile likelihood function $L(S(y_0),\hat{\beta},\hat{\sigma})$, where $\hat{\beta}_0$ and $\hat{\sigma}$ maximizes the likelihood function for each value of $S(y_0)$.

3 Comparison of Two Treatments

Consider $n=n_1+n_2$ patients randomly alocated to two different treatments. Thus, we have two samples: a sample 1 with the logarithms of the lifetimes $(y_{11},y_{12},\ldots,y_{1n1})$ of patients receiving treatment 1 and a sample 2 with the logarithms of the lifetimes $(y_{21}, y_{22}, \ldots, y_{2n_2})$ of patients receiving treatment 2, that usually is a new treatment under study.

Assuming the log-linear model (6) with σ and β_0 known; $x_{1i} = 0, i = 1, 2, \ldots, n_1$ for treatment 1 and $x_{2i} = 1, i = 1, 2, \ldots, n_2$ for treatment 2, the logarithm of the likelihood function for the survival function for a patient at a specified time $y_0 = log(t_0)$ in treatment 2 ($x_0 = 1$) is given (from (10)) by

$$l(S) \propto \quad n_2 log(-logS) + (logS) \sum_{i=1}^{n} exp\left(\frac{y_{2i} - y_0}{\sigma}\right) \tag{13}$$

where $S = S(y_0) = exp\left\{-exp\left(\frac{y_0 - \beta_0 - \beta_1}{\sigma}\right)\right\}$.

In this case, the maximum likelihood estimator for $S(y_0)$ is given by

$$\hat{S}(y_0) = exp\left\{-\frac{n_2 e^{y_0/\sigma}}{\sum_{i=1}^{n_2} e^{y_{2i}/\sigma}}\right\}, \tag{14}$$

and the standardized third derivative of $l(S)$ locally at $\hat{S}(y_0)$ (see (12)) is reduced to

$$STD\left(\hat{S}\right) = \left|n_2^{-1/2}\left(2 + 3log\hat{S}\right)\right| \tag{15}$$

Observe in (15), that we could have large values for $STD(\hat{S})$ if $S \cong 0$, especially for small values of n_2.

4 An Usefull Reparametrization for $S(y_0)$

When we have large values for $STD(\hat{S})$, we could explore different reparametrizations to improve the normality of the likelihood function. Some existing parametric families of transformations for proportions (see for example, Atkinson,

1985) could be used in this case. A special parametrization was introduced by Guerrero and Johnson (1982), and given by

$$\phi^{*}_{GJ}(\lambda) = \left\{ \left(\frac{S}{1-S} \right)^{\lambda} - 1 \right\} \Big/ \lambda \, . \tag{16}$$

Observe that the transformation is the Box and Cox(1964) transformation to the odds ratio $S/(1-S)$, which includes the logit parametrization $log[S/(1-S)]$ when $\lambda = 0$.

For a given value of λ, we could consider a simplified form of transformation given by

$$\phi_{GJ}(\lambda) = \left(\frac{S}{1-S} \right)^{\lambda} - 1, \tag{17}$$

which should not produce different results as considering (16).

Observe that, the inverse of transformation (17) is given by

$$S = \frac{(\phi_{GJ}+1)^{1/\lambda}}{1+(\phi_{GJ}+1)^{1/\lambda}} \, . \tag{18}$$

Assuming β_0 and σ known, the logarithm of the likelihood function for ϕ_{GJ}, where $S = S(y_0)$ with $x = x_0$, is (from (10)) given by

$$l(\phi_{GJ}) \;\propto\; \frac{n\overline{x}}{x_0} logB(\phi_{GJ}) - e^{\beta_0/\sigma} \sum_{i=1}^{n} e^{A_i(\phi_{GJ})} \tag{19}$$

where $A_i(\phi_{GJ}) = \frac{1}{\sigma}(y_i - \frac{y_0}{x_0}x_i) + \frac{\beta_0 x_i}{\sigma x_0} + \frac{x_i}{x_0} logB(\phi_{GJ})$ and $B(\phi_{GJ}) = log[1 + (\phi_{GJ}+1)^{-1/\lambda}]$.

In the special case of comparison of two treatments where $x_0 = 1$ (treatment 2), the logarithm of the likelihood function (19) is reduced to,

$$l(\phi_{GJ}) \;\propto\; n_2 log B(\phi_{GJ}) - B(\phi_{GJ})\sum_{i=1}^{n_2} exp\left(\frac{y_{2i}-y_0}{\sigma}\right) \qquad (20)$$

In this case, the standardized third derivative of the logarithm of the likelihood function $l(\phi_{GJ})$ given in (20), locally at the maximum likelihood estimator $\hat{\phi}_{GJ}(\lambda)$ (see appendix) is given by

$$STD\left(\hat{\phi}_{GJ}(\lambda)\right) \;=\; \left| n_2^{-1/2}\frac{B\left(\hat{\phi}_{GJ}\right)}{B'\left(\hat{\phi}_{GJ}\right)}\left(2 - \frac{3B''\left(\hat{\phi}_{GJ}\right)}{B'\left(\hat{\phi}_{GJ}\right)}\right)\right| \qquad (21)$$

where $\hat{\phi}_{GJ} + 1 = \left\{exp\left(\frac{n_2 e^{y_0/\sigma}}{\sum_{i=1}^{n_2} e^{y_{2i}/\sigma}}\right) - 1\right\}^{-\lambda}$.

From (21), we should search for the appropriate value of λ in the transformation (17) to improve the normality of the likelihood function in the two samples situation (see section 3), such that $STD(\hat{\phi}_{GJ}(\lambda)) \cong 0$.

5 An Example

In table 1, we have the survival times of two groups of patients submitted to two different treatments (data set introduced by Lee, 1980, page 294).

Treatment 1 (Control)	Treatment 2 (New Treatment)
5,10,17,32,32,33,34,36,	20.9,32.2,33.2,39.4,40.0,46.8,57.3,
43,44,44,48,48,61,64,65,	58.0,59.7,61.1,61.4,54.3,66.0,66.3,
65,66,67,68,82,85,90,92,92,	67.4,68.5,69.9,72.4,73.0,73.2,88.7,
102,103,106,107,114,	89.3,91.6,93.1,94.2,97.7,101.6,101.9,
114,116,117,124,139,142,	107.6,108.0,109.7,110.8,114.1,117.5,
143,151,158,195	119.2,120.3,133.0,133.8,163.3,165.1

Table 1 - Survival Times (in weeks) for Patients in Two Groups of Treatments

From graphical analises, it is possible to identify that the logarithms of the survival times of table 1 are well fitted by the extreme value distribution for both treatment groups (see for example, Achcar and Bolfarine, 1986).

Thus, assuming the log-linear model(6) with $x = 0$ for treatment $1(n_1 = 40$ patients) and $x = 1$ for treatment $2(n_2 = 40$ patients), the maximum likelihood estimators for β_0, β_1 and σ are given by $\widehat{\beta}_0 = 4.5496, \widehat{\beta}_1 = -0.0240$ and $\widehat{\sigma} = 0.4534$.

In table 2, we have the maximum likelihood estimators for $S(y_0)$ considering different values for $y_0 = log(t_0)$. We also have in table 2, the values of $STD(\widehat{S})$ (see (15)) assuming σ and β_0 known.

t_0	$\hat{S}(y_0)$	$STD(\hat{S})$
5	0.9984	0.3155
20	0.9663	0.3000
60	0.6795	0.1330
80	0.4826	0.0294
100	0.3037	0.2491
120	0.1683	0.5289
140	0.0818	0.8712
180	0.0128	1.7507
200	0.0041	2.2914
300	0.000001	6.0609
500	0.000000	19.3593

Table 2 - Maximum Likelihood Estimators for $S(y_0)$ with $y_0 = log(t_0)$ and Values for $STD(\hat{S})(\sigma = 0.4534$ and $\beta = 4.5496$ known)

In table 2, we observe some large values for $STD(\hat{S})$, especially for large values of t_0, which indicates bad normality for the likelihood function for $S(y_0)$.

In table 3, we have approximate 95% confidence intervals for $S(y_0)$ considering the asymptotical normality of $\hat{S}(y_0)$ and different values for $y_0 = log(t_0)$. We also have in table 3, approximate 95% confidence intervals for $S(y_0)$ considering the asymptotical normality of $\hat{\phi}_{GJ}(\lambda)$ with appropriate values for λ (from (21)) given by $STD(\hat{\phi}_{GJ}(\lambda)) \cong 0$. Observe that we find some very different confidence intervals for $S(y_0)$, especially for large values of t_0.

t_0	$\hat{S}(y_0)$	Parametrization $S(y_0)$	λ	Parametrization $\phi_{GJ}(\lambda)$
5	0.998400	0.997904;0.998895	-0.997300	0.997904;0.998895
20	0.966426	0.956198;0.976654	-0.939779	0.956217;0.976653
60	0.679835	0.598532;0.761136	-0.462257	0.600713;0.761890
80	0.482787	0.373839;0.591736	-0.287734	0.382270;0.598041
120	0.168308	0.075362;0.261253	-0.094786	0.096076;0.286619
180	0.012769	-0.004487;0.030025	-0.007566	0.003303;0.048714
200	0.004077	-0.002875;0.011029	-0.002447	0.000741;0.022258
300	0.000001	-0.000005;0.000007	-0.000001	0.000000;0.000092

Table 3 - Approximate 95% Confidence Intervals for $S(y_0)$ Considering Parametrizations $S(y_0)$ and $\phi_{GJ}(\lambda)$

In figures 1 and 2, we have the graphs of the "profile" likelihood functions for $S(y_0)$ and $\phi_{GJ}(\lambda)$ considering the values of t_0 and λ given in table 3. We observe, in general, an improvement in the normality of the "profile" likelihood function in the parametrization $\phi_{GJ}(\lambda)$, especially for large values of t_0.

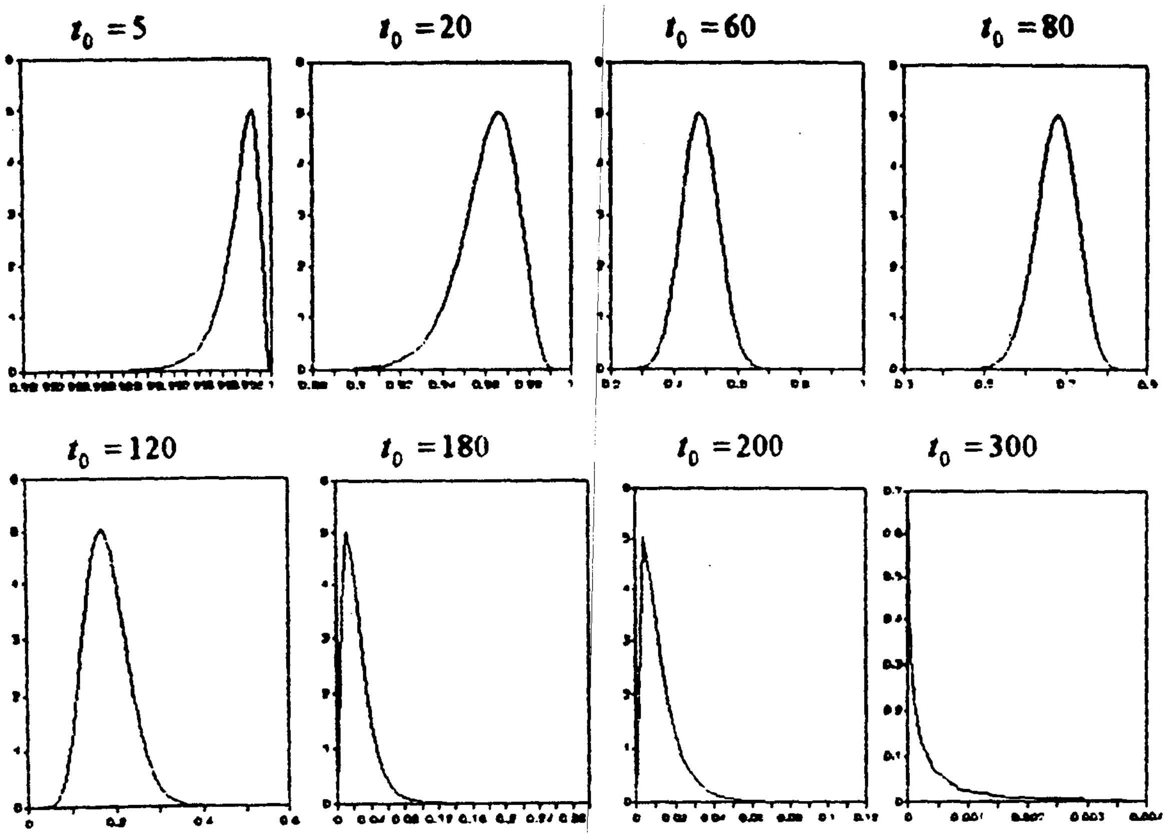

Figure 1 - "Profile" Likelihood Function for $S(y_0)$

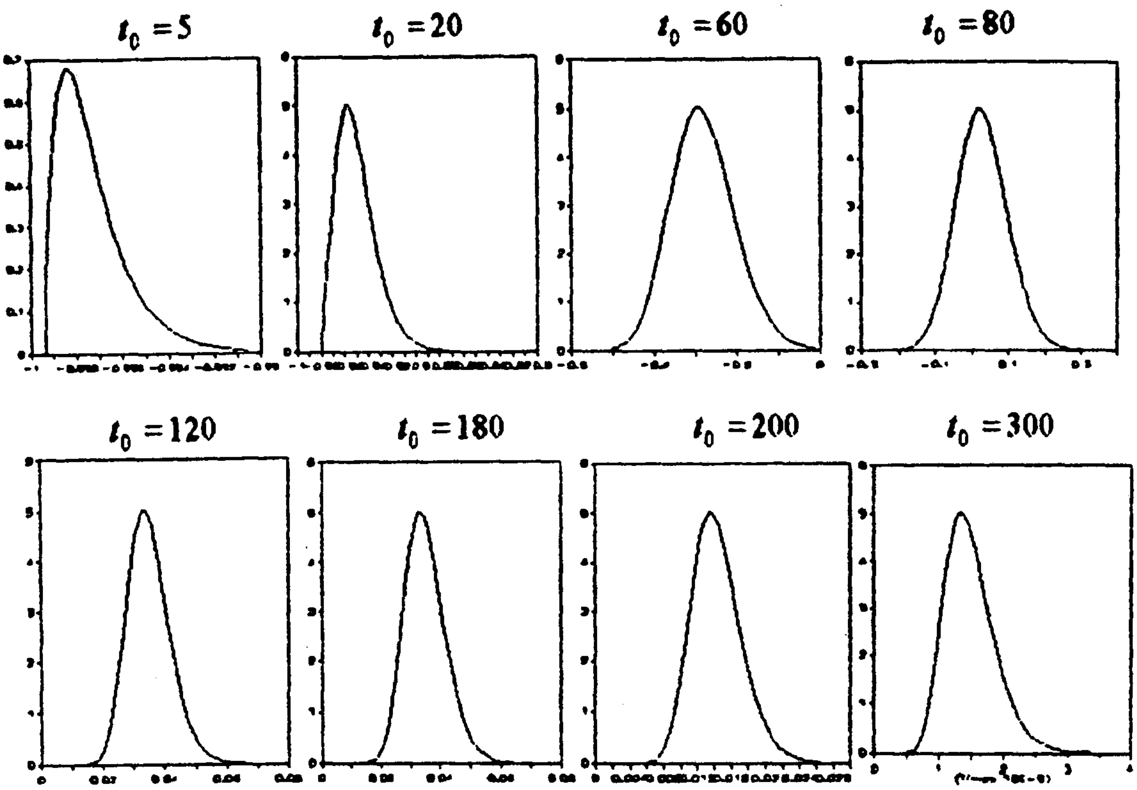

Figure 2 - "Profile" Likelihood Function for $\phi_{GJ}(\lambda)$

We also could check the adequability of the proposed reparametrization $\phi_{GJ}(\lambda)$ by using the t-plot (see Hills and Smith, 1993) of $T(\phi_{GJ})$ given by

$$T\left(\phi_{GJ}\right) = sgn\left(\phi_{GJ} - \hat{\phi}_{GJ}\right)\left\{-2l\left(\phi_{GJ}\right) + 2l\left(\hat{\phi}_{GJ}\right)\right\}^{1/2} \tag{22}$$

where $l(\phi_{GJ})$ is the logarithm of the likelihood function (20) assuming β_0 and σ known and $\hat{\phi}_{GJ}$ is the maximum likelihood estimator for $\phi_{GJ}(\lambda)$ (see figures 3 and 4). In figure 4, we observe an improvement in the linearity of the t-plots considering different values for t_0, which indicate improvements in the normality of the likelihood function for $\phi_{GJ}(\lambda)$.

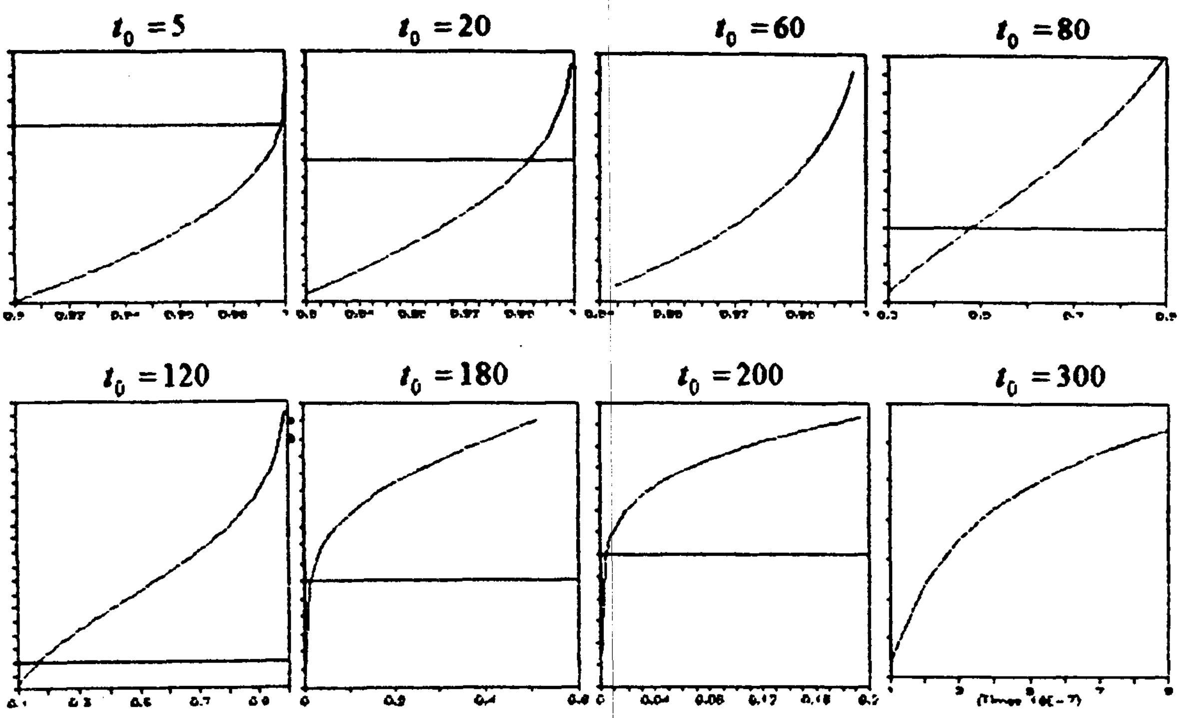

Figure 3 - t-plot of Hills and Smith for $S(y_0)$

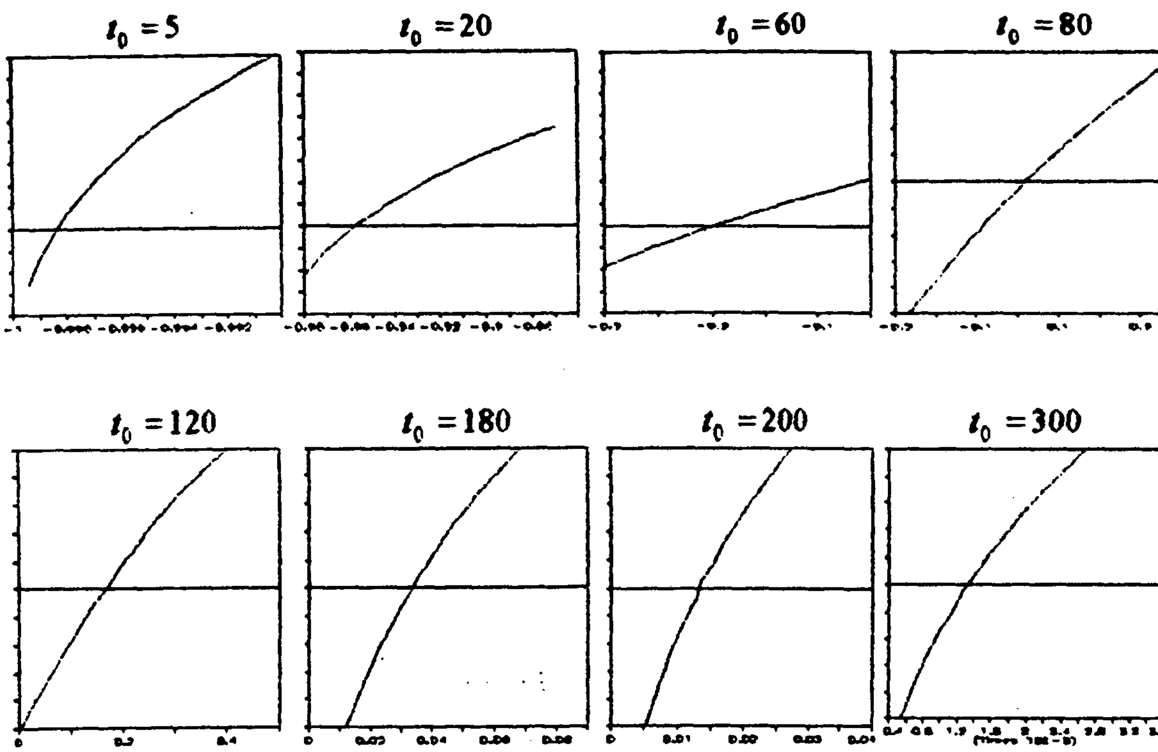

Figure 4 - t-plot of Hills and Smith for $\phi_{GJ}(\lambda)$

The proposed parametrization also could be very useful in a Bayesian analysis of the extreme value regression model. Usually, the statistician considers approximate integration methods to get the posterior summaries of interest, and the accuracy of these results usually depends on the choice of an appropriate reparametrization. As a special case, consider the use of Laplace's method (see for example, Tierney and Kadane, 1986) to aproximate the posterior moments for $S(y_0)$. Considering a noninformative prior density for β_0, β_1 and σ based on Jeffreys multiparameter rule (see for example, Box and Tiao, 1973), we have in table 4, Laplace's approximate posterior moments for $S(y_0)$ considering the joint posterior densities for $(S(y_0), \beta_0, \sigma)$ and $(\phi_{GJ}(\lambda), \beta_0, \sigma)$. We also have in table 4, numerically obtained posterior means for $S(y_0)$ using Simpson's rule. Observe that Laplace's approximations in the parametrization $\phi_{GJ}(\lambda)$, with values of λ given table 3, are very close to the numerically obtained posterior means.

t_0	Simpson's Rule	Laplace's in Parametrization $S(y_0)$	Laplace's in Parametrization $\phi_{GJ}(\lambda)$
5	0.998396	0.992666	0.996471
20	0.966432	0.897550	0.963142
60	0.681079	0.498351	0.675076
80	0.485988	0.361186	0.482784
120	0.174900	0.276818	0.176965

Table 4 - Posterior Means for $S(y_0)$ for Some Values of $t_0(y_0 = log t_0)$

6 Concluding Remarks

The use of transformation (17) for the survival function at a specified time t_0 in extreme value regression models considering an appropriate value for λ obtained from the standardized third derivative of the logarithm of the likelihood function locally at the maximum likelihood estimator of $\phi_{GJ}(\lambda)$ could be of great practical interest to improve the accuracy of the asymptotical inferences. As it was observed in an example, the appropriate invertible reparametrization is easily obtained for each application. We also could extend these results to other parametrical regression models.

APPENDIX

A Diagnostic Measure for Nonnormality of Likelihood Functions

An usefull diagnostic measure of nonnormality for likelihood functions is given by the standardized third derivative of the logarithm of the likelihood function locally at the maximum likelihood estimator (see for example, Sprott, 1973; or Kass and Slate, 1992).

In the one parameter case, this measure is given by

$$STD\left(\hat{\theta}\right) = \left| l'''\left(\hat{\theta}\right)\left(-l''\left(\hat{\theta}\right)\right)^{-3/2} \right| \qquad (A.1)$$

where $l(\theta)$ is the logarithm of the likelihood function and $\hat{\theta}$ is the maximum likelihood estimator.

Kass and Slate (1992) also present a generalization of (A.1) to be used as a diagnostic measure for nonnormality of likelihood functions and joint posterior densities of interest in the multiparameter case.

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