UNIVERSIDADE DE SÃO PAULO

Instituto de Ciências Matemáticas e de Computação

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O problema de corte de estoque com demanda estocástica

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Resumo
Esse trabalho apresenta uma extensão do problema de corte de estoque no caso em que a demanda é uma variável aleatória. O problema é formulado como um problema de programação estocástica não-linear de dois estágios com recurso. As variáveis de decisão de primeiro estágio são as quantidades de objetos cortados de acordo com os padrões de corte. As variáveis de decisão de segundo estágio, as quantidades de itens produzidos em excesso e em escassez. O objetivo do problema é minimizar o custo total esperado incorrido em ambos estágios, devido aos custos de produção e às penalidades por excesso e escassez de produção. Para resolver a relaxação linear do problema, foi proposto um método baseado no Simplex com geração de colunas. O método proposto foi avaliado segundo duas medidas dos efeitos das incertezas muito conhecidas em programação estocástica – o valor da solução estocástica (VSE) e o valor esperado de informação perfeita ($VEIP$). Mostra-se que a solução ótima do problema de dois estágios é mais efetiva em problemas em que a aleatoriedade é maior.
Palavras-chave: problemas de corte de estoque, programação estocástica, otimização linear, geração de colunas.

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On the cutting stock problem under stochastic demand

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Abstract
This paper addresses the one-dimensional cutting stock problem when demand is a random variable. The problem is formulated as a two-stage stochastic nonlinear program with recourse. The first stage decision variables are the number of objects to be cut according to a cutting pattern. The second stage decision variables are the number of holding or backordering items due to the decisions made in the first stage. The model objective is to minimize the total expected cost incurred in both stages, due to waste and holding or backordering penalties. A Simplex-based method with column generation is proposed for solving a linear relaxation of the resulting optimization problem. The method proposed is evaluated by using two well-known measures of uncertainty effects in stochastic programming – the value of stochastic solution (VSS) and the expected value of perfect information ($EVPI$). The optimal two-stage solution is shown to be more effective in problems where the degree of randomness is higher.
Keywords: cutting stock problems, stochastic programming, linear optimization, column generation.

Introduction

In the one-dimensional cutting stock problem, a stock of objects (bars, bobbins, reels, etc.) must be cut into smaller items in order to fulfill a customer demand known in advance, while attaining goals such as using as few stock objects as possible, and minimizing trim loss and production costs. Most of the papers in the literature on this subject deal with deterministic demand. This assumption is quite reasonable in make-to-order environments where no finished-goods inventories are considered, and customer demand is backlogged. In these product-market environments, production is not started until the order is known. On the other hand, in make-to-stock environments production starts before knowing the demand precisely. In such environments, standard products are commonly manufactured for future orders.

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Since future demand is affected by randomness, the end-items production can be overestimated or underestimated. In the former case, end-items are held with a holding cost (which includes opportunity, storage and handling costs). In the latter, lost demand incurs penalties (profits lost).

Under stochastic demand, the optimal solution of the cutting stock problem must provide not only the optimal set of cutting patterns, but also the optimal number of end-items needed to assure the feasibility of the problem over all possible demand realizations. Such realizations can be seen as *scenarios*, which are sequences of outcomes of random variables. In this paper, the cutting stock problem considered is formulated as a two-stage optimization model with recourse. According to the proposed model, the decision variables are partitioned into two distinct sets: the first-stage variables represent common decisions for all the scenarios; the second-stage variables depend both on the decisions made in the first stage and the scenarios considered. Cases in which randomness can be ignored or plays an important role are analyzed by using two well-known measures of uncertainty effects in stochastic programming – the value of stochastic solution (VSS) and the expected value of perfect information ($EVPI$).

To the best of our knowledge, only a few papers deal with stochastic cutting stock problems. Sculli (1981) proposed a stochastic analysis for knife positioning on insulating tape rolls in which the dimensions of the material to be cut are random variables. Krichagina et al. (1998) studied a cutting-stock and lot-sizing problem in the paper industry where the demand was stochastic. To solve the problem, they proposed a two-step procedure: the first step is performed by linear programming; the second step employs Brownian analysis. Chauhan et al. (2008) presented a model and two solution methods to determine parent roll size assortment and assignments to finished products in a paper mill industry, considering the demand for the finished products an independent stationary stochastic process with a given mean and variance. Although there is not a classification of cutting stock problems that explicitly differentiates between deterministic and stochastic demands, Trkman and Gradisar (2007) suggested an extension of Dyckhoff's typology (Dyckhoff, 1990) in which consecutive time periods and stochastic data are considered.

The paper is organized as follows. In Section 1 the two-stage recourse optimization model for cutting stock problems is described. In Section 2, a Simplex-based column generation method to solve the resulting optimization problem is developed. The measures of uncertainty effects for evaluating the optimal solutions provided by the method are discussed in Section 3. Section 4 presents some numerical experiments. Finally, in section 5, conclusions and possible extensions of the approach proposed are summarized.

1. Two-Stage Formulation

Consider the one-dimensional cutting stock problem (CSP) in which a sufficiently large stock of identical objects of the same length L must be cut into m different lengths $l_i = 1, \cdots, m$, ordered by customers in unknown quantities. The objects are cut according to predefined cutting patterns. To each cutting pattern is associated a vector

$$\mathbf{a}_j = (a_{1j}, a_{2j}, ..., a_{mj}),$$

where a_{ij} is the number of times that the order length l_i appears in the particular cutting pattern j, and therefore the following must be verified:

$$\sum_{i=1}^{m} l_i a_{ij} \leq L,$$
$$a_{ij} \in Z_+ \text{ for all } i,\ j.$$

The unknown demand is a random variable described by a measurable mapping $\mathbf{D} : \Omega \rightarrow \Re^m$ on some probability space $(\Omega; F; P)$. In order to take randomness into account, a scenario-based approach is adopted. It is assumed that the probability distribution of the demand has finite support $\Omega = \{1, 2, ..., S\}$, with corresponding probabilities $P = \{p_1, p_2, ..., p_S\}$, with $p_s > 0$ for all $s \in \Omega$. Each scenario $s \in \Omega$ corresponds to a particular realization of the demand vector $\mathbf{D}(s) = \mathbf{D}_s = (d_{1s}, d_{2s}, ..., d_{ms})$, in which the value d_{is} represents the demand of item i in scenario s.

In the approach proposed in this paper, the number of objects cut according to each cutting pattern is determined before the realization of the random demand, which corresponds to the adoption of the so-called *here-and-now* policy, as opposed to the alternative *wait-and-see* policy. After the demand becomes known, the number of items produced is likely different from those actually needed to fulfill the customer demand. Corrective or recourse decisions for each scenario $s \in \Omega$ are then made.

The decision variables of the model can be partitioned into two distinct sets:

- $\mathbf{x} \in Z_+^n$ is the vector of first-stage scenario-independent decision variables, in the sense that these variables are invariant with the scenarios $s \in \Omega$. The component x_j of $\mathbf{x}$ represents the number of objects cut according to the cutting pattern j, or, in other words, the frequency in which the cutting pattern j is used;

- $\mathbf{y} \in Z_+^m$ is the vector of second-stage, scenario-dependent decision variables. These variables represent recourse decisions that eventually need to be taken when a scenario is materialized. In this paper, two types of recourse decision variables are introduced. The first one, represented by y_{is}^+, is the number of holding end-items when the production overestimates the actual demand for item i in scenario s. The second one, represented by y_{is}^-, is the number of end-items needed to match the demand of item i when its production is underestimated in scenario s.

The corrective decisions result in penalties in the objective function in the form of extra costs due to differences between what is produced and what is actually demanded. The cutting stock optimization problem under stochastic demand is equivalent to the following deterministic nonlinear program:

$$\begin{aligned}
\min \quad & \sum_{j \in J} c_j x_j + \sum_{s \in \Omega} \sum_{i \in I} p_s \left(q_{is}^+ y_{is}^+ + q_{is}^- y_{is}^- \right) \\
\text{s.t.} \quad & \sum_{j \in J} a_{ij} x_j - y_{is}^+ + y_{is}^- = d_{is} \\
& y_{is}^+ \cdot y_{is}^- = 0 \\
& x_j, y_{is}^+, y_{is}^- \in Z_+ \quad \text{for all } i \in I, j \in J, s \in \Omega.
\end{aligned} \tag{1}$$

where

m, n, S: number of order types to be cut, cutting patterns and scenarios, respectively;

$I = \{1, ..., m\}$, $J = \{1, ..., n\}$, $\Omega = \{1, ..., S\}$;

c_j: unit cost of cutting a bar according to pattern j;

p_s: probability of scenario s;

d_{is}: demand of item i in scenario s;

q_{is}^+: holding cost per item i in scenario s;

q_{is}^-: shortage cost per item i in scenario s.

The objective in the optimization model (1) is to minimize the overall production cost, which includes the cost of cutting the objects and the expected cost of holding or backordering items. The linear constraints in (1) can be written in matrix form as

$$\mathbf{A}\mathbf{x} + \mathbf{W}_s \mathbf{y}_s = \mathbf{D}_s, \tag{2}$$

where $\mathbf{W}_s = [\mathbf{I} \ -\mathbf{I}]$ and each column of matrix $\mathbf{A} \in \Re^{m \times n}$ is the vector associated with a cutting pattern j. For notational convenience, vectors $\mathbf{y}_s^+ = [y_{1s}^+ \ y_{2s}^+ \ \cdots \ y_{ms}^+]^T$, $\mathbf{y}_s^- = [y_{1s}^- \ y_{2s}^- \ \cdots \ y_{ms}^-]^T$ and

$$\mathbf{y}_s = \begin{bmatrix} \mathbf{y}_s^+ \\ \mathbf{y}_s^- \end{bmatrix}$$

for all $s \in \Omega$ are defined.

The optimization model corresponds to a complete stochastic integer programming recourse model because the second stage problem is always feasible, independently of the decisions made in the first stage. Any number of items can be held or backordered. Therefore, the non-emptiness of the set

$$F = \{\mathbf{x} \in \mathbf{Z}_+^n \mid \exists \mathbf{y}_s \text{ satisfying } \mathbf{y}_s^+, \mathbf{y}_s^- \in Z_+^m,\ y_{is}^+ \cdot y_{is}^- = 0 \text{ and}$$
$$\mathbf{W}_s \mathbf{y}_s = \mathbf{D}_s - \mathbf{A}\mathbf{x} \text{ for all } i \in I \text{ and } s \in \Omega\},$$

is assured. Matrix $\mathbf{W}_s$ is known as a *recourse matrix* and gives arise to a special case of complete recourse called *simple recourse.* The linear constraints of (1) present the dual-angular structure (Lasdon, 1970):

$$\begin{bmatrix} \mathbf{A} & \mathbf{I} & -\mathbf{I} & & & & & & & \\ \mathbf{A} & & & \mathbf{I} & -\mathbf{I} & & & & & \\ \mathbf{A} & & & & & \mathbf{I} & -\mathbf{I} & & & \\ \vdots & & & & & & & \ddots & & \\ \mathbf{A} & & & & & & & & \mathbf{I} & -\mathbf{I} \end{bmatrix},$$

in which the common resource (cutting patterns) shared by all scenarios is explicitly represented.

Problem (1) exhibits $n + 2mS$ variables and $m \times S$ linear constraints. It is worth noting that (1) includes nonlinear constraints which are necessary in order to guarantee that at most one of the variables y_{is}^+ and y_{is}^- is positive for all $i \in I$ and $s \in \Omega$. Such nonlinear constraints are naturally satisfied when Simplex-based methods are applied, because y_{is}^+ and y_{is}^- cannot be both basic variables in any given basic feasible solution of (2).

The formulation of problem (1) employs elements of goal programming (Ignizio, 1982) and benefits from the mentioned property, common to any goal programming formulation.

2. Column Generation Approach

Problem (1) presents typical difficulties faced while solving practical cutting stock problems: the integrality of the decision variables and a large number of cutting patterns. By relaxing the integrality of the decision variables, the resulting problem can be solved by the Simplex Method with column generation; the pricing subproblems assume the structure of knapsack problems. The Simplex method with column generation was first used by Gilmore and Gomory (1961, 1963) to solve cutting stock problems and nowadays is one of the most successful methods for large scale integer-linear optimization problems (Lübbecke and Desrosiers, 2005).

A solution method for stochastic cutting stock problems based on the classical column generation approach introduced by Gilmore and Gomory (1961, 1963) is developed here. The linear relaxation of (1) is solved and efficient heuristic procedures can be devised to determine suboptimal integer solutions (Poldi and Arenales, 2005; Wäscher and Gau, 1996).

2.1 The Initial Basis

The order of a basic matrix of (1) is mS. A feasible basis can be built using S matrix blocks $\mathbf{H}$ associated with cutting patterns and $S-1$ matrix blocks $\mathbf{V}_s$ associated with the second-stage decision variables. Matrix $\mathbf{H}$ is a submatrix of $\mathbf{A}$ and $\mathbf{V}_s$ is a submatrix of $\mathbf{W}_s$. Without loss of generally, suppose that the first scenario has the largest probability, $p_1 \geq p_s, s = 2, \cdots, S$. Therefore, it is possible to make $\mathbf{y}_1 = 0$. The basic matrix of the problem has the following structure:

$$\mathbf{B} = \begin{bmatrix} \mathbf{H} & & & & & \\ \mathbf{H} & \mathbf{V}_2 & & & & \\ \vdots & & \ddots & & & \\ \mathbf{H} & & & \mathbf{V}_s & & \\ \vdots & & & & \ddots & \\ \mathbf{H} & & & & & \mathbf{V}_S \end{bmatrix},$$

where $\mathbf{H} = \text{diag}\left(\lfloor L/l_i \rfloor\right)$ is associated with the homogeneous cutting patterns and $\mathbf{V}_s = \text{diag}\left(v_{is}\right)$ is the second-stage decision variable matrix in scenario s, such that $v_{is} = 1$ if $d_{is} - d_{i1} \geq 0$ (i.e., y_{is}^- is basic variable), and $v_{is} = -1$, otherwise. The initial basic feasible solution is given by

$$\begin{cases} \mathbf{x_B} = \mathbf{H}^{-1}\mathbf{D}_1. \\ y_{i1}^+ = y_{i1}^- = 0. \\ y_{is}^- = \quad (d_{is} - d_{i1}), \text{ if } d_{is} \geq d_{i1}; \; y_{is}^- = 0, \text{ otherwise.} \\ y_{is}^+ = -(d_{is} - d_{i1}), \text{ if } d_{is} < d_{i1}; \; y_{is}^+ = 0, \text{ otherwise.} \end{cases}$$

where $\mathbf{x_B}$ represents the values of the first-stage decision variables, i.e., the frequency of the cutting patterns.

2.2 The Dual Vector

The initial basic costs vector $\mathbf{c_B}$ is composed of three different costs, depending on the nature of the basic columns: costs associated with cutting patterns, costs due to holding items and costs due to backordering items, the latter two multiplied by the probability of a given scenario. The dual vector π is computed by $\pi\mathbf{B} = \mathbf{c_B}$ and the set of dual vectors for all scenarios is

$$\pi = \left[\pi_{11}\ \pi_{21}\ \cdots\ \pi_{m1}\ \cdots\ \pi_{1S}\ \pi_{2S}\ \cdots\ \pi_{mS}\right],$$

where π_{is} is the dual vector associated with item i and scenario s.

2.3 Relative Costs (pricing subproblems)

The relative cost of each cutting pattern variable is obtained by solving the knapsack problem

$$\begin{aligned} \max \quad & \sum_{i \in I} \left(\sum_{s \in \Omega} \pi_{is} \right) a_i \\ \text{s.t.} \quad & \sum_{i \in I} l_i a_i \leq L, \\ & a_i \in Z_+ \qquad \text{for all } i \in I. \end{aligned}$$

Here, c_j is considered independent of the cutting pattern j. If c_j is related to waste (i.e., $c_j = L - \sum_{i \in I} l_i a_I$), the objective function of the pricing subproblems is slightly modified. The relative costs of second-stage decision variables are trivially determined.

2.4 Simplex Direction

The Simplex direction ϵ is determined by the column which enters the basis. If e represents the column associated with the entering variable (i.e., the variable with smallest relative cost), the simplex direction is given by $\mathbf{B}^{-1}e$. If the entering variable is second-stage typed, then e is zero except for the position $(s-1)i+1$, being 1 or -1. Otherwise, if a is the cutting pattern obtained by solving the knapsack problem, then $e = (a\ a\ \cdots\ a)^T$.

3. The Value of the Information and Stochastic Solution

A natural issue that arises in stochastic programming is to find out if it is necessary to solve the stochastic problem or it suffices to solve a simpler version of it. It is common, for example, to solve the deterministic version of the problem obtained by replacing all random variables by their expected values, or to solve a number of deterministic problems, each one associated with a particular scenario. Of course, it is essential to evaluate when these simpler approaches for stochastic programming problems can result in good (near optimal) solutions. In order to evaluate the effectiveness of these approaches, two well-known measures of uncertainty effects, the expected value of perfect information ($EVPI$) and the value of stochastic solution (VSS) are considered in this paper. For details about these measures, see Birge and Louveaux (1997), Kall and Wallace (1994), Madansky (1960), Avriel and Williams (1970), Escudero et al. (2007), and Bard et al. (2007) among others.

3.1 The Expected Value of Perfect Information

The expected value of perfect information measures the maximum amount that a decision maker is willing to pay in order to know the value of a random variable before making his decision (Birge and Louveaux, 1997; Avriel and Williams, 1970). In the cutting stock problem under stochastic demand, this might mean making a market survey or inducing the customers to commit themselves in advance to what their demand will be. In this case, the decision maker could produce items optimally, avoiding shortage and holding costs. For minimization problems, the evaluation of the $EVPI$ measure considers RP (recourse problem value; see Section 1) and *wait-and-see* solutions, WS, that is, the expected value of using the optimal solution for each scenario. For the cutting stock problem considered in this paper, the $EVPI$ is evaluated as follows:

<u>Step 1</u>: Determine the wait-and-see solutions for all $s \in \Omega$:

$$\begin{aligned}(WS)_s = \quad & \min \quad \sum_{j \in J} c_j x_j \\ & \text{s.t.} \quad \sum_{j \in J} a_{ij} x_j = d_{is} \\ & \qquad\quad x_j \in R_+ \quad \text{for all } i \in I, j \in J.\end{aligned}$$

<u>Step 2</u>: Evaluate the expected value of using the wait-and-see solution:

$$WS = \sum_{s \in \Omega} (WS)_s p_s.$$

<u>Step 3</u>: Determine the $EVPI$ measure defined by the difference:

$$EVPI = RP - WS.$$

A large $EVPI$ indicates that randomness plays an important role in the problem, and for this reason, it should not be ignored. Moreover, considering that the $EVPI$ represents how much it is reasonable to pay to obtain perfect information about the future, a large value shows that is not worthwhile to buy information about the random variable and change the strategy from here-and-now to wait-and-see. In this case, it is preferable to solve the stochastic model.

3.2 The Value of Stochastic Solution

The value of stochastic solution measures the cost of ignoring uncertainty when making a decision (Birge and Louveaux, 1997). In the cutting stock problem considered, it can be worthwhile for the decision maker to determine the possible gain obtained by solving the stochastic problem instead of a simpler version of it. Here, the simpler version is the expected value problem, EV, the problem obtained by replacing all random variables by their expected values. The VSS measure compares here-and-now and expected value approaches. Escudero et al. (2007) presents a general procedure to evaluate the VSS measure. For the cutting stock problem studied here, it could be determined as follows:

<u>Step 1</u>: For all $i \in I$, determine the expected value of demand $\bar{d}_i$ in the form

$$\bar{d}_i = \left\lfloor \sum_{s \in \Omega} p_s d_{is} \right\rfloor.$$

Step 2: Evaluate the EV:

$$\begin{aligned} EV = \quad & \min \sum_{j \in J} c_j x_j \\ & \text{s.t.} \sum_{j \in J} a_{ij} x_j = \bar{d}_i \\ & x_j \in R_+ \quad \text{for all } i \in I, j \in J. \end{aligned}$$

Step 3: For all $i \in I$ and $s \in \Omega$, do $y_{is} = \bar{d}_i - d_{is}$. If $y_{is} \geq 0$ then $y_{is}^+ = y_{is}$ and $y_{is}^- = 0$; else

$y_{is}^- = y_{is}$ and $y_{is}^+ = 0$. Step 4: Evaluate EEV:

$$EEV = EV + \sum_{s \in \Omega} p_s \left(\sum_{i \in I} q_{is}^+ y_{is}^+ + \sum_{i \in I} q_{is}^- y_{is}^- \right).$$

Step 5: Evaluate the difference:

$$VSS = EEV - RP.$$

In the expected value approach, when the quantity EEV is near the RP solution, it is very reasonable to consider the EV solution instead of the RP solution, in the sense that the approximation of the stochastic problem by the expected values problem is a good alternative in this situation. On the other hand, a large VSS means that the EV solution is not a good approximation of the RP solution.

4. Computational Issues

The main objective in this section is to provide some insight into the relationship between measures of uncertainty effects and the standard deviation μ of the demand, the number of order types to be cut m, the order lengths l_i, the number of scenarios S, and the costs c_j, q_{is}^+, and q_{is}^-. The idea is to obtain guidelines for deciding in which situations the recourse problem is more attractive, and when it is possible to solve the simpler version adopted (the expected value problem). To reach this goal, randomly generated instances have been used. The problem generator is based on Gau and Wäscher (1995) and Foerster and Wäscher (2000). The algorithms were written in C and the computational experiments were performed on a Pentium IV 2.67GHz, 2GB RAM and Windows Operating System. The sets of experiments were generated in such way that only one parameter was allowed to vary in each problem class; the others were assumed in their default settings, as follows.

- $L = 1000$,
- $l_i \in [10, 250]$,
- $m = 20$,
- $S = 10$,
- $d_{is} \sim N(100, \mu)$, where $\mu = 50$,
- $q_{is}^+ = q_{is}^- = c_j = 1$, for all $i \in I,\ j \in J,\ s \in \Omega$.

The classes were nominated according to the parameter allowed to vary, as shown in Table 1. The name of the class, the name of sub-classes, the number of instances and the parameter allowed to vary are represented in columns 1, 2, 3 and 4 of Table 1, respectively. Table 3 in the Appendix summarizes the main results for all classes. The number in parentheses indicates the ratio VSS/RP or $EVPI/RP$.

Name	Sub-classes	Instances	Parameter
SD	SD5, SD10, SD20, SD30, SD40, SD50, SD60, SD70, SD80	900	μ
SDm	SDm4, SDm10, SDm20, SDm30, SDm40	500	μ
OT	OT10, OT20, OT30, OT40, OT50, OT60, OT70, OT80	800	m
OL	OL250, OL500, OL750, OL1000	400	l_i
SN	SN10, SN20, SN30, SN40, SN50, SN60, SN70, SN80	800	S
Cs	Cs1, Cs2, Cs3, Cs4, Cs5, Cs6	600	c_j, q_{is}^{+}, q_{is}^{-}

Table 1: Problem classes.

Impact of the standard deviation. To analyze the impact of the standard deviation of the demand on the behaviour of VSS and $EVPI$, 9 problem classes with 100 instances each one were generated by fixing μ at 5, 10, 20, 30, 40, 50, 60, 70 and 80. See in Figure 1 that both measures increase substantially as the standard deviation increases; the largest VSS and $EVPI$ values occur when the standard deviation of the demand is the highest in the sub-class SD80, in which $\mu = 80$. In general, the standard deviation affects much more VSS than $EVPI$. Although VSS is not large, it varied from around 1.8 to 90 (0.54% to 6.2% of RP solution). This indicates a possible gain over the expected costs approach when RP is solved, instead of EV. See also that $EVPI$ varied from 83 to 1140 (24.24% to 78% of RP solution). This represents the cost of obtaining perfect information. In order to analyze other instances with very small VSS, 5 classes with 100 instances each were generated. The number of scenarios and demand values varied as follows: S was set to 4, 10, 20, 30 and 40. For each class, $d_{is} = d_i + Random(S)$, where d_i represents a seed for each d_{is} and $Random(S)$ randomly produces 1, 2, ..., or S. See in Figure 2 that the first class presented a null VSS, and that VSS is small, on average. This shows that the gain in solving RP instead of EV is negligible, and that the EV solution is a good approximation to the RP one.

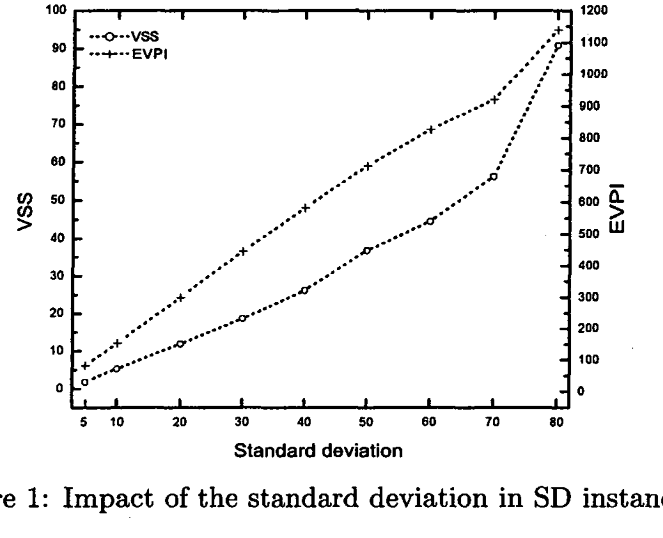

Figure 1: Impact of the standard deviation in SD instances.

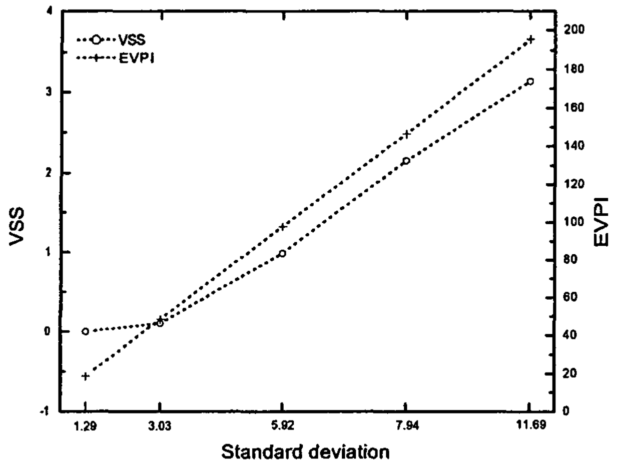

Figure 2: Impact of the standard deviation in SDm instances.

Impact of the number of order types to be cut. 8 problem classes with 100 instances each one were generated by fixing m at 10, 20, 30, 40, 50, 60, 70 and 80. Figure 3 depicts the behavior of VSS and $EVPI$ with respect to m. As m increases, both measures increase gradually. Thus, the number of order types to be cut has a very similar impact on the measures.

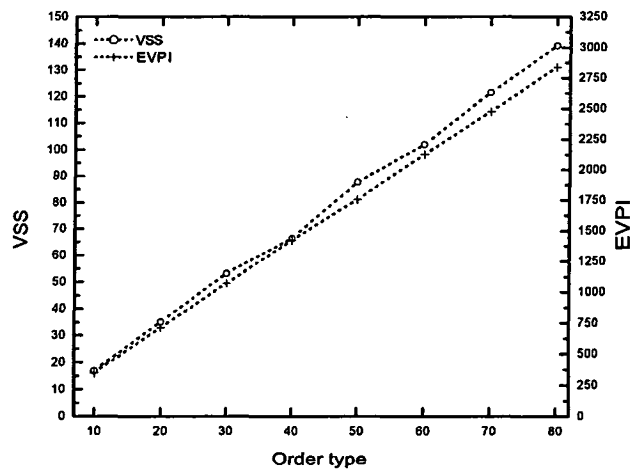

Figure 3: Impact of the number of order types.

Impact of the order length. 4 problem classes with 100 instances each one were generated by fixing the following intervals for l_i: [10,250], [10,500], [10,750], and [10,1000]. Note in Figure 4 that the order length has great effect on the behavior of VSS, and a minor effect on the behavior of $EVPI$. Larger values for VSS are obtained in classes where the order length vary substantially. Smaller values of $EVPI$ are associated with small order length classes.

Impact of the number of scenarios. 8 problem classes with 100 instances each were generated by fixing S at 10 ,20, 30, 40, 50, 60, 70 and 80. See in Figure 5 that as S increases, $EVPI$ increases gradually. The numerical results also suggest that VSS decreases slightly as S increases. In general, the influence of S on both measures is not significant.

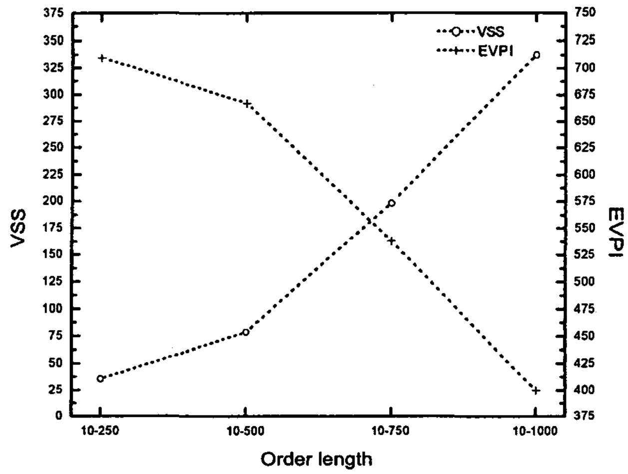

Figure 4: Impact of the order length.

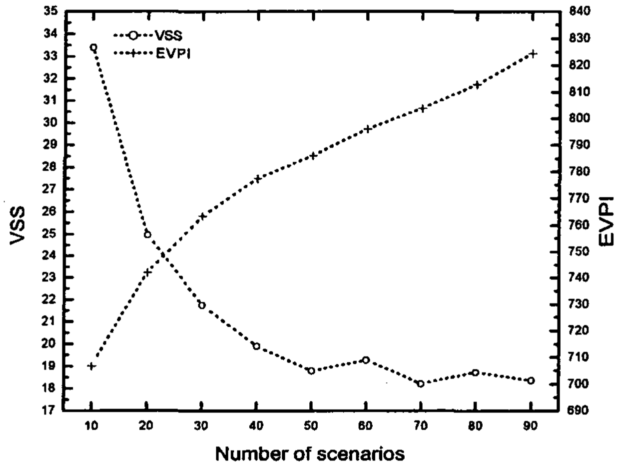

Figure 5: Impact of the number of the scenarios.

Impact of costs. To analyze the impact of costs (production, holding, and shortage) on the behavior of the measures, 6 problem classes with 100 instances each were randomly generated as follows.

- Cs1: $c_j = q_{is}^+ = q_{is}^- = 1$.
- Cs2: $c_j = 1$, $q_{is}^+ = \frac{\sum_i l_i/L}{m}$, and $q_{is}^- = 4q_{is}^+$.
- Cs3: c_j, q_{is}^+, $q_{is}^- \sim N(1.5, 0.25)$.
- Cs4: c_j, q_{is}^+, $q_{is}^- \sim N(1.5, 0.5)$.
- Cs5: c_j, q_{is}^+, $q_{is}^- \sim N(1.5, 0.75)$.
- Cs6: c_j, q_{is}^+, $q_{is}^- \sim N(1.5, 1.0)$.

Figure 6 ilustrates the behavior of the measures with respect to the Cs instances. There are good numerical evidences that relate a better behavior of VSS to more variation in the costs.

However, there are no conclusive numerical evidences about the behavior of $EVPI$ with respect to costs.

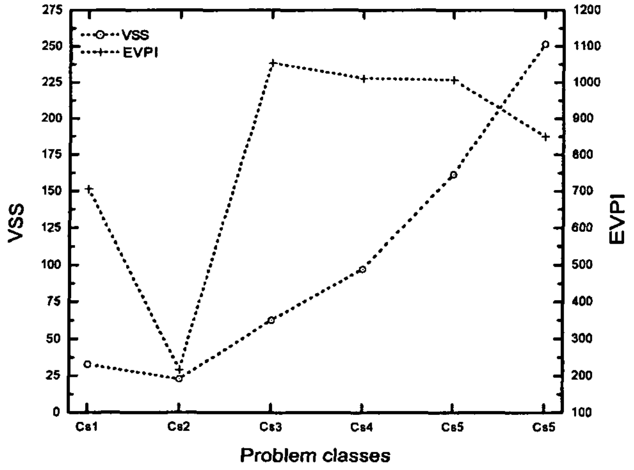

Figure 6: Impact of costs.

The numerical experiments suggest that VSS has a superior performance when μ and m increase, the order lengths and costs vary substantially, and the number of scenarios is small. In order to confirm this association, 250 additional instances were randomly generated by fixing the parameters at the values that produced the best VSS in each class, i.e., $\mu = 80$, $m = 80$, $l_i \in [10, 1000]$, c_j, q_{is}^+, $q_{is}^- \sim N(1.5, 1.0)$. The VSS obtained was 5353, which corresponds to 49.03% of the RP value. Note that the value obtained is the largest one as expected.

The computational effort (running time in seconds) needed for obtaining RP, EEV and WS solutions is presented in the last three columns of Table 3 (in the Appendix). Note that parameter S has greater impact in the running time, as evidenced in classes SDm and SN. On the other hand, as m increases, the running time increases as well, but relatively less (see for example class OT). Standard deviation of the demand, order lengths and costs do not have significant impacts in the running time. In general, the running time can be considered very small: all problem classes were solved in less than one second (on average). The computational times for obtaining the EEV and WS solutions were around 0.01 and 0.13 seconds, and are negligible for all practical purposes.

With respect to costs (production, holding and shortage costs), it is important to emphasize that when the shortage cost is smaller than the holding one, the solution of the recourse problem results in negligible production (in extreme situations, null production), independently of the production cost. Moreover, the following situations are analogous and produce results in which the backorder production tends to zero: null production cost, null holding cost, or very high shortage cost.

5. Concluding Remarks and Extensions

A two-stage stochastic programming approach with simple recourse for the cutting stock problem under stochastic demand was proposed in this paper. Each stage in the approach proposed is composed of a specific group of decision variables. The first group is composed of decision variables associated with the operational aspects involved in cutting objects; the second group incorporates decision variables related to the tactical planning of holding or backordering production. A relaxed version of the problem is solved by using the Simplex method with column generation, which provides non-integer first-stage optimal decisions. Due to the special structure of the stochastic constraints and integer data, integer optimal decisions are obtained in the second stage.

Considering the aim of analyzing in which situations the recourse problem is more suitable (i.e., situations where the recourse problem must be solved) and situations where solving simpler versions of the problem are good alternatives, 7 problem classes were randomly generated. The solution of the recourse problem was compared to the expected value and the wait-and-see approaches by using two measures of uncertainty effects: VSS and $EVPI$.

The analysis was based on 4250 randomly generated instances. For all classes, $WS \leq RP \leq EEV$, as expected, i.e., WS and EEV are lower and upper bound for the RP solution. These inequalities were established by Madansky (1960) and are easily proved (see Birge and Louveaux, 1997). The computational results indicate that, in general, the stochastic approach always introduces a gain with respect to the expected value approach in the instances considered, since VSS is positive in most of the classes. See in Table 3 that in the subclass SDm4 VSS is null, and in this situation the expected value approach is a very good alternative for solving the problem. In the same way, class SDm presents a very small VSS and, therefore, the optimal solutions of these instances can be reasonably approximated by the expected value approach.

It is also worth noting that $EVPI$ is positive in all classes, which indicates that randomness plays a role in the model. Large $EVPI$ (see classes SN, for example) means that randomness is important in the model, and small $EVPI$ (see class SDm) indicates that randomness plays a minor role in the model. Since perfect information about the future is usually not available, the VSS analysis become more attractive than the $EVPI$ one.

According to the numerical experiments, it was possible to observe that as the randomness in demand increases, VSS and $EVPI$ increase as well, and the recourse solution becomes more important. Moreover, the computational experiments indicated that the behavior of the measures adopted is influenced by the order length, number of order types, number of scenarios, and costs. Both measures presented good performances as m increased. The VSS measure presented best performance in wide-spread order length instances; $EVPI$ presented best performance in the small order length instances. However, as the number of scenarios increased, VSS decreased and $EVPI$ increased. The influence of costs in the measures was also analyzed. As the costs variation increased, VSS increased and $EVPI$ decreased. Besides, the computational efforts spent to obtaining RP, EEV, and WS solutions are very small. Numerical evidences suggest that, in general, the stochastic approach performs better (and, therefore, it is more useful) when the parameters of the problem vary significantly. Under this condition, random quantities must be explicitly considered, and it becomes expensive to change from the here-and-now situation to the wait-and-see one. Finally, the proposed model and solution can be used as an interesting management tool, since it makes possible to test different production strategies and scenarios, and to assess the advantages and disadvantages of decisions in each situation.

Among the possible extensions of the approach proposed in this paper, the following ones seem to be promising:

1. Introduce stochastic demand in cutting stock problems with limited and different stock lengths. In this kind of production model, $|K|$ types of stock objects are available in limited quantities e_k, $k \in K$. The problem is formulated as

$$
\begin{aligned}
\min \quad & \sum_{k\in K}\sum_{j\in J} c_{jk}x_{jk} + \sum_{s\in\Omega}\sum_{i\in I} p_s\left(q_{is}^{+}y_{is}^{+} + q_{is}^{-}y_{is}^{-}\right) \\
\text{s.t.} \quad & \sum_{k\in K}\sum_{j\in J} a_{ijk}x_{jk} - y_{is}^{+} + y_{is}^{-} = d_{is} \\
& \sum_{j\in J} x_{jk} \leq e_k \\
& x_{jk}, y_{is}^{+}, y_{is}^{-} \in Z_{+} \quad \text{for all } i \in I, j \in J, k \in K, s \in \Omega.
\end{aligned}
$$

It would be also interesting to extend the approach proposed to multi-period cutting stock problems with stochastic demand (see Trkman and Gradisar, 2007). In a multi-period context, it is quite reasonable to consider that future periods are more affected by randomness. Besides, it is also possible consider other parameters as random variables, such as capacities, costs, and setup time, among others.

2. Provide an integer solution to problem (1). It is possible to use some heuristic procedure to associate an integer solution to the solution obtained by linear relaxation (see Wäscher and Gau, 1996). Another possibility is to use some commercial solver to find an integer solution to the optimization problem. As a first step in this direction, some instances of the problem were solved by using the solver CPLEX 6.0 (used directly with default settings). Table 5 summarizes the data used and presents the value of a *gap* computed as

$$GAP = \frac{Sol(CPLEX) - Sol(Rel)}{Sol(Rel)} 100\%,$$

where $Sol(CPLEX)$ is the integer solution obtained by CPLEX and $Sol(Rel)$ is the relaxed solution obtained by the approach proposed. The values of GAP indicate that improvements in the two-stage optimization approach proposed may be necessary in order to obtain better lower bounds.

m	S	μ	Order length	$Sol(Rel)$	$Sol(CPLEX)$	GAP (%)
10	10	10	small	141.0	151.5	7.400
40	20	80	small	1077	1109	2.956
20	5	80	wide-spread	2065	2369	14.73
10	20	50	wide-spread	546.8	612.2	11.96

Table 2: Examples of integer solutions.

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6. Appendix Table of computational results

Class	*RP*	*EEV*	*WS*	*VSS*	*EVPI*	*RP* time	*EEV* time	*WS* time
SD5	341.3 ± 28.33	343.1 ± 28.53	258.2 ± 27.92	1.831	83.06	0.0294	0.0033	0.0261
SD10	414.4 ± 24.50	419.8 ± 24.79	260.6 ± 25.94	5.371	153.8	0.0306	0.0044	0.0294
SD20	560.5 ± 34.34	572.4 ± 35.36	261.2 ± 32.03	11.91	299.2	0.0281	0.0028	0.0294
SD30	702.1 ± 39.42	720.8 ± 41.06	256.6 ± 31.21	18.70	445.5	0.0297	0.0037	0.0308
SD40	848.0 ± 49.22	874.2 ± 50.93	265.5 ± 31.70	26.22	582.5	0.0299	0.0036	0.0325
SD50	976.8 ± 52.40	1014 ± 55.71	264.5 ± 31.72	36.83	712.3	0.0297	0.0038	0.1322
SD60	1092 ± 60.89	1136 ± 64.24	265.2 ± 32.06	44.55	826.5	0.0289	0.0038	0.0383
SD70	1192 ± 60.22	1248 ± 64.75	270.0 ± 33.92	56.31	921.6	0.0295	0.0023	0.0355
SD80	1451 ± 68.27	1542 ± 73.50	310.8 ± 34.22	90.83	1140	0.0302	0.0042	0.0441
Average	841.9 ± 46.40	874.4 ± 48.76	268.1 ± 31.19	32.51(0.0386)	573.8(0.6816)	0.0295	0.0035	0.0442
SDm4	154.6 ± 20.99	154.6 ± 20.99	135.9 ± 21.09	0.0000	18.67	0.0121	0.0335	0.0961
SDm10	194.5 ± 27.52	194.6 ± 27.54	145.9 ± 27.65	0.1050	48.60	0.0350	0.0039	0.0361
SDm20	250.3 ± 24.97	251.3 ± 25.10	152.6 ± 25.19	0.9893	97.73	0.2200	0.0050	0.0680
SDm30	318.8 ± 26.83	320.9 ± 27.15	172.3 ± 27.26	2.154	146.5	0.7280	0.0060	0.1150
SDm40	376.7 ± 28.42	379.8 ± 28.88	181.2 ± 28.98	3.139	195.5	1.543	0.0050	0.1400
Average	259.0 ± 25.75	260.2 ± 25.93	157.6 ± 26.03	1.277(0.0049)	101.4(0.3916)	0.5076	0.0107	0.0910
TO10	477.6 ± 34.67	494.5 ± 37.98	128.3 ± 22.12	16.89	349.3	0.0047	0.0011	0.0105
TO20	975.9 ± 47.84	1011 ± 52.24	261.9 ± 31.22	35.05	713.9	0.0297	0.0042	0.0339
TO30	1468 ± 62.56	1521 ± 64.97	395.8 ± 40.85	53.26	1072	0.1017	0.0064	0.0734
TO40	1941 ± 55.92	2008 ± 60.51	517.2 ± 40.94	66.41	1424	0.2581	0.0150	0.1556
TO50	2413 ± 84.44	2500 ± 89.00	652.8 ± 49.62	87.74	1760	0.5192	0.0242	0.2778
TO60	2914 ± 89.27	3016 ± 95.43	786.0 ± 55.25	102.0	2128	0.8797	0.0377	0.4301
TO70	3384 ± 78.18	3505 ± 83.11	907.9 ± 49.53	121.5	2476	1.520	0.0649	0.7914
TO80	3880 ± 94.60	4019 ± 101.6	1038.8 ± 54.39	139.1	2842	2.155	0.0952	1.152
Average	1455 ± 57.09	1507 ± 60.94	391.2 ± 36.95	51.87(0.0357)	1064(0.7311)	0.6835	0.0311	0.3656
OL250	971.0 ± 48.50	1007 ± 51.60	261.4 ± 30.14	35.68	709.6	0.0295	0.0037	0.0323
OL500	1186 ± 74.55	1265 ± 85.50	519.4 ± 73.92	78.71	666.9	0.0549	0.0101	0.0937
OL750	1381 ± 95.19	1579 ± 163.3	842.5 ± 158.9	198.5	538.4	0.0502	0.0030	0.0191
OL1000	1528 ± 98.84	1866 ± 181.6	1128 ± 176.8	337.7	400.1	0.0414	0.0014	0.0084
Average	1267 ± 79.27	1429 ± 120.5	687.9 ± 109.9	162.6(0.1284)	578.8(0.4569)	0.0440	0.0046	0.0384
SN10	968.1 ± 46.11	1002 ± 50.44	261.3 ± 34.00	33.40	706.8	0.0309	0.0037	0.0350
SN20	1006 ± 35.65	1031 ± 38.63	264.4 ± 27.48	24.96	742.0	0.1976	0.0036	0.0764
SN30	1026 ± 30.83	1048 ± 32.76	262.5 ± 27.58	21.75	763.4	0.5939	0.0047	0.1002
SN40	1036 ± 32.72	1055 ± 35.29	258.2 ± 28.22	19.90	777.4	1.317	0.0045	0.1359
SN50	1047 ± 32.99	1066 ± 35.63	261.2 ± 32.61	18.79	786.1	2.561	0.0045	0.1710
SN60	1065 ± 26.78	1084 ± 29.05	268.8 ± 26.24	19.28	796.1	4.212	0.0055	0.2419
SN70	1068 ± 29.61	1087 ± 32.52	264.5 ± 31.19	18.22	803.9	6.940	0.0059	0.2602
SN80	1078 ± 29.48	1096 ± 32.43	264.7 ± 29.18	18.73	812.8	9.914	0.0036	0.3075
SN90	1086 ± 28.58	1104 ± 31.20	261.5 ± 29.06	18.37	824.5	14.51	0.0072	0.3398
Average	1017 ± 35.66	1040 ± 38.55	261.5 ± 29.98	23.76(0.0234)	755.1(0.7428)	4.475	0.0048	0.1853
Cs1	970.9 ± 44.20	1004 ± 46.63	265.3 ± 31.35	32.74	705.7	0.0291	0.0038	0.0370
Cs1	477.2 ± 63.46	500 ± 64.40	259.8 ± 34.81	22.86	217.4	0.0297	0.0036	0.0350
Cs1	1448 ± 179.2	1510 ± 178.9	393.8 ± 71.47	62.50	1054	0.0283	0.0017	0.0322
Cs1	1403 ± 326.7	1500 ± 297.1	391.7 ± 136.6	97.20	1011	0.0289	0.0034	0.0330
Cs1	1403 ± 491.0	1564 ± 447.3	395.8 ± 201.9	161.5	1007	0.0303	0.0027	0.0333
Cs1	1288 ± 575.8	1540 ± 476.1	437.6 ± 243.1	251.5	850.9	0.0303	0.0030	0.0305
Average	1165 ± 280.1	1270 ± 251.7	357.3 ± 119.9	104.7(0.0899)	807.7(0.6933)	0.0294	0.0030	0.0335

Table 3: Computational results.

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