UNIVERSIDADE DE SÃO PAULO

Instituto de Ciências Matemáticas e de Computação

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Abstract

An important production programming problem arises in paper industries coupling multiple machine scheduling with cutting stock. From machine scheduling the problem of determining the quantity of jumbos (large rolls of paper) of different types of paper to be produced in each machine arises. These jumbos are then cut to meet the demand for items (smaller rolls of paper). Scheduling that minimizes setups and production costs may produce jumbos (each machine produces jumbos of a specific width) which may increase waste in the cutting process. On the other hand, the best number of jumbos in the point of view of minimizing waste in the cutting process may lead to high setup costs. Both problems are non-trivial combinatorial optimization problems, which have motivated extensive research in the last decades, however their combination is not well explored in the literature. In this paper, a coupled optimization modelling and heuristic solution methods are proposed. Computational experiments are devised in order to analyze the performance of the methods.
Keywords: cutting stock problems, lot sizing and scheduling problems, Lagrangian relaxation.

O problema integrado de corte de estoque e dimensionamento de lotes em indústrias de papel

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Resumo

Um importante problema de programação da produção surge em indústrias de papel acoplando o problema de planejamento em múltiplas máquinas paralelas com o problema de corte. O problema de dimensionamento de lotes deve determinar a quantidade de jumbos (bobinas grandes de papel) de diferentes tipos de papel a serem produzidos em cada máquina. Estes jumbos são então cortados para atender a demanda de itens (bobinas menores de papel). O planejamento, que minimiza custos de setup e produção, deve produzir jumbos (cada máquina produz jumbos de larguras diferentes) que diminuam a perda no processo de corte. Por outro lado, o melhor número de jumbos do ponto de vista de minimizar a perda no processo de corte pode acarretar em altos custos de setup. Ambos são problemas de otimização combinatória não trivial, o que tem motivado extensas pesquisas nas últimas décadas, entretanto, essa combinação não é bem explorada na literatura. Neste trabalho, são propostos um modelo de otimização integrado e métodos heurísticos de solução. São realizados experimentos computacionais com o intuito de analisar o desempenho dos métodos.

Palavras-chave: problema de corte de estoque, problema de dimensionamento de lotes, relaxação lagrangiana.

1. Introduction

In production programming of paper industries (and of many others) a fundamental stage which consists of cutting large rolls, called jumbos, of different widths to meet the demand for items over a number of time periods appears. Customers order for smaller rolls which are well defined by: i) width, ii) kind of paper, iii) demand, and iv) due date.

The jumbos are produced by different machines, each one with a specific production rate as well as specific setup times of changing from one type of paper to another one. Furthermore, the machines are capacitated and they produce different widths for the jumbos.

If, on one hand, one looks for a solution that produces low setups and stocks, no delaying, etc., on the other hand, one also tries to avoid waste in the cutting process. Jumbo production scheduling and cutting stage are interdependent from each other.

In the literature, there are many papers dealing with production programming and cutting stock, however independent of each other, such as Johnson and Montegomery (1974), Bahl *et al.* (1987), Maes *et al.* (1991), Kuik *et al.* (1994), Drexl and Kimms (1997), Karimi *et al.* (2003), Gilmore and Gomory (1961, 1963, 1965), Carvalho and Rodrigues (1995), Wäscher and Gau (1996), Hinxman (1980), Dowsland *et al.* (1992), Dyckhoff *et al.* (1985), Lodi *et al.* (2002), and so on.

Presently, as Thomas and Griffin (1996), and Drexl and Kimms (1997) have already pointed out, there is a tendency to deal with coupled problems. Lot sizing and scheduling problems are dependent on many other activities in an industry, such as distribution planning, cutting and packing problems, project scheduling, etc., and their coordination can help to greatly improve production efficiency and decrease costs.

Chandra and Fisher (1994) compare the coupled solution of a production-distribution system with the uncoupled solution and report improvements from 3% to 20%.

Farley (1988) was perhaps the first author to publish a coupled cutting stock and production programming problems in the clothing industry. Although this problem is in essence irregular two-dimensional cutting, this drawback was overcome by defining a number of stencils (i.e., smaller irregular two-dimensional cutting patterns, with the same width as the fabric - raw material, and different lengths) which are cut along the length of the fabric, i.e., a one-dimensional cutting problem. Actually, the author avoided to explicitly dealing with the imbedded cutting stock problem (i.e., cutting pattern generation), and the problem was formulated as production programming over a single period.

Nonas and Thorstenson (2000) considered a one-dimensional cutting stock problem with holding and setup costs associated with cutting patterns.

Based on a case study of a furniture industry, Gramani (2001) proposed a mathematical model for coupling lot sizing and two-dimensional cutting stock problems. The model is to determine which and how many final products (desks, shelves, wardrobes, etc.) should be produced in a same period. From this decision an order of a number of items (smaller rectangles) follows, which should be cut from available given plates (large rectangles). Therefore, the demand for items in the cutting stock problem was unknown beforehand. On the contrary, for the problem to be analyzed in this paper, the demand for items are given, but the stock is unknown, that is, the availability of objects to be cut is a decision variable.

Respício and Captivo (2002) integrated the one-dimensional cutting stock problem to a multi-item programming problem in paper industries. They considered a single paper machine without setup time or cost so that a simplified model was proposed and solved by a branch and price algorithm. Although their formulation can detect the influence of cutting process on jumbo production, it does not deal with the tradeoff between jumbo production and cutting process, since inventory variables are only implicit considered.

Hendry *et al.* (1996) studied a coupled cutting stock and the scheduling problem in the copper industry. The basic foundry operation consists of melting scrap copper in a furnace, casting it as bars (called 'logs') with specific diameters, which are cut into smaller items ordered by specific diameters and quantities. The authors presented two stage solution methods based on uncoupling the problems. In the first stage, the maximum number of logs of any diameter that can be produced per period is supposedly given, taking into account the furnace capacity, (in our approach this is a decision variable) and a one-dimensional cutting stock problem is solved. This solution gives the number of logs of specific diameters that should be produced per period. Some scheduling constraints are omitted at this stage. In the second stage, a daily production schedule is made for each period to meet the number of logs from the first stage.

Menon and Schrage (2002) consider the problem of allocating orders to machines in the paper industries. As usual in the cutting and packing area, they analyzed a single period of planning (6 weeks) and considered a kind of standardization for the produced jumbos, so that the demand for items (rolls) can be given as the number of items type i, i.e., width i, for example, 13 rolls of 21" width, and so on. Orders, that consist of a number of items required, can not be split across the machines (each order needs to be assigned to one of the machines), that make arising binary variables to allocate orders to machines. They proposed a bound-based approach to relax the problem which they claimed may be effective, in order to obtain good values for the binary variables, when the bounds are tight.

Recently, Correia et al.(2004) studied a cutting stock problem in a Portuguese paper mill. They proposed an enumerating scheme to generate cutting patterns feasible in terms of the technological and operational constraints imposed to the production process. In a second stage, the feasible cutting patterns are used as columns in a linear programming model. The problem is solved for a single period and for a single jumbo width.

2. Problem Definition

First of all, a lot sizing problem should be considered in order to decide which should be the weight of jumbos (large reels) to be produced in each period of a horizon planning. The jumbos are produced on parallel machines, which are capacitated and have different production rates.

The width and the maximum weight of each jumbo depend on the machine in which it is produced. For example, a machine can produce jumbos of 460 cm width, weighing up to 12 tons and another machine can produce jumbos of 540 cm width and up to 24 tons.

Different types of paper have to be produced to meet the demand, and changing from one type to another causes relevant waste of raw material and loss of machine time, and this setup cost and setup time are sequence dependent. For simplicity, we consider setup cost and time independent of sequence.

In practice, a jumbo may consist of different types of paper, including a low quality paper (considered waste) due to the transition of grades (see Fig. 1). However, for modelling purposes, one may consider that each jumbo consists of just one type of paper.

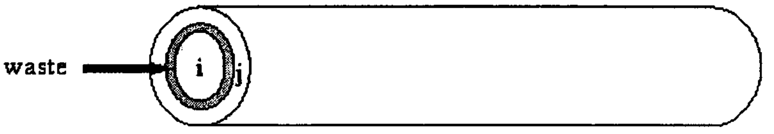

Figure 1: *Jumbo-reel: grade i, waste and grade j.*

After being made, the jumbos are cut into smaller reels of given widths (which may be cut in sequence into rectangles) in order to meet a given demand in such a way as to minimize the waste. In other words, an usual cutting stock problem.

Typically, the problem of jumbo production is empirically solved by expert production managers, who focus mainly on setup minimization. Therefore, lots of jumbos are produced without paying attention to the next production stage of cutting the jumbos which waste depends on the jumbos (widths and quantities) previously made.

Of course, the best widths and quantities of jumbos in terms of the cutting problem can introduce high setups when producing them. Therefore, these two problems, the lot sizing problem and the cutting stock problem, are interdependent and should be solved in an integrated manner.

In order to illustrate, Table 1 gives a piece of information of the client ordering with 5 different items (smaller reels: items 1 and 2, or rectangles: items 3, 4 and 5) and the relevant characteristics in a fixed time period t.

Table 1: *Client ordering in period t.*

Items	Size	Type	Quantity (ton)
1	l_1	1	d_{1t}
2	l_2	2	d_{2t}
3	$l_3 \times w_3$	2	d_{3t}
4	$l_4 \times w_4$	3	d_{4t}
5	$l_5 \times w_5$	1	d_{5t}

Note in Table 1 that some items are made of the same type of paper, e.g., items 2 and 3 are of type 2 paper. Therefore, the demand for type 2 paper in period t, according to Table 1, is $d_{1t}+d_{5t}$. This has to be taken into account in the jumbo production decision, however in which machine should the paper type jumbos be produced? (Machines produce particular widths, at different rates). Furthermore, in case of splitting the quantity, what should the quantity of paper of each type be in each machine? These questions should be answered in such a way as to minimize setup cost and time, production costs on one hand, and minimize waste in the cutting process, on the other.

3. Mathematical Modelling

In order to make it easier to write down a mathematical model we consider the jumbos of type k and width L_s cm (remember that this width depends on the machine the jumbo was made) as a number of *reference-reels* all of them of the same specific weight ρ_k previously defined as illustrated in Fig. 2.

For example, if we define previously that the reference-reels of type k should be such that its specific weight is $\rho_k = 0.1\frac{kg}{cm}$, then each reference-reel of 540 cm width would weigh 54 kg and each reference-reel of 460 cm width would weight 46 kg.

Therefore, supposing that x_s is the number of reference-reels of $L_s$cm width, the correspondent jumbo weight, say T_s, is given by $T_s = L_s\rho_k x_s$. For example, x_s=20 reference-reels of 540cm width means a jumbo of 540cm width weighing 1,080 kg (considering $\rho_k = 0.1\frac{kg}{cm}$.

Note that, if we cut an item of l_i=30 cm width from any reference-reel it would weigh $l_i\rho_k = 3$ kg. Therefore, if the demand for an item of 30 cm width of type k is 300 kg, then we will need 100 of them cut from reference-reels whatever their width. Moreover, each reference-reel is cut according to just one cutting pattern and if one has, like before, $x_s = 20$ reference-reels (i.e., a jumbo of 1,080 kg seen as 20 reference-reels) and 5 of them have been cut according to a cutting pattern and the 15 remaining cut according to another cutting pattern, then it means that the whole jumbo (that weighs 1,080 kg) had 270 kg cut according to the first cutting pattern and 810 kg cut according to the second cutting pattern. The smaller ρ_k is, the bigger x_s is, so that the discretization of the jumbo by reference-reels tends to represent the jumbo continuously.

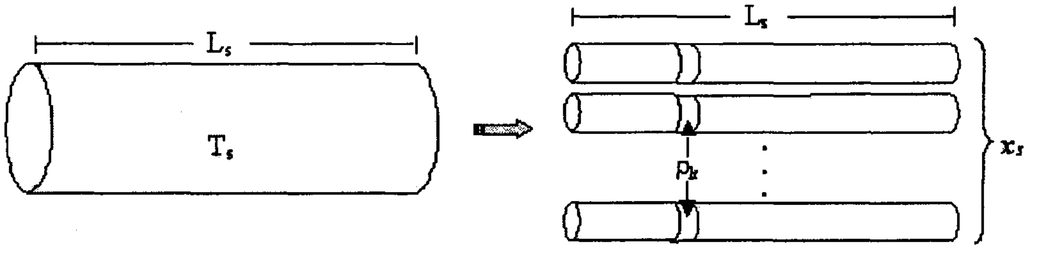

Figure 2: *A type k jumbo discretized as a number of reference-reels.*

In order to write down a mathematical model consider the following notation and data.

Indices:

$t = 1, \cdots, T$: number of the period in the horizon planning;

$k = 1, \cdots, K$: number of the type of paper;

$m = 1, \cdots, M$: number of the machine (machine m produces reference-reels of width L_m);

$j = 1, \cdots, N_m$: number of the cutting pattern for reference-reels type m;

$i = 1, \cdots, Nf$: number of the ordered item;

$\{1, \cdots, Nf\} = S(1) \cup S(2) \cup ... \cup S(K)$, where $S(k)$=$\{i$ such that item i is type k paper$\}$.

Parameters:

c_{kmt} : production cost for a type k reference-reel being made in machine m in period t;

h_{kt} : inventory cost for a type k reference-reel being held at the end of period t;

s_{kmt} : setup cost for machine m producing a type k reference-reel in period t;

cp_{kt} : cost for each centimeter of type k paper lost during the cutting process in period t;

σ_{it} : cost for holding a ton of final items i at the end of period t;

C_{mt} : capacity (ton) of machine m in period t;

$\mathbf{d}_{kt}$: vector of demand quantities for final items type k in period t. Its size is $|S(k)|$;

ρ_k : specific weight for type k reference-reel;

η_k : vector of weight of final items type k (the weight of final item i of type k and width l_i is given by $\eta_{ik} = \rho_k l_i$);

D_{kt} : demand (ton) for type k paper in period t;

b_{km} : weight of type k reference-reel produced in machine m ($b_{km} = L_m \rho_k$);

f_{km} : paper lost (ton) in setting up machine m to produce type k reference-reel;

$\mathbf{a}_{jm}$: vector associated to cutting pattern j for reference-reel of width L_m. Its size is $|S(k)|$ and each component a_{ijm} means the number of items i, $i \in S(k)$, cut according to cutting pattern j for the reference-reel of width L_m;

p_{jm} : paper waste (cm) in the cutting pattern j used to cut a reference-reel of width L_m;

Q : big number.

Decision variables:

x_{kmt} : number of type k reference-reels produced in machine m in period t;

w_{kmt} : number of type k reference-reels produced in machine m held at the end of period t;

z_{kmt} : binary variable that means if there was production or not of type k reference-reel in machine m in period t;

y^j_{kmt} : number of type k reference-reel produced in machine m in period t which are cut according to the cutting pattern j;

$\mathbf{e}_{kt}$: vector of final items of type k paper held at the end of period t. Its size is $|S(k)|$ and each component e_{ikt} means the number of final items i, $i \in S(k)$ held at the end of period t.

Remarks:

1. The parameter D_{kt}, which means the total quantity of type k paper that has to be available in period t, is not a proper parameter from the data problem, since it depends on the unknown waste in the cutting process. As defined, it should be: $D_{kt} = \sum_{i \in S(K)} \eta_{ik} d_{ikt} + waste$. As the waste is unknown beforehand, we introduce a new parameter θ as an estimate for waste.

2. On the other hand, the parameter D_{kt} can be expressed in terms of the variables which determine the number of reference-reels of type k paper that should be cut in period t, i.e., $D_{kt} = \sum_{m=1}^{M} \sum_{j=1}^{N_m} b_{km} y^j_{kmt}$.

Therefore, a coupled cutting stock and lot sizing problems can be written as:

$$\text{minimize} \quad \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}(c_{kmt}x_{kmt} + h_{kt}b_{km}w_{kmt} + s_{kmt}z_{kmt}) + \sum_{t=1}^{T}\sum_{k=1}^{K} cp_{kt}F(k,t) +$$

$$+\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i\in S(k)} \sigma_{it}\eta_{ik}e_{ikt} \tag{4.1}$$

$$\text{subject to} \quad \sum_{m=1}^{M}(b_{km}x_{kmt} + b_{km}w_{k,m,t-1} - b_{km}w_{kmt}) = D_{kt}, \qquad k=1,\cdots,K;\ t=1,\cdots,T \tag{4.2}$$

$$\sum_{k=1}^{K}(b_{km}x_{kmt} + f_{km}z_{kmt}) \leq C_{mt}, \qquad m=1,\cdots,M;\ t=1,\cdots,T \tag{4.3}$$

$$x_{kmt} \leq Qz_{kmt}, \qquad k=1,\cdots,K;\ m=1,\cdots,M;\ t=1,\cdots,T \tag{4.4}$$

$$\sum_{m=1}^{M}\sum_{j=1}^{N_m} a_{jm}y^j_{kmt} + e_{k,t-1} - e_{kt} = d_{kt}, \qquad k=1,\cdots,K;\ t=1,\cdots,T \tag{4.5}$$

$$\sum_{j=1}^{N_m} y^j_{kmt} = x_{kmt} + w_{k,m,t-1} - w_{kmt}, \qquad k=1,\cdots,K;\ m=1,\cdots,M;\ t=1,\cdots,T \tag{4.6}$$

$$w_{km0}=0,\ e_{k0}=0, \qquad k=1,\cdots,K;\ m=1,\cdots,M \tag{4.7}$$

$$x_{kmt} \geq 0,\ w_{kmt} \geq 0 \textit{ and integers}, \qquad k=1,\cdots,K;\ m=1,\cdots,M;\ t=1,\cdots,T \tag{4.8}$$

$$z_{kmt} \in \{0,\ 1\}, \qquad k=1,\cdots,K;\ m=1,\cdots,M;\ t=1,\cdots,T \tag{4.9}$$

$$y^j_{kmt} \geq 0,\ e_{kt} \geq 0 \textit{ and integers},$$

$$j=1,\cdots,N_M;\ k=1,\cdots,K;\ m=1,\cdots,M;\ t=1,\cdots,T. \tag{4.10}$$

The inventory balancing constraints (4.2) say that, for each type of paper and in each time period, the total quantity produced (ton) (in whatever machine m) plus the stock from the previous period have to meet the demand plus the stock for the next time period. Constraints (4.3) say that the total quantity of produced paper, whatever type k, plus the lost paper due to changing types, is limited by the capacity of the machine m in any period.

Constraints (4.4) are to set z_{kmt}=1 in case of $x_{kmt} > 0$. If x_{kmt}=0, then optimality criterium forces z_{kmt}=0. In (4.5) note that $\sum_{m=1}^{M}\sum_{j=1}^{N_m} \mathbf{a}_{jm}y^j_{kmt}$ gives a vector of the total the type k items cut in period t. Therefore, constraints in (4.5) are the item inventory balancing equations. Constraints (4.6) are to limit the number of reference-reels cut, which were produced before. These are the coupling constraints that involve decisions concerning the production of jumbos and the cutting of them.

Initial inventories are considered null in (4.7). The other constraints are to specify non-negativity, integer or binary for the decision variables.

The objective function (4.1) is a composition of a number of objectives such as jumbo production and inventory costs, setup costs, waste cost in cutting process, where the waste is given by $F(k,t) = \sum_{m=1}^{M}\sum_{j=1}^{N_m} p_{jm} y^j_{kmt}$ and final item inventory costs.

Imbedded in the coupled model, there is a capacited lot sizing problem for unrelated parallel machines. According to Maes *et al.* (1991), the problem of finding a feasible solution is NP-complete under setup time variables. Due to the difficulty of solving this problem exactly for medium scale, most of the methods in the literature are heuristics (Trigeiro *et al.*, 1989; Lozano *et al.*, (1991); Hindi *et al.*, 2003). Furthermore, the cutting constraints have two features that make the model difficult. The first one is the integrality condition on the variables y^j_{kmt}, and the second feature is the enormous number of variables (there is a variable for each cutting pattern). Gilmore and Gomory (1961, 1963) proposed a column generation technique in the simplex method for the integer condition relaxed problem, which is a powerful method for the cutting stock problem. Good heuristics to round the solution in order to obtain an integer solution can be easily devised (Wäscher and Gau, 1996). As we can see, even separately, both of these problems are hard to solve. In the next section we give two decomposition heuristics for the coupled problem.

4. Solution Methods

Since the model (4.1)-(4.10) couples two well-studied problems: the lot-sizing problem and the cutting stock problem, we have designed two decomposition heuristics which approach both problems separately one after the other and iteratively, so that we can use well known tools.

The first heuristic solves the lot sizing problem initially determining how the production of jumbos (which are written as multiple of reference-reels) should be and then the cutting stock problem is solved considering the produced jumbos. Just one cycle of this procedure, i.e., to solve lot sizing problem and then to solve cutting stock problem, tends to give a very good solution in terms of jumbo production, however not in terms of waste in the cutting process. The second heuristic solves a cutting stock problem at first, by supposing unlimited jumbos at hand, whatever the type of machine they were made from (remember that machines produce different widths). The solution to the problem above provides the need of jumbos (ton) per type of paper and period. Then, a lot sizing problem is solved to give a production plan. Now, just one cycle of this heuristic tends to produce a very good solution in terms of trim loss, but it can produce high setups. Therefore, in both heuristics we iterate in order to obtain better solutions.

4.1 The Lot-Cutting Heuristic

The solution approach to the coupled problem, that we call the lot-cutting heuristic, is based on a Lagrangian relaxation of the coupled constraints (4.6) which are added to the objective function (4.1) weighing by dual variables γ_{kmt}, $k = 1, \cdots, K$; $m = 1, \cdots, M$; $t = 1, \cdots, T$. Then, the objective function is changed for:

$$\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\Big(c_{km}x_{kmt} + h_{kt}b_{km}w_{kmt} + s_{kmt}z_{kmt}\Big) + \sum_{t=1}^{T}\sum_{k=1}^{K} cp_{kt}\sum_{m=1}^{M}\sum_{j=1}^{N_m} p_{jm}y^{j}_{kmt} +$$

$$+\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i\in S(k)} \sigma_{it}\eta_{ik}e_{ikt} - \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\gamma_{kmt}\Big(\sum_{j=1}^{Nm} y^{j}_{kmt} - x_{kmt} - w_{k,m,t-1} + w_{kmt}\Big)$$

$$= \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\Big(c_{km} + \gamma_{kmt}\Big)x_{kmt} + \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\Big[(h_{kt}b_{km} - \gamma_{kmt})w_{kmt} + \gamma_{kmt}w_{k,m,t-1}\Big] +$$

$$+\sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K} s_{kmt}z_{kmt} + \sum_{t=1}^{T}\sum_{m=1}^{M}\sum_{k=1}^{K}\Big(cp_{kt}\sum_{j=1}^{N_m} p_{jm} - \gamma_{kmt}\Big)\sum_{j=1}^{Nm} y^{j}_{kmt} +$$

$$\sum_{t=1}^{T}\sum_{k=1}^{K}\sum_{i\in S(k)} \sigma_{it}\eta_{ik}e_{ikt}. \tag{4.11}$$

Since the coupling constraints (4.6) have been relaxed, the problem (4.1)-(4.10) can be decomposed into two sub problems. One is a pure lot sizing problem LSP and the other is a pure cutting stock problem CSP. At first, the lot-cutting heuristic solves the LSP, which the objective function is to minimize the inventory and production costs and setup costs, which can be obtained from (4.11) by considering only the terms that involve inventory and production decisions of reference-reels, subject to constraints: (4.2)-(4.4) and (4.7)-(4.9).

The solution obtained from LSP provides the CSP with reference-reels to the CSP, which is solved by minimizing the waste and item inventory costs subject to (4.5)-(4.6) and (4.10).

The lot-cutting heuristic performs a maximum number of iterations p previously given. In the first iteration we set $\gamma_{kmt} = 0$, and in the other iterations γ_{kmt} is given by the dual variable associated to (4.6) obtained by CSP which have just been solved.

Note that the jumbo production in the lot sizing problem should match the quantity of type k paper in the period t enough to meet the demand for items in the cutting stage, including some additional percentage of the produced jumbos to cover trim loss. Since trim loss is unknown, we add a tolerance to the total of type k paper in the period t, that is, we consider $D_{kt} = (1+\theta)\sum_{i\in S(k)} \eta_{ik}d_{ikt},\ k = 1,\ldots,K,\ t = 1,\ldots,T$, where θ is an estimate of the waste in the cutting process.

Of course, if an estimate to cover the waste was not incorporated in the production of jumbos the cutting stock problem would be unable to meet the demand for items, that is, it would be infeasible. This would occur even if an estimate for waste were incorporated but not enough to actually cover the waste in the cutting process.

Note that the LSP is solved heuristically and its solution is simply rounded up (remember that the number of reference-reels is integer, and if their weights previously defined are low this rounding process becomes irrelevant). However, this rounding up is useful to cover the waste in the cutting process.

The algorithm starts with θ=0 and increases it step by step up to a given upper bound (e.g., 10%). The best solution is adopted. In the following, the lot-cutting heuristic is summarized.

Algorithm Lot-Cutting:

Step 1: Set $\gamma_{kmt} = 0$ and specify $\theta = 0$ (percentage of demand slack).

Step 2: For i=1 to p do:

2.1: Solve LSP by using the method in section 4.1.1, and D_{kt} in (4.2) as given above. If the method is able to produce a feasible solution go to Step 2.2. Otherwise Stop the algorithm: the infeasibility of LSP is caused by the excess demand.

2.2: Solve the CSP according to section 4.1.2.

If a feasible solution is obtained then go to Step 2.3. Otherwise, go to Step 3.

2.3: Update the Lagrangian multipliers γ_{kmt}.

Step 3: Increase θ (e.g., $\theta = \theta + 0.01$). If $\theta \leq$ *upperbound* (e.g., *upperbound* = 0.1) then go back to Step 2, else Stop the algorithm.

4.1.1 A Solution Method for the Lot Sizing Problem

In order to solve LSP as defined before by constraints (4.2)-(4.4), (4.7) and (4.8), which is a capacitated lot sizing problem for unrelated parallel machines we used a heuristic proposed by Toledo (1998). The author proposes a heuristic based on the Lagrangian relaxation of the capacity constraints and the subgradient method. The resulting Lagrangian problem is decomposed into K independent uncapacitated lot sizing sub problems, one for each item. At every step of the subgradient method the K uncapacitated lot sizing problems with unrelated parallel machines are solved. The author proposed an extension of the single machine dynamic programming algorithm proposed by Evans (1985) to solve those problems. This solution is often infeasible to the LSP and both a feasibility procedure and an improvement procedure are applied to this solution. Both procedures are based on shifts of production amounts between periods and machines to try to build a feasible solution and improve it. These procedures are similar to the smoothing and improvement procedures in Trigeiro *et al.* (1989). The following algorithm summarizes the heuristic.

Algorithm for LSP:

Step 1: Set the Lagrangian multipliers to zero.

Step 2: Solve the K uncapacitated lot sizing sub problems with unrelated parallel machines.

Step 3: If this solution is feasible for the LSP go to Step 6.

Step 4: Apply the Feasibility Procedure.

Step 5: If a feasible solution is found, apply the Improvement Procedure.

Step 6: Update the incumbent solution.

Step 7: Update the Lagrangian multipliers according to the subgradient method.

Step 8: If the limit to iterations (e.g. 100) is not exceeded, nor optimality is obtained, go back to Step 2.

The solution obtained by this heuristic provides the CSP with the jumbos to be cut. The stock of reference-reels in LSP was aggregated whatever the machine they were produced, so that a new aggregated variable was defined: $w_{kt} = \sum_{m=1}^{M} b_{km} w_{kmt}$. So, w_{kt} means the quantity (ton) of type k paper in stock at the end of the period t. The value of w_{kmt} is recovered when solving CSP, described as follows.

4.1.2 Solving the Cutting Stock Problem over Horizon Planning

The cutting stock problem over horizon planning consists of the second subproblem resulting from the decomposition heuristic and involves constraints (4.5)-(4.6). Note that this problem is uncoupled in k, that is, there are K independent subproblems. Moreover, note that, for each k, the constraints are coupled because of the item inventory variables e_{ikt} (which are components of the vector $\mathbf{e}_{kt}$). Then, if we ignore these coupling variables it leads to KT independent ordinary CSP sub problems (one for each type of paper and each period). This means that no inventory of items is made, that is, no item is produced before it is ordered. This null item inventory solution does not allow better combinations of items in a period are analyzed in order to decrease waste. The CSP are solved from $t = 1, \ldots, T$, and the remaining number of reference-reels in the period t is added to $x_{k,m,t+1}$ providing the stock of reference-reels to be cut in the *t+1* period. These ideas are summarized as follows:

Algorithm for CSP over Horizon Planning:

For *k=1* to K do:

Let $w_{km0} = 0$, for all m

For *t=1* to T do:

Solve the CSP, obtaining y^j_{kmt} and w_{kmt}. (Note that in (4.6), $w_{k,m,t-1}$ is given from *t-1*).

Remark: The above algorithm determines the values to w_{kmt}, which is the remaining of reference-reels held to the period *t+1*. This variable was aggregated in LSP per period, now it is recovered.

4.2 The Cutting-Lot heuristic

Instead of producing jumbos based on the item ordering directly, and then increasing the demand of paper to cover trim losses as done before, now the Cutting-Lot heuristic firstly solves the cutting stock problem based on the item ordering directly, but with a free number of jumbos supposedly available. Therefore, the solution of the cutting stock problem provides the number of reference-reels necessary to meet the demand for any type of paper. Then, a lot sizing problem, which consider setups, machine capacities, production and setup costs, etc., is solved and provides a new production programming, which is input for the next cutting stock problem, and so on.

The heuristic is summarized as follows.

Algorithm Cutting-Lot:

Step 0: Let Iteration = 1

Step 1: Solve CSP over Horizon Planning, according to section 4.1.2. If Iteration = 1, then the constraints (4.6) are omitted.

The solution of this step provides: $\sum_j y^j_{kmt}$ that is the necessary number of reference-reels of type k paper and L_m width (i.e., produced in machine m) in the period t to meet the demand. Therefore, the demand for type k paper in the period t is given by:

$$D_{kt} = \sum_m b_{km} \sum_j y^j_{kmt}.$$

(Note that the number of reference-reels of type k paper with L_m width in the period t given by the cutting stock problem is the best solution in terms of the cutting process, however it is not the best solution concerning the production of the jumbos. Therefore, the aggregated demand for jumbos type k in the period t provides the production of jumbos with the opportunity of better allocating the types of paper on the machines concerning production.)

Step 2: For $k = 1$ to K, do:

Solve the LSP with the demand for type k paper in the period t, D_{kt}, given in Step 1, basically formed by (4.2)-(4.4) and (4.8)-(4.9), and the terms of objective function related to the jumbo production process. The LSP solution provides x_{kmt}, the number of reference-reels of type k produced on machine m (i.e., L_m width), for $t = 1$ to T.

Iteration = Iteration + 1.

Analogous to the Lot-Cutting heuristic, if the CSP is infeasible then D_{kt} is increased by a tolerance θ, and the CSP is solved again until finding a feasible solution, or until θ

equals an upper bound.

Step 3: Repeat Step 1 and Step 2 for a limited number of times, or stopped if no improvement is made from an iteration to the next.

5. Computational Experiments

In order to analyze both heuristics given in Sections 4.1 and 4.2, a number of instances were randomly generated, by varying $Nf = 5, 10, 20$ (the number of item types ordered), $K = 2, 4, 6$ (the number of different types of paper), and $T = 8, 10, 12$ (the total number of time periods). This provides 27 classes, with 10 instances in each class. For all classes, we set $M = 2$ (i.e., two machines are considered), and machine 1 produces jumbos of $L_1 = 540$ cm width, and machine 2 $L_2 = 460$ cm width. We considered the specific weight for type k reference-reel: $\rho = 2$ kg/cm.

For each class, the instances were randomly generated with cost parameters and machine capacity constant over time. Extra input parameters which have to be generated are the following.

- production cost of reference-reel: $c_{kmt} \in [0.015 \quad 0.025] \cdot b_{km}$;
- setup cost: $s_{kmt} \in [0.03 \quad 0.05] \cdot c_{kmt}$;
- inventory cost for reference-reels: $h_{kt} \in [0.0000075 \quad 0.0000125]$;
- paper lost due to the production process: $f_{km} \in [0.01 \quad 0.05] \cdot b_{km}$;
- trim loss cost due to the cutting process: $cp_{kt} = \frac{\sum_{m=1}^{M} c_{kmt}}{M} \cdot 10$;
- inventory cost for final items i at the end of period t: $\sigma_{it} = 0.5 \cdot h_{kt}, \quad i \in S(k)$;
- item lengths: $l_i \in [0.1 \quad 0.3] \cdot \frac{\sum_{m=1}^{M} L_m}{M}$;
- demand for final items: $d_{ikt} \in [0 \quad 300]$. If $d_{ikt} \leq 50$ then set $d_{ikt} = 0$.

The capacity of the production machine was generated as follows.

$$Cap = 1.24 \frac{\sum_{t=1}^{T} \sum_{m=1}^{M} \sum_{k=1}^{K} \left(\frac{D_{kt}}{M} + f_{km} \right)}{T}$$

$$C_{mt} = \frac{b_{km}}{\sum_{m=1}^{M} b_{km}} \cdot Cap$$

In Table 3, the results of Cutting-Lot and Lot-Cutting heuristics to 270 instances grouped into 27 classes are depicted. The column 'OFV', for each heuristic, contains the average of the objective function values of the instances which were well-solved. The number of well-solved instances (10 instances were run for each class) is shown in the column "Solved'. The column 'Victory' gives the number of instances that one heuristic performed better than the other. If an instance is solved by one heuristic and it is not by the other, a victory is counted for the successful one. However, this instance is not considered when calculating averages. Since the algorithms iterate, we also provide the number of the iteration, on average, in which the best solution was obtained, in the column 'Iteration' (maximum is 30). The column 'GAP' shows $\frac{OFV(LC)-OFV(CL)}{OFV(CL)} \times 100$, so that positive values give us the percentage of how better the Cutting-Lot heuristic was.

Note that, in Table 3, the Cutting-Lot heuristic performed better than the Lot-Cutting heuristic, except for Class 7 ($Nf = 5, K = 2, T = 12$). However, for this class, the Cutting-Lot heuristic wins in terms of the number of victories. The overall number of victories obtained by the Cutting-Lot heuristic was 155 for the 270 instances (i.e., 57%), and the heuristics tie for the same the objective function values in about 33% of the instances. Both heuristics solved about 90% of all instances. The Cutting-Lot heuristic, in general, obtained the best solution earlier than the Lot-Cutting heuristic, on average, 1.7 and 2.5 iterations respectively. It is worth nothing that the Cutting-Lot heuristic stops with the best solution obtained performing on average 1.7 iterations. However, the Lot-Cutting heuristic run p iterations (we used p=30) and the best solutions is obtained, on average, at iteration number 2.5.

6. Conclusions and Future Research

In this article a large mixed linear integer optimization model to couple the jumbo (i.e., large paper reel) production programming and the cutting process in the paper industry was proposed. The jumbo production variables provide the availability of jumbos to be cut, leading to a typical constraint for the cutting stock problem (usually, stock is a given data for cutting stock problems). In order to solve the coupled problem, we used two heuristics. The first one is based on Lagrangian relaxation to decompose the problem, and an iteration consisting of firstly solving the production jumbo programming and then the cutting stock problem. The dual variables associated to the coupling constraints are used to redefine costs to the production jumbo programming. In the second heuristic firstly we solve the cutting stock problem to meet the demand, assuming no limits on the jumbos. This determines the need of types of paper for each period and then we solve the lot sizing problem. The computational experiments show that the second heuristic performs better than the first one.

For future research we may suggest:

(i) anteceding items from the period t+1 to the period t. It could be done by taking into account negative simplex multipliers concerning the subproblem in the period t. For example, if the simplex multiplier of the subproblem of type k in the period t associated to item i, say π^i_{kt}, is negative, then it roughly means that the subproblem objective function decreases at the rate π^i_{kt} when demand for item i increases (this is just a tendency, because one unit more of item i may be sufficient to make the basis infeasible);

(ii) improve the LSP heuristic by using meta-heuristics, for example;

Table 2: *Computational results.*

		Cutting-Lot (CL)				Lot-Cutting (L/C)				Gap
Class	Nf, T, K	OFV	Victory	Solved	Iteration	OFV	Victory	Solved	Iteration	$\frac{LC-CL}{CL}$ %
1	5,8,2	62256	7	8	1.8	64848	2	10	3.7	4.0
2	10,8,2	85829	8	10	1.8	86866	2	10	2.7	1.2
3	20,8,2	175596	4	10	1.3	176741	0	10	3.1	0.6
4	5,10,2	60455	5	8	2.0	60860	3	9	3.6	0.7
5	10,10,2	109741	8	10	2.1	110528	2	10	1.7	0.7
6	20,10,2	222816	1	10	1.1	222874	1	10	2.3	0.0
7	5,12,2	83828	4	6	2.5	82422	2	6	8.3	-1.7
8	10,12,2	128425	9	9	2.1	131339	0	9	1.0	2.2
9	20,12,2	267435	1	10	1.2	267344	2	10	1.9	0.0
10	5,8,4	102251	10	10	2.1	105364	0	10	6.0	3.0
11	10,8,4	190181	9	9	2.0	193986	0	9	2.3	2.0
12	20,8,4	327226	3	10	1.4	328218	1	10	1.3	0.3
13	5,10,4	127388	6	8	1.5	130450	2	8	4.5	2.3
14	10,10,4	235800	7	10	2.0	241009	1	9	3.8	2.2
15	20,10,4	420768	1	10	1.1	421249	1	10	1.0	0.1
16	5,12,4	158357	6	9	1.6	158753	2	9	3.9	0.2
17	10,12,4	283428	8	9	1.8	286423	1	9	1.0	1.0
18	20,12,4	504548	1	10	1.1	504714	2	10	1.1	0.0
19	5,8,6	154196	7	7	2.0	162500	0	7	4.0	5.1
20	10,8,6	272086	10	10	2.2	278005	0	10	1.0	2.1
21	20,8,6	455619	3	10	1.3	457566	1	10	1.0	0.4
22	5,10,6	179063	5	6	1.5	182602	1	6	3.5	1.9
23	10,10,6	323868	10	10	2.2	328492	0	10	1.0	1.4
24	20,10,6	636062	4	10	1.4	639104	0	10	1.0	0.5
25	5,12,6	250838	4	7	1.6	253934	3	7	1.0	1.2
26	10,12,6	386629	10	10	2.3	395894	0	10	1.0	2.3
27	20,12,6	761624	3	10	1.3	762182	1	10	1.0	0.1

(iii) solve the whole cutting stock problem over the horizon planning by considering the item inventory variables all together.

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