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Abstract

This work describes a new methodology for approximating smooth curves from sample points. Using a topological framework based on Morse theory for cell complexes, this new approach avoids geometrical calculations during the reconstruction process, increasing the robustness and the computational performance of the algorithm. Furthermore, under an adequate sampling rate, shown in the paper, our approach ensures the correct reconstruction of the original curve.

Resumo

Este trabalho descreve uma nova metodologia para aproximação de curvas a partir de pontos não organizados. Utilizando uma fundamentação topológica baseada em teoria de Morse para complexos simpliciais, esta nova abordagem evita cálculos geométricos durante o processo de reconstrução, aumentando a robustez e a performance computacional. Além disso, sob uma taxa de amostragem adequada, nossa abordagem assegura a correta reconstrução da curva original.

Key words. Curve reconstruction, discrete Morse theory, Delaunay triangulation

1 Introduction

The problem of reconstructing a curve or surface from scattered points arises in different fields. This reconstruction aims at fitting a piecewise linear curve (surface) on a set of sample points taken from a smooth curve (surface).

Despite the inherent lack of geometrical and topological information in a point cloud, several algorithms have gone some way towards solving this reconstruction problem [7, 6, 8, 1, 4]. In fact, some reconstruction methods, as the ones by Amenta et al. [1, 2, 3, 4] for example, can guarantee, under an adequate sampling rate, the correct reconstruction of the original object.

Although theoretical results have been attained in the field, little has been achieved in overcoming the geometrical difficulties present in almost every reconstruction technique. Besides having high the computational cost, geometrical calculations make the algorithms less reliable, or even prohibitive in applications that deal with dense datasets.

In this work we present a new technique for reconstructing curves from sample points which reduces geometrical calculations drastically. In fact, geometrical tests are only employed in the initial steps, as for example in the Delaunay triangulation. Based on discrete Morse theory [12, 13], the algorithm uses topology for deciding which edges belong to the polygonal approximation. A discrete gradient field, which is derived from a discrete Morse function defined on the Delaunay triangulation, allows the correct edges to be chosen in an efficient way.

As in the most reliable approaches, our algorithm also ensures the correct reconstruction under an adequate sampling rate. Furthermore, its sampling rate is better than that of the others described in the literature. Although two-dimensional, the framework presented here introduces a promising methodology for reconstruction from scattered points, whose applications go beyond the one presented in this paper, such as surface reconstruction.

This work is organized as follows: section 2 presents a short review of previous works. The basic concepts, necessary to a better understanding of the following sections, are introduced in section 3. Section 4 presents the algorithm and proves its correctness. Results and comparisons are given in section 5. Conclusions and further work are discussed in section 6.

2 Previous Work

In this section we summarize the main algorithms devoted to reconstructing curves from sample points, emphasizing their advantages and drawbacks. In order to put our work in context, we describe algorithms devoted to reconstructing objects in the two-dimensional case. Some of the algorithms presented bellow have a three-dimensional version, which will not be discussed.

In few words, two-dimensional reconstruction algorithms aim at finding, from a set of sample points S, a polygon (or a set of polygons) that approximates the original curve.

One of the first attempts to approximating curves (and surfaces) from sample points is the α-shape algorithm by Edelsbrunner [9]. Starting with the Delaunay triangulation of the sample points, the α-shape removes all simplices that are not contained in an empty ball with radius $\frac{1}{\alpha}$. Although α-shape is simple to implement, it only works properly for evenly sampled points, as the same value of α is applied for the whole data set.

In the same way as α-shape, the β-skeleton algorithm [15] extracts the polygons that fit the sample points using empty balls and the Delaunay triangulation. In this approach, forbidden regions defined from the empty balls give rise to the approximation. β-skeleton, which is defined only in the two-dimensional case, is easy to implement, presenting the same disadvantages of α-shape.

Making use of a minimum spanning tree, the algorithm by Figueiredo and Miranda [11] gives theoretical guarantees of correct reconstruction for some classes of curves. This algorithm is one of the first to present a theoretical background for correct reconstruction. The main problems with this approach is that the theory is not valid for all kind of curves and the algorithm only works under a dense sample of points.

The first satisfactory algorithm theoretically guaranteed to approximate a smooth curve is the Crust algorithm by Amenta et al. [1]. Under an adequate sampling rate, it ensures that it is possible to construct a correct polygonal approximation for a closed curve using, besides the sample points, the Voronoi vertices computed from this points. Crust works in a set of points that are not evenly sampled, reducing user intervention during the reconstruction process. The instability of geometrical calculations needed for computing the Voronoi vertices is the main drawback of this technique. Furthermore, the algorithm must build two Delaunay triangulation, one to compute the Voronoi vertices and another to generate the Crust, increasing the computational cost. Another contribution presented by Amenta et al. [1] is the improvement in the β-skeleton algorithm, imposing a theoretical framework that ensures, for a specific value of β, the correct reconstruction.

The Power Crust [4] is an improvement on the Crust algorithm. This approach employs a version of the weighted Voronoi diagram, called Power Diagram, to compute a piecewise linear approximation of a smooth curve. Power Crust is also theoretically guaranteed to generate a correct reconstruction, presenting a better computational performance than Crust. A problem with Power Crust is the numerical instability due to geometical calculations in the Power Diagram. In practical applications, as shown in section 5, different results can be obtained by perturbing the sample points.

Based on Crust, Dey et al. [8] present an algorithm that reconstructs closed curves as well as arcs. Although their algorithm makes use of a less rigid sample rate, it may not ensure a correct approximation for the original object. Another disadvantage of this method is the large number of geometrical tests performed by the algorithm, since besides the Delaunay triangulation, the algorithm refines a Gabriel graph, which takes many geometrical operations.

The first work to guarantee correct reconstruction for non-smooth curves is due to Giesen [14], which ensures the correct reconstruction by using a computationally expensive traveling salesman tour. Althaus and Mehlhorn [5] improved Giesen's algorithm by showing that, under an appropriated sampling rate, the Traveling Salesman tour can be obtained in polynomial time.

Edelsbrunner makes use of the classical Morse theory to derive an algorithm to fitting a polygonal curve to a set of sample points [10]. Although Edelsbrunner's technique has a topological background, the topology is used just to deduce the geometrical calculations performed by the algorithm. Another disadvantage is that this methodology does not present any guarantee for the correct reconstruction. This two features distinguish his approach from ours, which proposes a reconstruction algorithm based on discrete Morse theory that employs topological tools in all but the initial step. Our approach, in fact, avoids geometrical calculations during the reconstruction process, making it simpler, faster, and robust. Furthermore, we prove that under an adequate sampling rate our algorithm produces a correct approximation for the original object.

3 Basic Concepts

This section introduces the basic concepts and terminology used in the remaining of the text.

Let C be a smooth closed curve in $\mathbb{R}^2$. The center of the maximal empty circles touching C in at least two points make up the *medial axis* of C. The *local feature size* of a point p in C, denoted $lfs(p)$, is the distance from p to the medial axis of C. A set of points $S \subset C$ is an *r-sample* of C if the distance of any point $p \in C$ to the closest point in S is at most $r \cdot lfs(p)$ (in this case C is said r-sampled).

Two sample points a and b of S are called adjacent iff there is an arc of C which does not contain any point of S other than a and b.

Given a set of points S in $\mathbb{R}^2$, a *triangulation* of S is a two-dimensional simplicial complex K whose vertices are the points in S and the union of the simplices in K is the convex hull of S. If the circuncircle of each triangle in K does not contain any point of S in its interior, K is called a *Delaunay triangulation*.

Let K be a simplicial complex and $f : K \longrightarrow \mathbb{R}$ a real function defined on K. f is a *discrete Morse function* [12] if for every d-dimensional simplex $\sigma \in K$:

$$\begin{aligned} \#\{\beta \supset \sigma \,|\, f(\beta) \leq f(\sigma)\} \leq 1 \\ \#\{\gamma \subset \sigma \,|\, f(\gamma) \geq f(\sigma)\} \leq 1 \end{aligned} \tag{1}$$

where β is a $(d+1)$-dimensional simplex containing σ, and γ is a $(d-1)$-dimensional simplex contained in σ.

The above definition tells that a discrete Morse function must assign higher numbers to higher dimensional simplices, with at most one exception at each simplex.

Another important concept regarding our approach is the notion of critical simplices. A d-dimensional simplex $\sigma \in K$ is *critical* if

$$\begin{aligned} \#\{\beta \supset \sigma \,|\, f(\beta) \leq f(\sigma)\} = 0 \\ \#\{\gamma \subset \sigma \,|\, f(\gamma) \geq f(\sigma)\} = 0 \end{aligned} \tag{2}$$

meaning that, in the "neighborhood" of σ, the discrete Morse function always assigns higher numbers to higher dimensional simplices, without exceptions, i.e., σ is either a maximum or a minimum of f.

Notice that regular simplices (a simplex is regular if it is not critical) appear in pairs, i.e., if σ is a d-dimensional regular simplex then either there is a $(d+1)$-dimensional simplex $\beta \supset \sigma$ such that $f(\beta) \leq f(\sigma)$ or there is a $(d-1)$-dimensional simplex $\gamma \subset \sigma$ such that $f(\gamma) \geq f(\sigma)$. Given a discrete Morse function f, we denote the pair formed by two simplices σ and β, $\sigma \subset \beta$ by $(\sigma, \beta)_f$.

An important property proved by Forman [13] is that a regular simplex forms a pair with only one simplex, i.e., the pairs $(\sigma, \beta)_f$ and $(\gamma, \sigma)_f$ do not exist simultaneously. It is not difficult to see that critical simplices do not possesses a pairing mate.

Let V and E be the sets of vertices and edges of a simplicial complex K and f a discrete Morse function defined on K. The set $Im\,(V) = \{e \in E \,|\, \exists\, v \in V \text{and } (v, e)_f\}$, called *vertex gradient field,* plays an essential role in our algorithm. $Im\,(V)$ is the set of regular edges associated with regular vertices and, as we show in next section, for an appropriate discrete Morse function, $Im\,(V)$ may generate a first polygonal approximation for the curve C.

4 Reconstruction by Discrete Morse Function

As mentioned above, the new curve reconstruction algorithm proposed in this work is based on discrete Morse theory. The idea behind this algorithm is to obtain the polygonal approximation for the original curve by analyzing the regular and critical edges of an adequate discrete Morse function defined on the Delaunay triangulation of the samples.

Before presenting the algorithm itself we formally define the discrete Morse function employed by our technique.

Let S be a set of sample points on a smooth curve $C \subset \mathbb{R}^2$, DT be the Delaunay triangulation of S, and $l(e)$ be the length of an edge $e \in DT$. Let also V, E and T be the sets of vertices, edges, and triangles of DT, respectively. We can define the discrete Morse function $f : DT \longrightarrow \mathbb{R}$ as follows:

1. for each triangle $t \in T$, $f(t) = \max\{l(e) \,|\, e \in t\}$
2. for each edge $e \in E$, $f(e) = l(e)$.
3. for each vertex $v \in V$, $f(v) = \min\{l(e) \,|\, v \in e\}$.

It is not difficult to see that the definition above satisfies the discrete Morse function properties (see the definition (1)). The ambiguous values, which might appear in isoceles or equilateral triangles for example, are eliminated by perturbing $l(e)$.

An interesting fact derived from the definition above is that the vertices of S are either critical or form pairs with edges in DT so that $Im\,(V)$ is the set of the "shortest edges" in DT. As we will see, although essential, these edges are not enough to guarantee a correct reconstruction of the original object, so, besides $Im\,(V)$, we make use of some critical edges, as is shown in the following algorithm:

Algorithm 4.1 *(For a set of sample points S)*

1. *Compute the Delaunay triangulation DT of S.*

2. *Define the discrete Morse function f as decribed above.*

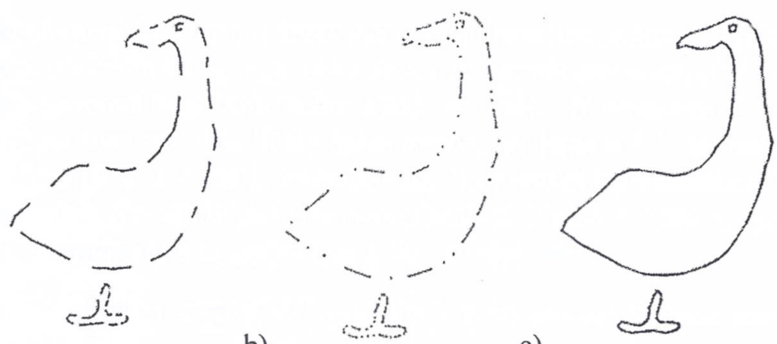
3. *Compute $Im\,(V)$.*

4. *For each sample point s contained in only one edge of $Im\,(V)$, compute the smallest critical edge incident to s. Let N be the set of all these critical edges.*

5. *Return $G = N \cup Im\,(V)$*

The set of edges G returned from the algorithm 4.1 is the polygonal reconstruction of the original smooth curve C. As in section 3, we are supposing that the original smooth curve C is a simple closed curve, thus, the polygonal approximation G must also be simple (without self-intersection) and closed.

Figure 1 shows the result of the algorithm. Figure 1a) presents the set of regular edges, i.e., the edges in $Im\,(V)$. The critical edges completing the reconstruction and the final approximation are shown in figures 1b) and 1c), respectively.



a) b) c)

Figure 1: Reconstruction of a duck. a) edges in $Im\,(V)$; b) critical edges; c) final reconstruction.

The computational complexity of the algorithm 4.1 is dominated by the Delaunay triangulation, as the discrete Morse function, regular elements, and critical elements can be computed in $\Theta(n)$, where n is the number of sample points.

The remaining of this section is devoted to proving that, under an adequate sample rate, G is actually the correct reconstruction of the original curve C.

4.2 Proof of correctness

In order to guarantee the correct polygonal reconstruction we assume that the set of sample points S is a r-sample of a smooth simple closed curve C.

We start this section stating three results from Amenta et al. [2], where their proofs can be found.

Lemma 4.3 *Let C be a smooth closed curve in $\mathbb{R}^2$. Any disc B that intersects C in at least two connected components contains at least one point of the medial axis of C.*

Lemma 4.4 *Let C be a r-sampled smooth curve in $\mathbb{R}^2$ with $r < 1$. The angle among three adjacent samples is at least $\pi - 4\arcsin(r/2)$.*

Theorem 4.5 *The distance from a point p in a r-sampled smooth curve C to points in the polygonal reconstruction of C is at most $(\frac{r^2}{2})lfs(p)$.*

Next lemma presents an important property relating the distance of sample points to their lfs.

Lemma 4.6 *$lfs(p) \leq lfs(q) + l(\overline{pq})$ for any two points p and q in S.*

Proof. Let $\tilde{q}$ be the point of the medial axis such that $lfs(q) = l(\overline{q\tilde{q}})$, so $lfs(p) \leq l(\overline{p\tilde{q}}) \leq lfs(q) + l(\overline{pq})$. ■

Lemma 4.7 *Let $e = \overline{ab}$ be an edge connecting two adjacent samples a and b. If $r < 1$ then $l(e) \leq \frac{2r}{1-r} lfs(a)$.*

Proof. The perpendicular bisector of e intersects C in at least two points. Let p be the intersection point for which the arc of C defined by a, p, and b does not contain any point of S other than a and b.

Let B be the disc centered in p with radius $l(\overline{pa}) = l(\overline{pb})$. B intersects C either in only one connected arc or in more than one arc. If the latter is true, by lemma 4.3, the radius of B must be greater than $lfs(p)$, thus, $l(\overline{pa}) \geq lfs(p)$, contradicting the r-sampling condition. That way, $B \cap C$ generates only one arc of C. By lemma 4.6 we have $lfs(p) \leq lfs(a) + l(\overline{pa}) \leq lfs(a) + r \cdot lfs(p)$, so $lfs(p) \leq \frac{lfs(a)}{1-r}$. The lemma follows since $l(e) \leq 2r \cdot lfs(p)$. ■

Proposition 4.8 *Let $e = \overline{ab}$ be an edge of DT such that b is the closest adjacent sample of a. If $r < \frac{1}{3}$ then $e \in Im(V)$.*

Proof. Let p be as in the proof of lemma 4.7. Without losing generality, we can suppose $lfs(p) = 1$. Theorem 4.5 ensures that the distance from p to e is at most $\frac{r^2}{2}$, thus, $l(e) \leq 2r\sqrt{1 - \frac{r^2}{4}}$. The ball B' centered in a with radius $l(e)$ does not have any samples of S in its interior other than a and b. In fact, if there is a sample c inside B' either c is adjacent to b or c belongs to an arc of C intersected by B'. In the first case the angle defined by abc is smaller than $\pi/2$, contradicting lemma 4.4. If c belongs to an arc of C, by lemma 4.3, B' contains a point of the medial axis of C in its interior. Notice that the distance from p to a is smaller than $r \cdot lfs(p) = r$ and that $r + l(e) = r + 2r\sqrt{1 - \frac{r^2}{4}} < 1$, thus, B' is inside the unitary ball centered in p (figure 2), which does not contain any point of the medial axis in its interior. Hence, e is the smallest edge containing a, ensuring that $e \in Im(V)$. ■

The proposition above is extremely important, since it ensures that each vertex is connected to its closest adjacent vertex. Furthermore, the proposition also guarantees that every sample point is contained in at least one edge in $Im(V)$.

The next proposition is the final ingredient for ensuring the correctness of our algorithm.

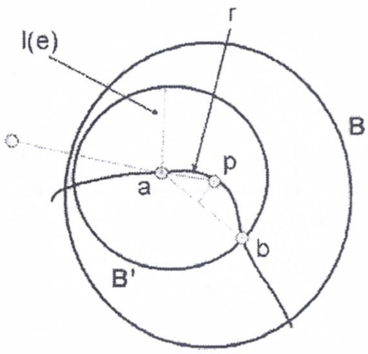


Figure 2: Edge in $Im(V)$

Proposition 4.9 *Let* $e = \overline{ab}$ *be an edge connecting two adjacent samples in a* r*-sampled smooth curve* C*, with* $r < \frac{1}{3}$*. If* $e \notin Im(V)$ *then* e *is a critical edge of* f*. Furthermore,* e *is the smaller critical edge incident on* a *and* b*.*

Proof. Suppose that $e \notin Im(V)$ is not a critical edge, i.e., e is the longest edge of a triangle $t \in DT$. Let c be the vertex of t which is not incident on e and B be the circuncircle of t (figure 3). By lemma 4.4, we know that c is adjacent neither to a nor to b. Then, B intersects C in more than one component, so, by lemma 4.3, B contains a point x of the medial axis of C in its interior. As $r < 1$, x must be in the region of B between e and c, thus, $l(e) > lfs(a)$. As $l(e) \leq \frac{2r}{1-r} lfs(a)$ (lemma 4.7), we have $lfs(a) < l(e) \leq \frac{2r}{1-r} lfs(a)$ contradicting the hypothesis $r < \frac{1}{3}$.

Suppose now that there is another critical edge e', smaller than e, containing a. The ball whose diameter is e' contains a point of the medial axis in its interior, thus, $lfs(a) < l(e')$. As $l(e') < l(e) \leq \frac{2r}{1-r} lfs(a)$, then $r > \frac{1}{3}$, contradicting the hypothesis. ■

The next theorem, which ensures the correctness of our algorithm, follows straightforwardly from propositions 4.8 and 4.9.

Theorem 4.10 *If* $r < \frac{1}{3}$ *then the edges of* G *connect exactly each pair of adjacent samples of* S*.*

It is important to note that the sampling required by theorem 4.10 is better than those described in the literature, making our algorithm more reliable in practical applications, as presented in next section.

5 Results

In this section we present results of our algorithm when applied to some practical problems. We also compare those results with other 2D reconstruction methods described in the literature, namely,

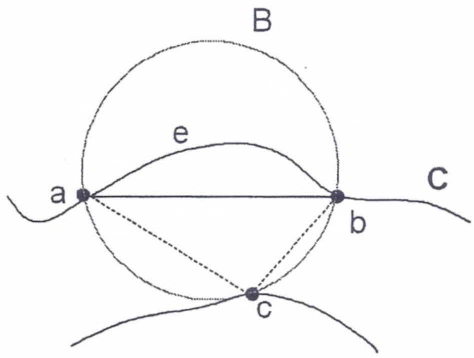


Figure 3: Proposition 4.9

Algorithm	Time (ms)
Our algorithm	32
Power-Crust	484
Crust	359
Beta-Skeleton	32

Table 1: Time consumption of the algorithms.

Crust, Beta-skeleton, and Power-Crust. The algorithms were implemented in C++ and a set of points sampled from the Brazil map was employed as input during the comparisons.

Figure 4 shows the results obtained with each algorithm. The best results were obtained by our algorithm and by Power-Crust, as observed in figures 4b) and d), respectively. As can be seen from figure 4c), Crust misses some edges, generating a set of open curves. Beta-skeleton gives a very unsatisfactory reconstruction (figure 4e)), leaving many undesired edges.

Table 1 presents the computational times (in milliseconds) spent by the algorithms. We can see that our algorithm is about fifteen times faster than the Power-Crust, the only one, besides ours, to produce a satisfactory reconstruction. Although Beta-skeleton presents a performance similar to our algorithm, its results are not satisfactory.

Another important fact that deserves mentioning is the numerical instability presented by Power-Crust. Perturbing some points close to Marajó's island (see figure 5a)), Power-Crust connects the continent to the island, as shown in figure 5b). With the same perturbation, our algorithm produces the correct reconstruction, as in figure 4b).

The results are conclusive of the fact that reconstruction by discrete Morse functions represents an advance for 2D reconstruction from sample points, as it shares all the good properties of the best algorithms, namely: theoretical guarantees, robustness, and computational performance.

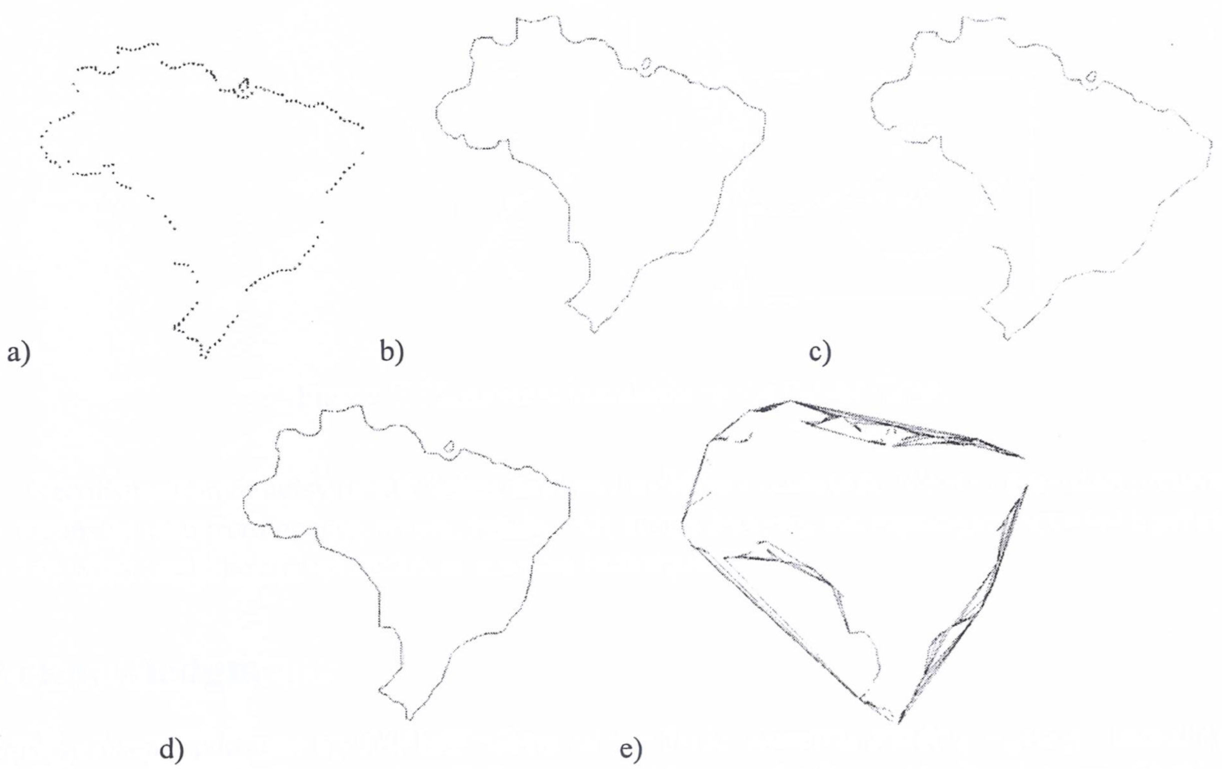


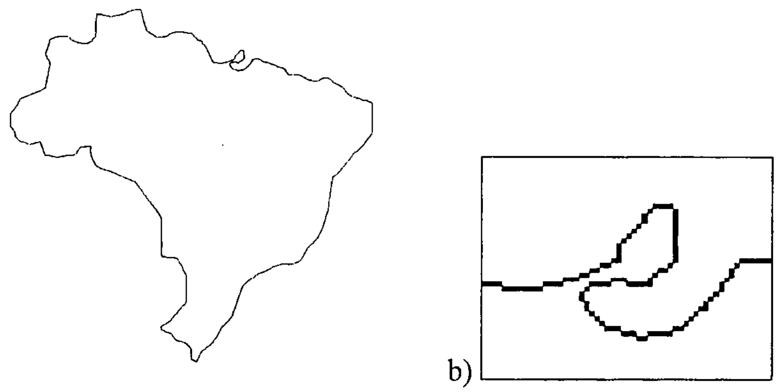
Figure 4: Brazil map reconstruction: a) sample points; b) our algorithm; c) Crust; d) Power-Crust; e) Beta-skeleton

6 Conclusion and Future Work

This paper presents a new algorithm for approximating smooth simple closed curves in $\mathbb{R}^2$ from a set of sample points on this curve that improves existing solutions in robustness and performance without the need for additional sampling. Based on discrete Morse theory, our approach presents a new methodology which avoids geometrical calculation, handling the reconstruction problem in a simple and robust way. Furthermore, our algorithm is easy to implement and offers theoretical guarantees similar to the well known algorithms, with better computational performance.

Two important aspects are under investigation: how to extend the theory to surface reconstruction and how to adapt the discrete Morse function to deal with noisy data.

As the current methods for surface reconstruction from sample points are unstable and, in several cases, not cost effective, the extension of this topological approach to surface reconstruction will represent an important contribution for this research field.



a) b)

Figure 5: Numerical instability of the Power-Crust.

Reconstruction of noisy data is being explored in different contexts. We are particularly interested in reconstruction from images where, jointly with image filtering, the topological approach presented in this work could be an alternative to existing techniques.

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