# UNIVERSIDADE DE SÃO PAULO

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## NOTAS

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# LOT SIZING AND SCHEDULING IN AN AUTOMATED FOUNDRY

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**Abstract:** The studied foundry has only a furnace and several molding machines in order to produce a known demand of different types of items which can be made of different alloys. There are two important and linked decision levels: for each programming period, which alloy should be produced in the furnace, and the quantity of items to be produced in each molding machine. Two cases are distinguished: a single alloy can produce all items, and different alloys are needed in order to produce the items. Assuming that the furnace is the production bottleneck, efficient problem-based solution methods are proposed.

Keywords: linear and integer programming, lot sizing, scheduling.

**Resumo:** Este trabalho consiste num estudo de planejamento de uma fundição automatizada, para a qual desenvolvemos um modelo matemático e um método de solução. A fundição tem um único forno e várias máquinas paralelas de moldagem para a produção de uma demanda conhecida de diferentes tipos de itens, os quais podem ser feitos de diferentes ligas metálicas. Existem dois níveis importantes de decisão que são interligados: para cada período, qual liga deve ser fundida no forno e, quais devem ser as quantidades dos itens produzidos em cada máquina?. Distinguimos dois casos: uma única liga pode produzir todos os items e, diferentes ligas são necessárias para a produção dos itens. Supondo que o forno seja o gargalo de produção, propomos métodos heurísticos baseados no problema.

Palavras-chaves: programação linear inteira, dimensionamento de lotes, programação da produção.

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# 1. INTRODUCTION

Lot sizing and scheduling problems arise in planning the production of several items on different machines over a number of time periods in such a way as to optimize a certain objective function (see Bahl, *et al.,* 1987; Berretta, 1997; Billington, *et al.*, 1983; Drexl and Kimms, 1997, França, *et al.*, 1997; Shapiro, 1993). Several of these problems are in class NP, so that exact algorithms generally fail to obtain optimality for large instances. Particularly, Maes, *et al.*, (1986) showed that the feasibility problem for lot sizing problems which include setup time (0-1 variables are explicity used in the capacity constraints) are NP-complete, which means that no easy method to check feasibility is available. This is the reason that most algorithms are heuristic. Although setup times are not considered in the problem studied in this paper, it is worth noting that one of the mathematical models also includes 0-1 variables in some constraints, but with a different meaning.

This article is concerned with a single-level, multiitem, capacitated, parallel machines lot sizing and scheduling problem that arises in an automated foundry. The basic equipment in a foundry are a furnace and a set of molding machines. Figure 1 shows a simplified scheme of the main activities in a foundry.

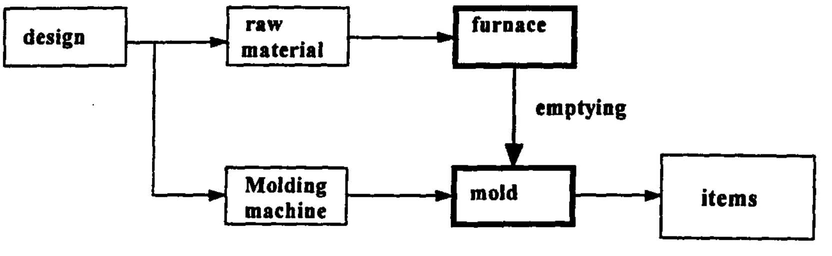


Fig. 1. Main Activities in a Foundry

The furnace is fed with raw materials producing a type of alloy, and has limited production capacity per hour. The liquid alloy is poured out, and then the furnace is fed again in order to produce a new (or the same) type of alloy.

The alloy produced is poured into sand molds which are the outcome of molding machines whose production capacities per hour are known. These parallel molding machines can produce different types of molds within a programming period, but these molds are for producing items that use the same alloy, since just one type of alloy can be produced within a period. This shows that the production planning in the studied foundry has two important and linked levels: the scheduling of the furnace, where the type of the alloy must be decided for a programming period and the scheduling of the molding machines by specifying which and how many of each item should be produced in that period, i.e., lot sizing.

It should be pointed out that the solution methods proposed in this work are valid when the furnace is the production bottleneck, that is, the molding machine capacities are sufficiently larger than the furnace capacity in a programming period.

Two cases are studied: i) a single alloy produces all items, and ii) different types of alloys have to be produced. The mathematical models can be very large in practice, mainly depending on the number of items and programming periods.

## 2 THE FIRST PROBLEM: A SINGLE ALLOY

This section considers the problem where only a single alloy is needed in order to produce all demanded items. Therefore, there is no furnace scheduling problem and it is modeled as a linear programming problem. We use the following symbols:

*Indices:*

$m = 1,....,M$ molding machines;
$t = 1,....,T$ periods;
$i = 1,....,N$ items.

*Data:*

$a_{im}$ the maximum quantity of item $i$ that molding machine $m$ can produce per hour, (*ton/h*). ($a_{im} = 0$ means that item $i$ cannot be produced by machine $m$);

$F_t$ the maximum quantity of alloy produced by the furnace per hour in period $t$, (*ton/h*);

$h_t$ number of hours in period $t$, ($h$);

$d_{it}$ demand of item $i$ in period $t$, (*ton*);

$c_{it}$ production cost of item $i$ in period $t$ (*\$/ton*).

*Decision Variables:*

$X_{imt}$ the fraction of period $t$ used by molding machine $m$, producing item $i$;

$I_{it}$ inventory of item $i$ at the end of period $t$, (*ton*).

Note that $a_{im}h_tX_{imt}$ means the quantity of item $i$ produced in period $t$ in machine $m$. The mathematical model follows:

$$Minimize \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{m=1}^{M} c_{it}\, a_{im} h_t\, X_{imt} \qquad (1)$$

*Subject to*:

$$\sum_{m=1}^{M} a_{im} h_t X_{imt} - I_{it} + I_{i\,(t-1)} = d_{it} \qquad i = 1,...,N \quad t = 1,...,T \qquad (2)$$

$$\sum_{i=1}^{N}\sum_{m=1}^{M} a_{im}\, h_t X_{imt} \le F_t\, h_t \qquad t = 1,...,T \qquad (3)$$

$$\sum_{i=1}^{N} X_{imt} \le 1 \qquad t = 1,...,T \quad m = 1,...,M \qquad (4)$$

$$X_{imt} \ge 0 \quad i=1,...,N,\ m=1,...,M,\ t=1,...,T \qquad (5)$$

In the previous model, the objective function (1) is to minimize the total production costs ($c_{it}$ can be defined as in Billington, *et al.*, (1986): $c_{it}=c_i(T\text{-}t+1)$, where $c_i$ is a cost added to the holding cost of each unit of product $i$. Of course, one can straightforwardly price the variables $I_{it}$ with no change to the algorithm). Equations (2) represent the inventory balances;

inequalities (3) define the furnace production capacity; inequalities (4) are due to $X_{imt}$ being a time ratio. Figure 2 depicts the production flow chart in the foundry at any period *t*.

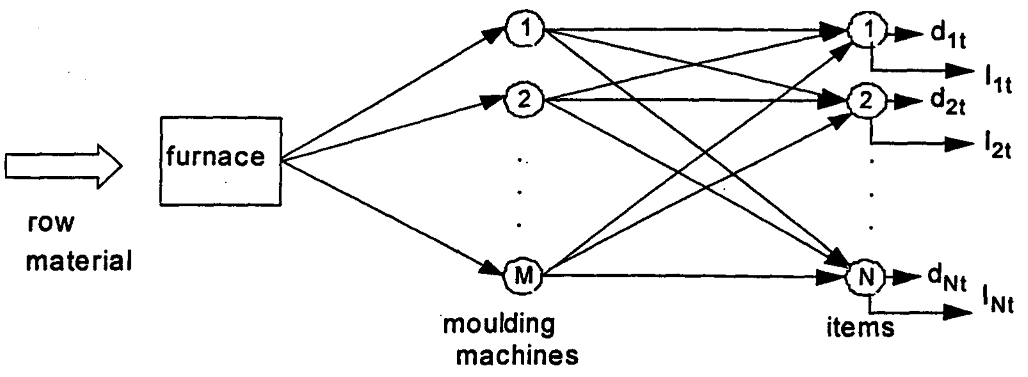


Fig. 2. Production Flow

### *2.1 Solution method*

An optimal solution method is presented for when the production bottleneck is the furnace. In this case, constraints (4) can be relaxed without loss of optimality.

Consider the model (1)-(5) relaxing (4) and let $P_{it}$ be a new variable defined as:

$$P_{it} = \sum_{m=1}^{M} a_{im} h_t X_{imt},$$

which means the quantity of item *i* to be produced in period *t*. The relaxed problem can be rewritten as a single-level capacitated lot sizing problem (Trigeiro *et al.*, 1989):

*The relaxed problem*

$$\text{Minimize} \quad \sum_{t=1}^{T}\sum_{i=1}^{N} c_{it} P_{it} \qquad (6)$$

*Subject to*:

$$\sum_{i=1}^{N} P_{it} \le F_t\, h_t \qquad t = 1,\dots,T \qquad (7)$$

$$P_{it} + I_{i\,(t-1)} - I_{it} = d_{it} \qquad t = 1,\dots,T,\ i = 1,\dots,N \qquad (8)$$

$$P_{it} \ge 0 \qquad t = 1,\dots,T,\ i = 1,\dots,N \qquad (9)$$

The relaxed problem can be seen as a network flow problem as illustrated in Fig. 3.

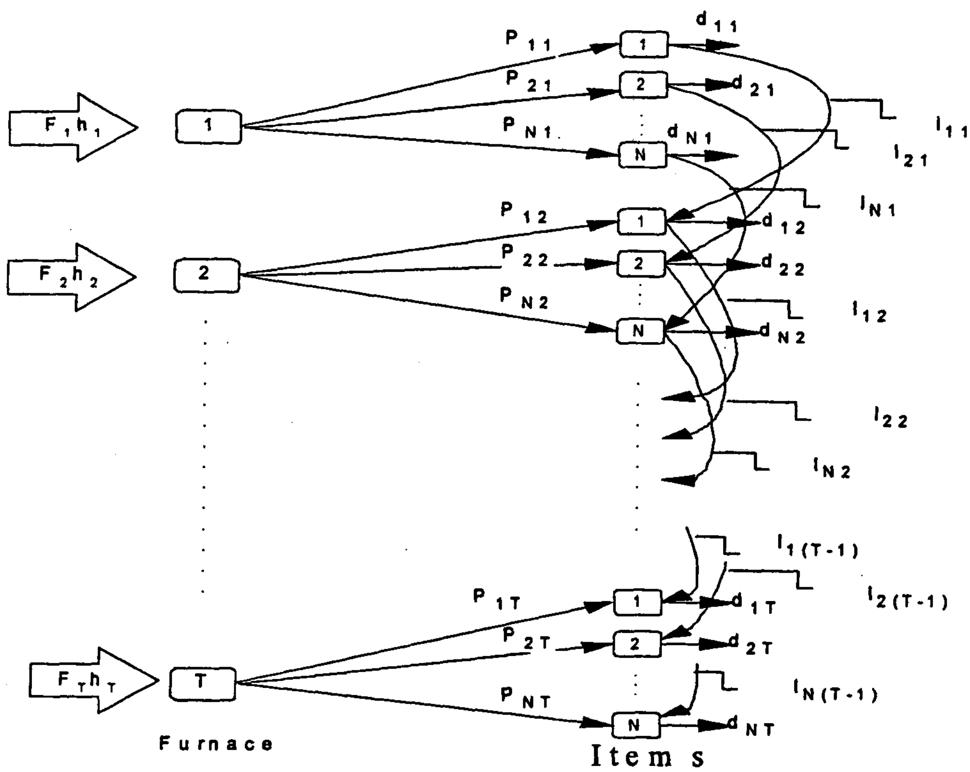


Fig. 3. The relaxed problem

After the relaxed problem is solved (which provides the quantity of item $i$ that should be produced in each programming period), the molding machine can be scheduled. Therefore, the following generalized transportation problem should be solved, for each $t$.

$$\text{Minimize} \quad \sum_{m=1}^{M}\sum_{i=1}^{N} X_{imt} \tag{10}$$

*Subject to:*

$$\sum_{i=1}^{N} X_{imt} \le 1 \qquad m = 1,\dots,M \tag{11}$$

$$\sum_{m=1}^{M} a_{im} h_t X_{imt} = P_{it} \qquad i = 1,\dots,N \tag{12}$$

$$X_{imt} \ge 0 \qquad m=1,\dots,M \;\; i=1,\dots,N \tag{13}$$

An alternative objective for the machine scheduling can be:

$$\min \max_{m=1,\dots,M} \left\{ \sum_{i=1}^{N} X_{imt} \right\} \tag{14}$$

which balances the production of items among the molding machines better than (10). In this case, constraints (11) are not necessary and the problem could be modeled as the following. Let

$$Z_t = \max_{m=1,\dots,M} \left\{ \sum_{i=1}^{N} X_{imt} \right\} \tag{15}$$

be a new variable. Then the balanced schedule of machines is given by:

$$minimize\ Z_t \tag{16}$$

*Subject to:*

$$\sum_{i=1}^{N} X_{imt} \le Z_t \qquad m = 1,...,M \tag{17}$$

$$\sum_{m=1}^{M} a_{im} h_t X_{imt} = P_{it} \qquad i = 1,...,N \tag{18}$$

$$X_{imt} \ge 0 \qquad m=1,...,M \quad i=1,...,N \tag{19}$$

Note that if the production bottleneck was not at the furnace, the problem (10)-(13) could be unfeasible, or $Z_t > 1$ in (16)-(19). If this is the case, it is possible to devise an iterative procedure by first identifying periods where the production of items is very large yielding, for example, $Z_t > 1$, and then going back to the relaxed problem (6)-(9) and tightening constraint (7).

### *2.2 The Algorithm*

Here the proposed method is summarized which is exact under the assumption that the capacities of the molding machines are large enough.

*1. Solve the problem (6)-(9) determining $P_{it}$, the quantity of the item i to be produced in period t.*

*2. For each period t, t=1,...,T, solve (10)-(13) (or (16)-(19)) determining $X_{imt}$, the fraction of period t that the molding machine m is used to produce item i. End.*

## 3 THE SECOND PROBLEM: DIFFERENT ALLOYS

In this section, different types of alloys must be produced because the demanded items utilize different alloys. But in any programming period only a single alloy can be produced, since there is just one furnace in operation. In addition to the problem in Section 2, consider:

*The Additional Index:*

$j = 1,...., L$ alloys.

*The Additional Set:*

$L_j$ the set of items that can be produced by alloy $j$ (an item can be in two or more of such sets).

*The Additional Variable:*

$$Y_{jt} = \begin{cases} 1, & \text{if the alloy } j \text{ is produced in period } t \text{, or} \\ 0, & \text{otherwise.} \end{cases}$$

$\lambda$ a large number.

The mathematical mode is follows.

$$Minimize \quad \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{m=1}^{M} c_{it} a_{im} h_t X_{imt} \qquad (20)$$

*subject to:*

$$\sum_{m=1}^{M} a_{im} h_t X_{imt} - I_{it} + I_{i\ (t-1)} = d_{it} \qquad i = 1,\ldots,N \quad t = 1,\ldots T \qquad (21)$$

$$\sum_{i=1}^{N}\sum_{m=1}^{M} a_{im} h_t X_{imt} \leq F_t\ h_t \qquad t=1,\ldots,T \qquad (22)$$

$$\sum_{i \in L_j} X_{imt} \leq \left(1 - Y_{jt}\right)\lambda + 1 \qquad j=1,\ldots,L,\ m=1,\ldots,M,\ t=1,\ldots,T \qquad (23)$$

$$\sum_{i \notin L_j} X_{imt} \leq \left(1 - Y_{jt}\right)\lambda \qquad j=1,\ldots,L,\ m=1,\ldots,M,\ t=1,\ldots,T \qquad (24)$$

$$\sum_{j=1}^{L} Y_{jt} = 1 \qquad t=1,\ldots,T \qquad (25)$$

$$X_{imt} \geq 0 \qquad i=1,\ldots,N,\ m=1,\ldots,M,\ t=1,\ldots,T \qquad (26)$$

$$Y_{jt} \in \left\{0,1\right\} \qquad j=1,\ldots,L,\ t=1,\ldots,T \qquad (27)$$

In the model above constraints (23) and (24) assure that only items associated with the chosen alloy are produced, and (25) are logical constraints to assure that only a single alloy can be produced in each programming period $t$.

### *3.1 A heuristic approach*

A relaxation strategy is devised in order to guide the search in the solution space. Consider the relaxation of the set $L_j$ such that every alloy can produce all items, i.e., $\bar{L}_j = \{1,\ 2,\ldots,N\}$ (Gonçalves-Vianna and Arenales, 1995). Therefore, constraints (23), (24), and (25) become unnecessary, and the problem is transformed into one of a single alloy (1)-(5) which can be solved by the algorithm 2.2.

The solution of the relaxed problem (6)-(9) provides $P_{it}$ the quantity of item $i$ in period $t$, but now, before the molding machine can be scheduled, it is necessary to verify if there is an alloy capable of producing such items for each period. Therefore, one should search for alloys $j_t$, $t=1\ldots T$, such that

$$\left\{i:\ P_{it} > 0, i = 1,2,\ldots,N\right\} \subseteq L_{j_t}.$$

Otherwise, the following heuristic is used to choose an alloy.

*Heuristic to choose an alloy in period $t>1$:*

*Suppose $j_1,...,j_{t-1}$ are the indices of alloys already chosen for periods $1,...,t-1$, respectively (note that the items in sets $L_{j_k}$, $k=1,...,t-1$, can therefore be produced before $t$).*

*1. Determine $A_{t-1}$ the set of items which can be produced before $t$: $A_o=\varnothing$,*

$$A_{t-1} = L_{j_1} \cup L_{j_2} \cup ....\cup L_{j_{t-1}}.$$

*2. Determine $B_t$ the set of items with the highest priority in $t$:*

$$B_t = \{i \mid i \notin A_{t-1} \text{ and } d_{it} > 0\}$$

*3. If $B_t \neq \varnothing$*

*then*

*3.1 Determine $C_t$ the set of alloys with high priority in $t$:*

$$C_t = \{j : L_j \subseteq B_t\}$$

*(If $C_t=\varnothing$ then the heuristic fails or the original problem is unfeasible).*

*Choose the alloy $j_t$ for the period $t$ such that:*

$$\max_{j \in C_t} \left\{ \sum_{i \in L_j} P_{it} \right\}$$

*else*

*3.2 Choose the alloy $j_t$ for the period $t$ such that:*

$$\max_{j=1,...,L} \left\{ \sum_{i \in L_j} P_{it} \right\}$$

End of the heuristic.

If the chosen alloy $j_t$ is such that there exists $i \notin L_{j_t}$ with $P_{it}>0$, the relaxed problem has to be solved once more before choosing the alloy for the next period, fixing $P_{it}=0$, for all $i \notin L_{j_t}$. Note that the old solution is no longer feasible.

Once the alloys are determined for all periods, one should schedule the molding machines to obtain, for each period, the quantity of each item to be produced by each machine.

The machine scheduling problem for each period $t$ is the same as (10)-(13) or (16)-(19), but with the additional conditions:

$$X_{imt} \geq 0 \quad m = 1,...,M \quad i \in L_{j_t} \tag{28}$$

$$X_{imt} = 0 \quad m = 1,...,M \quad i \notin L_{j_t} \tag{29}$$

where $j_t$ is the alloy chosen for period $t$.

In terms of the model (20)-(27), $Y_{j_t t}=1$ and $Y_{jt}=0$ for all $j \neq j_t$, which satisfies (25), constraints (24) are held due to (29), and (23) become (4). The other constraints have already been considered in problem (1)-(5), which is the relaxed problem of (20)-(27), obtained by the relaxation of set $L_j$. Therefore, the procedure produces a feasible solution, but, of course, the optimality may be lost. Some computational performances of this heuristic are reported in section 3.3. Next the algorithm is summarized.

*3.2 The Algorithm*

*Let $t = 1$.*

1. *While $t \leq T$ do,*

   *1.1. Solve the relaxed problem (6)-(9).*

   *1.2. Choose the alloy $j_t$ for period $t$ using the heuristic in section 3.1.*

   *If $\{i \mid P_{it} > 0,\ i=1,\dots,N\} \subseteq L_{j_t}$*

   *then: $t=t+1$ and repeat step 1.2.*

   *else: Fix $P_{it} = 0$, $i \notin L_{j_t}$, $t=t+1$,*

   *and go to step 1.1.*

2. *Solve the relaxed problem. (Recall that the alloys $j_1, j_2, \dots, j_T$ were chosen for the periods 1,2,....,T, respectively). For each period $t$, $t=1,\dots,T$, solve the problem (10)-(13) (or (16)-(19)) together with (28)-(29), determining $X_{imt}$, the fraction of period $t$ that machine $m$ has to produce item $i$. End.*

*3.3 Computational Experiments.*

The heuristic algorithm in section 3.2 to solve problem (20)-(27) was implemented with C language using the CPLEX library for solving the problems (6)-(9) and (10)-(13) (or (16)-(19)) with the additional constraints (27) and (28) and run on a SUN/Sparc/Classic.

For a medium size problem such as $N=25$, $T=8$, $L=8$ and $M=10$, the Cplex package failed to find a feasible solution after running for approximately 5 hours. It should be pointed out that the CPLEX package uses a starting heuristic that consists of fixing a set of integer variables whose values in the LP relaxation are integer or almost integer. Although this heuristic may be efficient in general, it can lead to a mistake in chosing some alloys, in the present study, yielding unfeasibility that can ruin the performance of a branch and bound method because no upper bound will ever be available. On the other hand, the proposed heuristic method was able to obtain a feasible solution in 30 seconds.

In order to analyze the quality of the heuristic solutions, were randomly generated dozens of small scale problems and run using the CPLEX and the proposed heuristic. The CPLEX was able to obtain the optimal solutions and the heuristic solutions were approximately 0.5%-optimal.

## 4. FINAL REMARKS

This study shows the importance of developing problem-based heuristic methods in order to solve large practical integer programming problems, which are, in general, NP. General heuristics, such as the one used by the CPLEX package (see comments in section 3.3) to find an initial solution can be useful, of course, if one does not have any ideas of how to solve the problem, but they will scarcely be better than problem-based heuristic procedures, or even, they can fail in order to obtain an acceptable solution.

Foundry production planning problems are not, in general, uncoupled problems. Indeed, the casting stage is the last stage in multistage assembly systems (see Afentakis and Gavish, 1986; Afentakis, *et al.,* 1984; Santos-Meza, 1997). Therefore, it is necessary to have a rapid and efficient solution method for the foundry planning problem to solve the overall production planning problem. How to integrate the foundry problem into a multistage system (*coordination problem*) is a topic for future research (actually, directions are given in Santos-Meza, 1997), which is probably the most crucial goal for future work, according to Drexl and Kimms (1997).

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