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Abstract

In this paper the convergence of a simple product integration method which solves a non-linear integral equation arising from non-linear waves in shock tubes is analysed. Simple numerical results are given.

Resumo

Neste trabalho a convergência de um método simples do tipo integração produto, o qual resolve uma equação integral não linear proveniente de ondas de choque não lineares em tubos, é analisada. Resultados numéricos são apresentados.

Keywords: Integral Equations, Product Integration Method, Convergence.

Palavras-Chaves: Equações Integrais, Método Integração Produto, Convergência.

1 Introduction

Knowledge of the boundary-layer development behind a moving shock is important for evaluating the non-ideal behavior of shock tubes. Keller (1981), restricted his consideration to relatively weak waves when the fluid velocities are small and assumed the boundary-layer thickness to be small compared with the diameter of the tube. By considering the conservation of mass and momentum and using Chester's (1964) boundary-layer parameter

$$\beta = \frac{2}{R} \quad \left(\frac{\nu}{\pi}\right)^{\frac{1}{2}} \quad \left(1 + \frac{\gamma - 1}{Pr^{\frac{1}{2}}}\right),$$

where ν, γ, Pr and R denote the kinematic viscosity, the ratio of specific heats, the Prandtl number and the diameter of the tube, respectively, he was able to derive the following integral equation

$$\left(\frac{\gamma+1}{2c_0}\right) u^2 = \beta \int_0^\tau u(\tau-\xi)\xi^{-\frac{1}{2}} d\xi,$$

where c_0 denotes the undisturbed sound speed. In a later paper Schneider(1982) considered the generalised equation

$$y(t)^{1+\frac{\beta}{\alpha}} = \frac{\Gamma(1+\alpha+\beta)}{\Gamma(1+\alpha)\Gamma(\beta)} \quad \int_0^t (t-s)^{\beta-1} y(s) ds, \tag{1.1}$$

where $0 < \beta < 1$ and α is positive constant, and showed rigorously that

$$y(t) = \begin{cases} 0, & 0 \le t \le c, \\ (t-c)^\alpha, & t \ge c, \end{cases} \tag{1.2}$$

with arbitrary $c \ge 0$, was, rather remarkably, the unique solution of equation (1.1).

Since a simple unique solution exists it may well be argued that there is little point in studying numerical methods for its solution and for this particular problem this is true. However it does contain some interesting and difficult features for the numerical analyst. For example it has a descontinuous first derivative at $t = c$, it is non-linear and has an integrable singularity. It is therefore a difficult problem to design a numerical algorithm for its solution. Thus a working numerical method and especially a convergence proof would be useful in understanding generally problems with the difficult features. The fact that a solution exists us in checking the validity of such an algorithm.

The purpose of this note is to present a numerical method of product integration type, obtained by replacing the integrand's unknown functions by constants, and demonstrate it to be convergent. Although being a low-order method, it has practical interest because of its ease of implementation. Futhermore, by reducing the mesh spacing sufficiently, results which are as accurate as is desired can be obtained. Many authors (among them, Weiss(1972), Eggermont (1979), Franco(1983) e Tang(1992)), have employed this method to successfully solve integral equations.

2 The numerical method

The method which we shall propose here is to replaces $y(s)$ by $y(t_j)$ for $s \in [t_j, t_{j+1}]$. It is therefore a simple product integration method.

We define for all $h \in (0, h_0), h_0 > 0$, such that $Nh = t_N = 1$ for some integer N the function

$$\Delta_h : C[0,1] \to I\!R^{N+1},$$

so that

$$\Delta_h \phi(t) = (\phi(0), \phi(h), \ldots, \phi(1-h), \phi(1))^T.$$

Let $\phi_h y = 0$ define all algorithms such that

$$\phi_h : I\!R^{N+1} \to I\!R^{N+1}.$$

Let $t_i = t_{i-1} + h$ with $t_0 = 0$.

We can rewrite equation (1.1) in the form

$$[y(t_i)]^{1+\frac{\beta}{\alpha}} = \frac{\Gamma(1+\alpha+\beta)}{\Gamma(1+\alpha)\Gamma(\beta)} \sum_{j=0}^{i-1} \int_{t_j}^{t_{j+1}} (t_i - s)^{\beta-1} y(s) ds. \tag{2.1}$$

Thus if y_i denotes an approximation to $y(t_i)$ at $t = t_i$, $i = 1, 2, \ldots, N$, we can write

$$y_i{}^{1+\frac{\beta}{\alpha}} = \frac{\Gamma(1+\alpha+\beta)}{\Gamma(1+\alpha)\Gamma(\beta)} \sum_{j=0}^{i-1} y_j \int_{t_j}^{t_{j+1}} (t_i - s)^{\beta-1} ds.$$

Computing the integral we obtain

$$y_i{}^{1+\frac{\beta}{\alpha}} = \mu_1 h^\beta \sum_{j=0}^{i-1} w_{i-j} y_j, \tag{2.2}$$

where

$$\mu_1 = \frac{\Gamma(1+\alpha+\beta)}{\beta\Gamma(1+\alpha)\Gamma(\beta)},$$

and

$$w_{i-j} = (i-j)^\beta - (i-j-1)^\beta. \tag{2.3}$$

Now, we can define, more formally, the discretization algorithm $\phi_h \mathbf{y} = 0$, where

$$[\phi_h \mathbf{y}]_i = \begin{cases} y_i - 0, & 0 \le i \le k, \\ y_i - h^\alpha, & i = k, \\ y_i^{1+\frac{\beta}{\alpha}} - \mu_1 h^\beta \quad \sum_{j=0}^{i-1} w_{i-j} y_j, & k+1 \le i \le N, \end{cases} \tag{2.4}$$

where y_i, $0 \le i \le k$, are necessary given values for a uniqueness solution (Schneider(1982)) and y_i, $k+1 \le i \le N$, are approximation for $y(t_i)$. Note that the algorithm (2.4) would be solved for We can y_i, $i = k+1, \ldots, N$, in a step-by-step fashion.

In matrix notation (2.4) becomes

$$\phi_h y = y^{1+\frac{\beta}{\alpha}} - b_1 - \mu_1 h^\beta A_N y, \tag{2.5}$$

where

$$y^{1+\frac{\beta}{\alpha}} = (y_0^{1+\frac{\beta}{\alpha}}, y_1^{1+\frac{\beta}{\alpha}}, \ldots, y_N^{1+\frac{\beta}{\alpha}})^T,$$

$$b_1 = (0, \ldots, 0, h^{\alpha+\beta}, 0, \ldots, 0)^T,$$

with $h^{\alpha+\beta}$ in the k-th position, and

$$(A_N)_{ij} = \begin{cases} w_{i-j}, & k \le i \le N, \quad k \le j \le i-1, \\ 0, & \text{otherwise.} \end{cases} \tag{2.6}$$

3 Preliminary lemmas

The following discrete Gronwall lemmas we be required.

Lemma 3.1: (Henrici (1962)) If x_j, $j = 0, 1, \ldots, N$ is a sequence of real numbers with

$$| x_i | \le hM \sum_{j=0}^{i-1} | x_j | + \delta; \quad i = 1, 2, \ldots, N,$$

where $\delta > 0$ and $M > 0$ is usually independent of h, then

$$| x_i | \le (hM | x_0 | + \delta) e^{Mih}; \quad i = 1, 2, \ldots, N.$$

Lemma 3.2: Let x_j, y_j, $j = 0, 1, \ldots, N$ be sequences of real numbers satisfying:

$$| x_i^{\gamma} - y_i^{\gamma} | \leq Mh^{\beta} \sum_{j=0}^{i-1} \frac{1}{(i-j)^{1-\beta}} | x_j - y_j | + \delta, \quad i = 1, 2, \ldots, N, \tag{3.1}$$

where $\gamma > 1, 0 < \beta < 1, \delta > 0$ and $M > 0$ is independent of h, then

$$| x_i^{\gamma} - y_i^{\gamma} | \leq (hM' | x_0 - y_0 | + \delta') e^{M'ih}, \quad i = 1, 2, \ldots, N, \tag{3.2}$$

where

$$M' = MB(\beta, \beta),$$

with $B(a, b)$ the beta function defined for $Re(a) > 0$, $Re(b) > 0$ by

$$\int_0^1 \frac{du}{u^{1-a}(1-u)^{1-b}},$$

and

$$\delta' = \delta(1 + \frac{M}{\beta}).$$

Proof

We can given that

$$| x_k^{\gamma} - y_k^{\gamma} | \leq M_1 h^{\beta} \sum_{i=0}^{k-1} \frac{1}{(k-i)^{1-\beta}} | x_i - y_i | + \delta, \quad k = 1, 2, \ldots, N,$$

Let

$$L = \max_{0 \leq i \leq N-1} | x_i - y_i |,$$

and

$$l = \min_{\substack{0 \leq i \leq N-1 \\ x_i \neq y_i}} | x_i^{\gamma} - y_i^{\gamma} | .$$

Then, we can determine C so that

$$L \leq Cl.$$

Therefore, exists C_1, so that

$$| x_i - y_i | \leq L \leq Cl \leq C_1 | x_i^\gamma - y_i^\gamma |; \quad 0 \leq i \leq N-1.$$

Thus for $k = 1, 2, \ldots, N$ we have

$$\begin{aligned} | x_k{}^\gamma - y_k{}^\gamma | &\leq M_1 h^\beta \sum_{i=0}^{k-1} \frac{1}{(k-i)^{1-\beta}} C_1 | x_i^\gamma - y_i^\gamma | + \delta \\ &= M h^\beta \sum_{i=0}^{k-1} \frac{1}{(k-i)^{1-\beta}} | x_i^\gamma - y_i^\gamma | + \delta, \end{aligned}$$

where $M = M_1 C_1$.

Using (3.1), we obtain, for $k = 1, 2, \ldots, N$, that

$$| x_k{}^\gamma - y_k{}^\gamma | \leq M_1 h^\beta \sum_{i=0}^{k-1} \frac{1}{(k-i)^{1-\beta}} \left(M h^\beta \sum_{j=0}^{i-1} \frac{1}{(i-j)^{1-\beta}} | x_j - y_j | + \delta \right) + \delta.$$

Collecting terms and noting that

$$\sum_{i=1}^{k-1} \sum_{j=0}^{i-1} \frac{1}{(k-i)^{1-\beta}(i-j)^{1-\beta}} = \sum_{j=0}^{k-2} \sum_{i=j+1}^{k-1} \frac{1}{(k-i)^{1-\beta}(i-j)^{1-\beta}}$$

we obtain

$$\begin{aligned} | x_k{}^\gamma - y_k{}^\gamma | &\leq M^2 h \sum_{j=0}^{k-2} \left(h^{2\beta-1} \sum_{i=j+1}^{k-1} \frac{1}{(k-i)^{1-\beta}(i-j)^{1-\beta}} \right) | x_j - y_j | \\ &+ M h^\beta \sum_{i=0}^{k-1} \frac{1}{(k-i)^{1-\beta}} \delta + \delta. \end{aligned}$$

We now observe that

$$h^\beta \sum_{i=0}^{k-1} \frac{1}{(k-i)^{1-\beta}} = h \sum_{i=0}^{k-1} \frac{1}{(t_k - t_i)^{1-\beta}} \leq \int_0^{t_k} \frac{ds}{(t_k - s)^{1-\beta}} = \frac{1}{\beta} t_k^\beta,$$

where $t_j = jh$.

The last inequality is clear when the sum is regard as a (partial) Riemann sum. And finally

$$
\begin{aligned}
h^{2\beta-1}\sum_{i=j+1}^{k-1}\frac{1}{(k-i)^{1-\beta}(i-j)^{1-\beta}} &= h\sum_{i=j+1}^{k-1}\frac{1}{(t_k-t_i)^{1-\beta}(t_i-t_j)^{1-\beta}} \\
&= h\sum_{l=1}^{k-j-1}\frac{1}{(t_{k-j}-t_l)^{1-\beta}(t_l)^{1-\beta}} \\
&\leq \int_0^{t_{k-j}}\frac{ds}{(t_{k-j}-s)^{1-\beta}s^{1-\beta}} \\
&= t_{k-j}^{2\beta-1}\int_0^1\frac{du}{(1-u)^{1-\beta}u^{1-\beta}} \\
&= t_{k-j}^{2\beta-1}B(\beta,\beta).
\end{aligned}
$$

The last inequality is also deduced by considering the sum as a (partial) Riemann sum, but some subtility is required (see McKee(1979)).

Therefore

$$
\begin{aligned}
|\, x_k{}^\gamma - y_k{}^\gamma \,| &\leq MhB(\beta,\beta)t_k^{2\beta-1}\sum_{j=0}^{k-1}|\, x_j - y_j \,| \\
&+ \delta(1+\frac{M}{\beta}t_k^\beta).
\end{aligned}
$$

Replacing k by i, we can write

$$
|\, x_i{}^\gamma - y_i{}^\gamma \,| \leq hM'\sum_{j=0}^{i-1}|\, x_j - y_j \,| + \delta' \tag{3.3}
$$

where

$$
M' = MB(\beta,\beta),
$$

and

$$
\delta' = \delta(1+\frac{M}{\beta}),
$$

since $t_i \leq t_n = 1$.

The inequality (3.3) is now in a form in which lemma 3.1 can be applied to give the required result.

4 Consistency and convergence

Theorem 4.1: The discretisation ϕ_h is consistent of order α if exists M, independent of h and N, such that for all $i \geq k+1$ we have

$$| \, [\Delta_h(y(t))^{1+\frac{\beta}{\alpha}} - b_1 - \mu_1 h^\beta A_N \Delta_h y(t)]_i \, | \leq M h^\alpha$$

Proof

We need only consider

$$\begin{aligned} | \int_0^{t_i} (t_i - s)^{\beta-1} y(s) ds \; - \; & \sum_{j=0}^{i-1} \int_{t_j}^{t_{j+1}} (t_i - s)^{\beta-1} y(t_j) ds \; | = \\ &= | \sum_{j=0}^{i-1} \int_{t_j}^{t_{j+1}} (t_i - s)^{\beta-1} [y(s) - y(t_j)] ds \; | \\ &\leq \sum_{j=0}^{i-1} \int_{t_j}^{t_{j+1}} (t_i - s)^{\beta-1} \; | \; y(s) - y(t_j) \; | \; ds \\ &= \sum_{j=0}^{l-1} I_j(s) + I_l(s) + \sum_{j=l+1}^{i-1} I_j(s), \end{aligned}$$

where

$$I_k(s) = \int_{t_k}^{t_{k+1}} (t_i - s)^{\beta-1} \; | \; y(s) - y(t_j) \; | \; ds.$$

Assuming that $c \in [t_l, t_{l+1}]$, we have that for $j \neq l$ we can suppose that $y(s) \in C^1[0,1]$ and then we obtain

$$\begin{aligned} I_j(s) \; &= \; \int_{t_j}^{t_{j+1}} (t_i - s)^{\beta-1} (s - t_j) \; | \; y'(\xi_j(s)) \; | \; ds \\ &\leq \; \max_{s \in [t_j, t_{j+1}]} | \; y' \xi_j) \; | \int_{t_j}^{t_{j+1}} (t_i - s)^{\beta-1} ds, \end{aligned}$$

where $t_j < \xi_j < t_{j+1}$.

Hence on evaluation of the integral

$$I_j(s) \leq M_1 h^{\beta+1} [(i-j)^\beta - (i-j-1)^\beta],$$

where $M_1 = \frac{|y'(\xi_j)|}{\beta}$.

For $j = l$ we note that

$$I_l \leq \quad 2 \quad \max_{s\in[t_l,t_{l+1}]}\{|\, y(s)\,|\}\int_{t_l}^{t_{l+1}}(t_i - s)^{\beta-1}ds.$$

If we make the assumption that

$$\max_{s\in[t_l,t_{l+1}]}|\, y(s)\,| \leq M'h^{\alpha},$$

which we know to be valid for we know the exact solution, we have

$$I_l \leq M_2 h^{\alpha+\beta}[(i-l)^{\beta} - (i-l-1)^{\beta}]$$

where

$$M_2 = \frac{2M'}{\beta}$$

Therefore

$$\begin{aligned}\sum_{j=0}^{l-1} I_j(s) + I_l(s) + \sum_{j=l+1}^{i-1} I_j(s) \;&\leq\; Mh^{\beta}\Bigg\{h\sum_{j=0}^{l-1}[(i-j)^{\beta} - (i-j-1)^{\beta}] \\ &+\; h^{\alpha}[(i-l)^{\beta} - (i-l-1)^{\beta}] \\ &+\; h\sum_{j=l+1}^{i-1}[(i-j)^{\beta} - (i-j-1)^{\beta}]\Bigg\}\end{aligned}$$

where $M = max \quad \{M_1, M_2\}$. Thus,

$$\begin{aligned}\sum_{j=0}^{l-1} I_j(s) + I_l(s) + \sum_{j=l+1}^{i-1} I_j(s) \;&=\; Mh^{\beta}\Big\{h[(i)^{\beta} - (i-l)^{\beta}] \\ &+\; h^{\alpha}[(i-l)^{\beta} - (i-l-1)^{\beta}] \\ &+\; h[(i-l-1)^{\beta}]\Big\} \\ &\leq\; Mh^{\alpha+\beta}i^{\beta},\end{aligned}$$

since that for h sufficiently small, and $\alpha < 1$, we have that $h < h^{\alpha}$.

Therefore

$$\sum_{j=0}^{l-1} I_j(s) + I_l(s) + \sum_{j=l+1}^{i-1} I_j(s) \leq M h^{\alpha},$$

since $ih \leq Nh = 1$.

Then we can conclude that the method is consistent of order h^{α} at last. This does not mean that convergence of order h will not be obtained since consistency of order h^{α} only occurs once, (see Elliot, McKee(1981)).

Before stating a convergence theorem a bound is required on the quadrature weights w_{ij}.

Lemma 4.1: The quadrature weights w_{ij} are bounded as follows:

$$\mid w_{i-j} \mid = w_{i-j} \leq \frac{M'}{(i-j)^{1-\beta}}; \quad 0 \leq i \leq N, \quad 0 < \beta < 1,$$

where M' is some constant independent of i, j and h.

Proof

For definition we have

$$w_{i-j} = \frac{\beta}{h^{\beta}} \int_{t_j}^{t_{j+1}} (t_i - s)^{\beta-1} ds \leq \frac{1}{h^{\beta}} (t_{j+1} - t_j)(t_i - t_{j+1})^{\beta-1}, \tag{4.1}$$

since

$$\max_{\substack{s \in [t_j, t_{j+1}] \\ 0<\beta<1}} \{(t_i - s)^{\beta-1}\} = (t_i - t_{j+1})^{\beta-1}.$$

Therefore

$$\mid w_{i-j} \mid = w_{i-j} \leq h^{1-\beta} (t_i - t_{j+1})^{\beta-1}.$$

Note that this to be valid only for $(i, j) \in S$, where

$$S \equiv \{(i,j)/ \quad 0 \leq j < i \leq N \quad \text{and} \quad j \neq i-1\}.$$

Then

$$\mid w_{i-j} \mid \leq \frac{M'}{(i-j)^{1-\beta}},$$

where

$$M' = \max_S \left\{ \left(\frac{i-j}{i-j-1} \right)^{1-\beta} \right\},$$

is bounded; since

$$max_S \left\{ \left(\frac{i-j}{i-j-1} \right)^{1-\beta} \right\} = 2^{1-\beta}.$$

Now, consider (4.1) for $j = i-1$. We have

$$w_1 = \frac{\beta}{h^\beta} \int_{t_{i-1}}^{t_i} (t_i - s)^{\beta-1} ds.$$

Computing the integral we obtain

$$w_1 = 1 \leq \frac{M'}{1^{1-\beta}}.$$

This complete the proof of lemma.

Theorem 4.2: The discretisation ϕ_h is convergent of order α if it is consistent of order α.

Proof

Using (2.5) and adding and subtracting $\mu_1 h^\beta A_N \Delta_h y(t)$ and using the triangle inequality, we obtain:

$$\begin{aligned} | \, [y^{1+\frac{\beta}{\alpha}} - \Delta_h [y(t)]^{1+\frac{\beta}{\alpha}}]_i \, | \; &\leq \; \mu_1 h^\beta \, | \, [A_N (y - \Delta_h y(t))]_i \, | \\ &+ \; | \, [\Delta_h [y(t)]^{1+\frac{\beta}{\alpha}} - b_1 - \mu_1 h^\beta A_N \Delta_h y(t)]_i \, | \\ &\leq \; \mu_1 h^\beta \, | \, A_N (y - \Delta_h y(t)]_i \, | + M h^\alpha, \end{aligned}$$

using consistency.

Thus

$$\begin{aligned} | \, [y^{1+\frac{\beta}{\alpha}} - \Delta_h [y(t)]^{1+\frac{\beta}{\alpha}}]_i \, | \; &\leq \; \mu_1 h^\beta \sum_{j=0}^{i-1} | \, (A_N)_{ij} \, | \, | \, [(y - \Delta_h y(t)]_j \, | + M h^\alpha \\ &\leq \; \mu_1 M' h^\beta \sum_{j=0}^{i-1} \frac{1}{(i-j)^{1-\beta}} \, | \, [(y - \Delta_h y(t)]_j \, | + \delta, \end{aligned}$$

since

$$| (A_N)_{ij} | = | w_{i-j} | = w_{i-j} \leq \frac{M'}{(i-j)^{1-\beta}} \quad \text{and} \delta = Mh^{\alpha}.$$

Therefore

$$| [y^{1+\frac{\beta}{\alpha}} - \Delta_h[y(t)]^{1+\frac{\beta}{\alpha}}]_i | \leq Mh^{\beta} \sum_{j=0}^{i-1} \frac{1}{(i-j)^{1-\beta}} | [(y - \Delta_h y(t)]_j | + \delta.$$

Using the lemma 3.2 we can conclude that convergence of order α is obtained.

5 Numerical results

In order to illustrate the theoretical results of Section 4, the equation (1.1) was solved by means of (2.5) with

i) $\alpha = \beta = \frac{1}{2}$ and $c = 1.0$

Table 5.1: *Error*

t_i	1.2	1.3	1.4	1.5
$h = 0.1$	$9.0D-2$	$1.0D-1$	$1.0D-1$	$1.0D-1$
$h = 0.01$	$2.2D-2$	$2.0D-2$	$1.8D-2$	$1.6D-2$
$h = 0.001$	$3.2D-3$	$2.7D-3$	$2.4D-3$	$2.2D-3$

ii) $\alpha = \beta = \frac{1}{2}$ and $c = 0.1$

Table 5.2: *Error*

t_i	1.2	1.3	1.4	1.5
$h = 0.1$	$8.5D-2$	$8.3D-2$	$8.1D-2$	$7.9D-2$
$h = 0.01$	$1.2D-2$	$1.2D-2$	$1.2D-2$	$1.1D-2$
$h = 0.001$	$1.6D-3$	$1.6D-3$	$1.5D-3$	$1.5D-3$

iii) $\alpha = 1, \beta = \frac{1}{2}$ and $c = 1.0$

Table 5.3: *Error*

t_i	1.2	1.3	1.4	1.5
$h = 0.1$	$6.9D-2$	$1.1D-1$	$1.4D-1$	$1.7D-1$
$h = 0.01$	$3.1D-2$	$3.5D-2$	$3.8D-2$	$4.0D-2$
$h = 0.001$	$5.2D-3$	$5.5D-3$	$5.8D-3$	$6.0D-3$

iv) $\alpha = 1, \beta = \frac{1}{2}$ and $c = 0.1$

Table 5.4: *Error*

t_i	1.2	1.3	1.4	1.5
$h = 0.1$	$2.5D-1$	$2.6D-1$	$2.7D-1$	$2.8D-1$
$h = 0.01$	$4.7D-2$	$4.8D-2$	$4.8D-2$	$4.9D-2$
$h = 0.001$	$6.6D-3$	$6.7D-3$	$6.8D-3$	$6.8D-3$

6 Conclusion

An integral equation arising from the propagation of non-linear waves has been studied. An analysis of convergence has been presented and this depends upon the use of non-linear discrete Gronwall lemma which is new. Finally numerical results are presented.

References

Chester, W. 1964 *Ressonat oscilations in closed tubes*, J. Fluid. Mech.**18**, 44.

Eggermont, P.P.B.sl A new analysis of the Euler, midpoint and trapeziodal discretization methods for the numerical solution of Abel type integral equations, tech.Rep. MIPG34, (1979), 1-54.

Elliott, C.; McKee, S. 1981 *On the numerical solution of an integro-differential equation arising from wave-power hidraulics*, Bit **21**, 318-325.

Franco, N.B.; McKee, S.; Dixon,J. *A numerical solution of Lighthill's integral equation for surface temperature distribution of a projectile*, Mat. Apl. Comput. **2**, (1983), 257-271.

Henrici, P. 1962 *Discrete variable methods in ordinary differential equations*, John Wiley and Sons Ltd.

Keller, J. 1981 *Propagation of simple non-linear waves in gas filled tubes with friction*, Z. Angew. Math.Phys. **32**, 170-181.

McKee, S. 1979 *The analysis of a variable step, variable coefficient linear multistep method for solving a singular integro-differential equation arising from the diffusion of discrete particles in a turbulent fluid*, J. Inst. Math. Applics. **23**, 373-388

Schneider, W.R. 1982 *The general solution of a non-linear integral equation of convolution type*, Z. Angew. Math.Phys., **33**, 140-142.

Tang, T.; Mckee, S.; Diogo, T.*Product integration methods for an integral equations with a logarithmic singular kernel*, Appl. Numer. Math. **9**, (1992), 259-266.

Weiss, R.; Anderssen, R.S. *A product integration method for a class of first kind Volterra equations*, Numer. Math. **18**, (1972), 442-456.

NOTAS DO ICMSC

SÉRIE COMPUTAÇÃO

034/97 MARTINS, A.L.; OLIVEIRA, M.C.F.; MINGHIM, R. - Visualização Científica em mecânica dos fluidos.

033/97 NUNES, M.G.V.; HASEGAWA, R.; VIEIRA, F.M.C.; SANTOS, G.H.R.; FORTES, R.P.M. - SASHE: Sistema de autoria e suporte hipermídia para ensino [1].

032/97 DIOGO, T.; FRANCO, N..F. - High order product integration methods for Volterra integral equation with weakly singular Kernel.

031/96 ARENALES, M.N.; MORABITO, R. - A theory of the generation of cutting and packing patterns Part I: foundations.

030/96 TOMÉ, M..F.; CASTELO FILHO, A. ; MURAKAMI, J.; CUMINATO, J.A.; MINGHIM, R.; OLIVEIRA, M.C.F. - Numerical simulation of axisymmetric free surface flows.

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