UNIVERSIDADE DE SÃO PAULO

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ISSN - 0103-2577

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NOTAS DO ICMSC
Série Computação

São Carlos
Set./1996

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Abstract

This report presents a finite difference technique for solving three-dimensional Newtonian free surface flows. The technique is an extension of that employed by the GENSMAC code for calculating two-dimensional incompressible flows in arbitrary domains. The method is based on the SMAC (Simplified Marker-and-Cell) method using primitive variables for an incompressible fluid. The full methodology and a detailed description of the finite-difference equations are given.

Resumo

Neste trabalho apresentamos uma técnica numérica utilizando diferenças finitas para resolver escoamentos tridimensionais com superfícies livres para um fluido Newtoniano incompressivel. Esta técnica é uma extensão daquela empregada pelo método GENSMAC para calcular escoamentos bidimensionais em domínios arbitrários. O método utilizado é baseado no método SMAC (Simplified Marker-and-Cell) usando variáveis primitivas para um fluido incompressivel. Uma detalhada descrição do método é apresentado.

3. Procedure

To solve equations (3) - (6) we employ the following procedure which is the 3D-version of GENSMAC (see Tome and McKee [1]).
Let us suppose that at a given time, say t_0, the velocity field $\mathbf{u}(\mathbf{x}, t_0)$ is known and boundary conditions for the velocity and pressure are given. To compute the velocity field and the pressure field at the advanced time $t = t_0 + \delta t$, we proceed as follows:

Step 1: Let $\tilde{p}(\mathbf{x}, t_0)$ be a pressure field which satisfies the correct pressure condition on the free surface.

Step 2: Calculate the intermediate velocity field, $\tilde{\mathbf{u}}(\mathbf{x}, t)$, from

$$\frac{\partial \tilde{u}}{\partial t} = -\frac{\partial u^2}{\partial x} - \frac{\partial (uv)}{\partial y} - \frac{\partial (uw)}{\partial z} - \frac{\partial \tilde{p}}{\partial x} + \frac{1}{Re}\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + (1/F_r^2) g_x \quad (7)$$

$$\frac{\partial \tilde{v}}{\partial t} = -\frac{\partial (uv)}{\partial x} - \frac{\partial (v^2)}{\partial y} - \frac{\partial (vw)}{\partial z} - \frac{\partial \tilde{p}}{\partial y} + \frac{1}{Re}\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] + (1/F_r^2) g_y \quad (8)$$

$$\frac{\partial \tilde{w}}{\partial t} = -\frac{\partial (uw)}{\partial x} - \frac{\partial (vw)}{\partial y} - \frac{\partial (w^2)}{\partial z} - \frac{\partial \tilde{p}}{\partial z} + \frac{1}{Re}\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] + (1/F_r^2) g_z \quad (9)$$

with $\tilde{\mathbf{u}}(\mathbf{x}, t_0) = \mathbf{u}(\mathbf{x}, t_0)$ using the correct boundary conditions for $\mathbf{u}(\mathbf{x}, t_0)$. Equations (7) - (9) are solved by a finite difference method.

Step 3: Solve the Poisson equation

$$\nabla^2 \psi(\mathbf{x}, t) = \nabla . \tilde{\mathbf{u}}(\mathbf{x}, t) \quad (10)$$

Step 4: Compute the velocity field

$$\mathbf{u}(\mathbf{x}, t) = \tilde{\mathbf{u}}(\mathbf{x}, t) - \nabla \psi(\mathbf{x}, t) \quad (11)$$

Step 5: Compute the pressure

$$p(\mathbf{x}, t) = \tilde{p}(\mathbf{x}, t_0) + \frac{\psi(\mathbf{x}, t)}{\delta t} \quad (12)$$

2. Basic Equations

The basic equations governing the flow of a incompressible Newtonian fluid are the non-dimensional Navier-Stokes equations which can be written as

$$\frac{\partial \mathbf{u}}{\partial t} + \nabla.(\mathbf{uu}) = -\nabla p + \frac{1}{Re}\nabla^2\mathbf{u} + \frac{1}{F_r^2}\mathbf{g} \tag{1}$$

and the mass conservation equation

$$\nabla.\mathbf{u} = 0 \tag{2}$$

where $Re = UL/\nu$ and $F_r^2 = U/\sqrt{Lg}$ are the associated Reynolds number and Froude number respectively. U and L are typical velocity and length scales, g is the gravitational constant and $\mathbf{g}$ is the unit gravitational field vector, $\mathbf{u}$ is the velocity field and p is the non-dimensional pressure.

If we consider three-dimensional Cartesian coordinates then the equations above take the form

$$\begin{aligned}\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial (uv)}{\partial y} + \frac{\partial (uw)}{\partial z} &= -\frac{\partial p}{\partial x} + (1/R_e)\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] \\ &+(1/F_r^2)g_x \end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial v}{\partial t} + \frac{\partial (uv)}{\partial x} + \frac{\partial (v^2)}{\partial y} + \frac{\partial (vw)}{\partial z} &= -\frac{\partial p}{\partial y} + (1/R_e)\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right] \\ &+(1/F_r^2)g_y \end{aligned} \tag{4}$$

$$\begin{aligned}\frac{\partial w}{\partial t} + \frac{\partial (uw)}{\partial x} + \frac{\partial (vw)}{\partial y} + \frac{\partial (w^2)}{\partial z} &= -\frac{\partial p}{\partial z} + (1/R_e)\left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right] \\ &+(1/F_r^2)g_z \end{aligned} \tag{5}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{6}$$

where $\mathbf{u} = (u, v, w)$.

With suitable initial and boundary conditions, equations (3) - (6) consist of a system of partial differential equations for the unknowns $\mathbf{u}$ and p.

Thus, we solve the momentum equations explicitly followed by a sparse symmetric system (the discrete Poisson equation) for the potential function ψ. For cavity filling problems the order of this system is continually increasing (since one only solves for $\mathbf{u}$ and p within the bulk fluid).

4. Boundary conditions

The boundary conditions at the mesh boundary can be of several types. Let u_n, u_{m1} and u_{m2} denote the normal and tangential velocities to the boundary, respectively. Then the following boundary conditions can be applied on the mesh boundary:

- No-slip boundary

$$\mathrm{u_n} = 0 \ , \qquad \mathrm{u_{m1}} = 0 \ , \qquad \mathrm{u_{m2}} = 0$$

- Free-slip boundary

$$\mathrm{u_n} = 0 \ , \qquad \frac{\partial \mathrm{u_{m1}}}{\partial \mathrm{n}} = 0 \ , \qquad \frac{\partial \mathrm{u_{m2}}}{\partial \mathrm{n}} = 0$$

- Prescribed inflow boundary

$$\mathrm{u_n} = \mathrm{U_{inf}} \ , \qquad \mathrm{u_{m1}} = 0 \ , \qquad \mathrm{u_{m2}} = 0$$

- Prescribed outflow boundary

$$\mathrm{u_n} = \mathrm{U_{out}} \ , \qquad \mathrm{u_{m1}} = 0 \ , \qquad \mathrm{u_{m2}} = 0$$

- Continuative outflow boundary

$$\frac{\partial \mathrm{u_n}}{\partial \mathrm{n}} = 0 \ , \qquad \frac{\partial \mathrm{u_{m1}}}{\partial \mathrm{n}} = 0 \ , \qquad \frac{\partial \mathrm{u_{m2}}}{\partial \mathrm{n}} = 0$$

For the Poisson equation we require

$$\frac{\partial \psi}{\partial \mathrm{n}} = 0$$

on rigid boundaries and

$$\psi = 0$$

on the free surface. In the equations above, the subscript n, $m1$ and $m2$ denote normal and tangential directions to the boundary respectively.

5. Free Surface Stress Conditions

The boundary conditions on the free surface, in the absence of surface tension, are (see Batchelor [2])

$$\mathbf{n} \cdot \sigma \cdot \mathbf{n} = 0$$

$$\mathbf{m1} \cdot \sigma \cdot \mathbf{n} = 0$$

$$\mathbf{m2} \cdot \sigma \cdot \mathbf{n} = 0$$

where $\sigma = \sigma_{i,j}$ is the stress tensor given by

$$\sigma_{i,j} = -p_{i,j} + \frac{1}{Re}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right], \qquad i,j = 1,2,3$$

and $\mathbf{n} = (n_x, n_y, n_z)$ is the local outward unit normal vector to the surface; $\mathbf{m1}, \mathbf{m2}$ are the local tangential vectors. If we take Cartesian coordinates then equations above become:

$$p - \frac{2}{Re}\left[\frac{\partial u}{\partial x}n_x^2 + \frac{\partial v}{\partial y}n_y^2 + \frac{\partial w}{\partial z}n_z^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)n_x n_y + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)n_x n_z \right.$$
$$\left. + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)n_y n_z\right] = 0 \qquad (13)$$

$$\frac{1}{Re}\left[2\frac{\partial u}{\partial x}m1_x n_x + 2\frac{\partial v}{\partial y}m1_y n_y + 2\frac{\partial w}{\partial z}m1_z n_z + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)(m1_x n_y + m1_y n_x)\right.$$
$$\left. + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)(m1_x n_z + m1_z n_x) + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)(m1_y n_z + m1_z n_y)\right] = 0 \qquad (14)$$

$$\frac{1}{Re}\left[2\frac{\partial u}{\partial x}m2_x n_x + 2\frac{\partial v}{\partial y}m2_y n_y + 2\frac{\partial w}{\partial z}m2_z n_z + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)(m2_x n_y + m2_y n_x)\right.$$
$$\left. + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)(m2_x n_z + m2_z n_x) + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)(m2_y n_z + m2_z n_y)\right] = 0 \qquad (15)$$

Equations (13)–(15) represent the boundary conditions at the free surface of the fluid. The finite difference approximation to these equations will be given in the next Section by considering various local surface orientations.

6. Finite Difference Approximation

For solving equations (7)–(12) we employ the following approach. A staggered grid is used. A typical cell is shown in figure 1. The velocity $\tilde{\mathbf{u}}$ is discretized at u, v and w-nodes respectively.

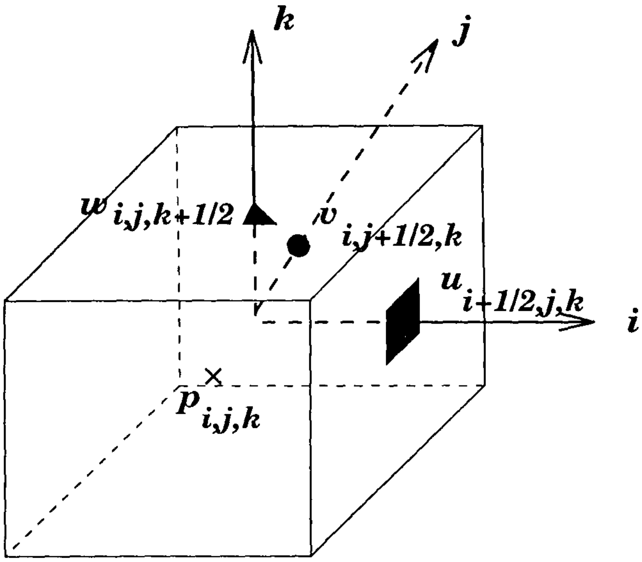

Fig. 1. Typical cell in a GENSMAC3D calculation.

For instance, considering eq. (7) for $\tilde{u}$, the discretization is performed as follows: the time derivative is discretized explicitly while the spatial derivatives are approximated by central differences. The convective terms are first averaged and then central differences applied, namely

$$
\begin{aligned}
\frac{\partial \tilde{u}}{\partial t}\Big|_{i+\frac{1}{2},j,k} &= \frac{u^{n+1}_{i+\frac{1}{2},j,k} - u^{n}_{i+\frac{1}{2},j,k}}{\delta t} + O(\delta t) \\
\frac{\partial^2 u}{\partial x^2}\Big|_{i+\frac{1}{2},j,k} &= \frac{u_{i-\frac{1}{2},j,k} - 2u_{i+\frac{1}{2},j,k} + u_{i+\frac{3}{2},j,k}}{\delta x^2} + O(\delta x^2) \\
\frac{\partial^2 u}{\partial y^2}\Big|_{i+\frac{1}{2},j,k} &= \frac{u_{i+\frac{1}{2},j-1,k} - 2u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j+1,k}}{\delta y^2} + O(\delta y^2) \\
\frac{\partial^2 u}{\partial z^2}\Big|_{i+\frac{1}{2},j,k} &= \frac{u_{i+\frac{1}{2},j,k-1} - 2u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j,k+1}}{\delta z^2} + O(\delta z^2) \\
\frac{\partial \tilde{p}}{\partial x}\Big|_{i+\frac{1}{2},j,k} &= \frac{\tilde{p}_{i+1,j,k} - \tilde{p}_{i,j,k}}{\delta x} + O(\delta x^2) \\
\frac{\partial u^2}{\partial x} = 2u\frac{\partial u}{\partial x}\Big|_{i+\frac{1}{2}} &= u_{i+\frac{1}{2},j,k}\frac{u_{i+\frac{3}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + O(\delta x^2) \\
\frac{\partial (uv)}{\partial y}\Big|_{i+\frac{1}{2},j,k} &= \frac{(uv)_{i+\frac{1}{2},j+\frac{1}{2},k} - (uv)_{i+\frac{1}{2},j-\frac{1}{2},k}}{\delta y} + O(\delta y^2) \\
\frac{\partial (uw)}{\partial z}\Big|_{i+\frac{1}{2},j,k} &= \frac{(uw)_{i+\frac{1}{2},j,k+\frac{1}{2}} - (uw)_{i+\frac{1}{2},j,k-\frac{1}{2}}}{\delta z} + O(\delta z^2)
\end{aligned}
$$

where

$$(uv)_{i+\frac{1}{2},j+\frac{1}{2},k} = \frac{\left(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j+1,k}\right)}{2} \frac{\left(v_{i,j+\frac{1}{2},k} + v_{i+1,j+\frac{1}{2},k}\right)}{2}$$

$$(uv)_{i+\frac{1}{2},j-\frac{1}{2},k} = \frac{\left(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j-1,k}\right)}{2} \frac{\left(v_{i,j-\frac{1}{2},k} + v_{i+1,j-\frac{1}{2},k}\right)}{2}$$

and

$$(uw)_{i+\frac{1}{2},j,k+\frac{1}{2}} = \frac{\left(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j,,k+1}\right)}{2} \frac{\left(w_{i,j,k+\frac{1}{2}} + w_{i+1,j,k+\frac{1}{2}}\right)}{2}$$

$$(uw)_{i+\frac{1}{2},j,k-\frac{1}{2}} = \frac{\left(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j,k-1}\right)}{2} \frac{\left(w_{i,j,k-\frac{1}{2}} + w_{i+1,j,k-\frac{1}{2}}\right)}{2} .$$

Similarly for the derivatives in (8) we obtain

$$\begin{aligned}
\frac{\partial \tilde{v}}{\partial t}\Big|_{i,j+\frac{1}{2},k} &= \frac{v^{n+1}_{i,j+\frac{1}{2},k} - v^{n}_{i,j+\frac{1}{2},k}}{\delta t} + O(\delta t) \\
\frac{\partial^2 v}{\partial x^2}\Big|_{i,j+\frac{1}{2},k} &= \frac{v_{i-1,j+\frac{1}{2},k} - 2v_{i,j+\frac{1}{2},k} + v_{i+1,j+\frac{1}{2},k}}{\delta x^2} + O(\delta x^2) \\
\frac{\partial^2 v}{\partial y^2}\Big|_{i,j+\frac{1}{2},k} &= \frac{v_{i,j-\frac{1}{2},k} - 2v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{3}{2},k}}{\delta y^2} + O(\delta y^2) \\
\frac{\partial^2 v}{\partial z^2}\Big|_{i,j+\frac{1}{2},k} &= \frac{v_{i,j+\frac{1}{2},k-1} - 2v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{1}{2},k+1}}{\delta z^2} + O(\delta z^2) \\
\frac{\partial \tilde{p}}{\partial y}\Big|_{i,j+\frac{1}{2},k} &= \frac{\tilde{p}_{i,j+1,k} - \tilde{p}_{i,j,k}}{\delta y} + O(\delta y^2) \\
\frac{\partial v^2}{\partial y} = 2v\frac{\partial v}{\partial y}\Big|_{i,j+\frac{1}{2},k} &= v_{i,j+\frac{1}{2},k}\frac{v_{i,j+\frac{3}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y} + O(\delta y^2) \\
\frac{\partial (uv)}{\partial x}\Big|_{i,j+\frac{1}{2},k} &= \frac{(uv)_{i+\frac{1}{2},j+\frac{1}{2},k} - (uv)_{i-\frac{1}{2},j+\frac{1}{2},k}}{\delta x} + O(\delta x^2) \\
\frac{\partial (vw)}{\partial z}\Big|_{i,j+\frac{1}{2},k} &= \frac{(vw)_{i,j+\frac{1}{2},k+\frac{1}{2}} - (vw)_{i,j+\frac{1}{2},k-\frac{1}{2}}}{\delta z} + O(\delta z^2)
\end{aligned}$$

where

$$(uv)_{i+\frac{1}{2},j+\frac{1}{2},k} = \frac{\left(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j+1,k}\right)}{2} \frac{\left(v_{i,j+\frac{1}{2},k} + v_{i+1,j+\frac{1}{2},k}\right)}{2}$$

$$(uv)_{i-\frac{1}{2},j+\frac{1}{2},k} = \frac{\left(u_{i-\frac{1}{2},j,k} + u_{i-\frac{1}{2},j+1,k}\right)}{2} \frac{\left(v_{i,j+\frac{1}{2},k} + v_{i-1,j+\frac{1}{2},k}\right)}{2}$$

and

$$(vw)_{i,j+\frac{1}{2},k+\frac{1}{2}} = \frac{\left(v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{1}{2},k+1}\right)}{2} \frac{\left(w_{i,j,k+\frac{1}{2}} + w_{i,j+1,k+\frac{1}{2}}\right)}{2}$$

$$(vw)_{i,j+\frac{1}{2},j,k-\frac{1}{2}} = \frac{\left(v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{1}{2},k-1}\right)}{2} \frac{\left(w_{i,j,k-\frac{1}{2}} + w_{i+1,j+1,k-\frac{1}{2}}\right)}{2}$$

and for the derivatives in (9) we have

$$
\begin{aligned}
\frac{\partial \tilde{w}}{\partial t}\Big|_{i,j,k+\frac{1}{2}} &= \frac{w^{n+1}_{i,j,k+\frac{1}{2}} - w^{n}_{i,j,k+\frac{1}{2}}}{\delta t} + O(\delta t) \\
\frac{\partial^2 w}{\partial x^2}\Big|_{i,j,k+\frac{1}{2}} &= \frac{w_{i-1,j,k+\frac{1}{2}} - 2w_{i,j,k+\frac{1}{2}} + w_{i+1,j,k+\frac{1}{2}}}{\delta x^2} + O(\delta x^2) \\
\frac{\partial^2 w}{\partial y^2}\Big|_{i,j,k+\frac{1}{2}} &= \frac{w_{i,j-1,k+\frac{1}{2}} - 2w_{i,j,k+\frac{1}{2}} + w_{i,j+1,k+\frac{1}{2}}}{\delta y^2} + O(\delta y^2) \\
\frac{\partial^2 w}{\partial z^2}\Big|_{i,j,k+\frac{1}{2}} &= \frac{w_{i,j,k-\frac{1}{2}} - 2w_{i,j,k+\frac{1}{2}} + w_{i,j,k+\frac{3}{2}}}{\delta z^2} + O(\delta z^2) \\
\frac{\partial \tilde{p}}{\partial z}\Big|_{i,j,k+\frac{1}{2}} &= \frac{\tilde{p}_{i,j,k+1} - \tilde{p}_{i,j,k}}{\delta z} + O(\delta z^2) \\
\frac{\partial w^2}{\partial z} = 2w\frac{\partial w}{\partial z}\Big|_{i,j,k+\frac{1}{2}} &= w_{i,j,k+\frac{1}{2}}\frac{w_{i,j,k+\frac{3}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z} + O(\delta z^2) \\
\frac{\partial (uw)}{\partial x}\Big|_{i,j,k+\frac{1}{2}} &= \frac{(uw)_{i+\frac{1}{2},j,k+\frac{1}{2}} - (uw)_{i-\frac{1}{2},j,k+\frac{1}{2}}}{\delta x} + O(\delta x^2) \\
\frac{\partial (vw)}{\partial y}\Big|_{i,j,k+\frac{1}{2}} &= \frac{(vw)_{i,j+\frac{1}{2},k+\frac{1}{2}} - (vw)_{i,j-\frac{1}{2},k+\frac{1}{2}}}{\delta y} + O(\delta y^2)
\end{aligned}
$$

where

$$
\begin{aligned}
(uw)_{i+\frac{1}{2},j,k+\frac{1}{2}} &= \frac{\left(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j,k+1}\right)}{2}\frac{\left(w_{i,j,k+\frac{1}{2}} + w_{i+1,j,k+\frac{1}{2}}\right)}{2} \\
(uw)_{i-\frac{1}{2},j+\frac{1}{2},k} &= \frac{\left(u_{i-\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k+1}\right)}{2}\frac{\left(w_{i,j,k+\frac{1}{2}} + v_{i-1,j,k+\frac{1}{2}}\right)}{2}
\end{aligned}
$$

and

$$
\begin{aligned}
(vw)_{i,j+\frac{1}{2},k+\frac{1}{2}} &= \frac{\left(v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{1}{2},k+1}\right)}{2}\frac{\left(w_{i,j,k+\frac{1}{2}} + w_{i,j+1,k+\frac{1}{2}}\right)}{2} \\
(vw)_{i+\frac{1}{2},j,k-\frac{1}{2}} &= \frac{\left(v_{i,j-\frac{1}{2},k} + v_{i,j-\frac{1}{2},k+1}\right)}{2}\frac{\left(w_{i,j,k+\frac{1}{2}} + w_{i,j-1,k+\frac{1}{2}}\right)}{2}.
\end{aligned}
$$

With these approximations, equations (7)–(9) become

$$
\begin{aligned}
&\tilde{u}_{i+\frac{1}{2},j,k} = u_{i+\frac{1}{2},j,k} - \delta t \Bigg[u_{i+\frac{1}{2},j,k}\left(\frac{u_{i+\frac{3}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x}\right) + \frac{\tilde{p}_{i+1,j,k} - \tilde{p}_{i,j,k}}{\delta x} \\
&\frac{1}{4\delta y}\Big(\big(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j+1,k}\big)\big(v_{i,j+\frac{1}{2},k} + v_{i+1,j+\frac{1}{2},k}\big) - \big(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j-1,k}\big) \\
&\big(v_{i,j-\frac{1}{2},k} + v_{i+1,j-\frac{1}{2},k}\big)\Big) + \frac{1}{4\delta z}\Big(\big(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j,k+1}\big)\big(w_{i,j,k+\frac{1}{2}} + w_{i+1,j,k+\frac{1}{2}}\big) \\
&-\big(u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j,k-1}\big)\big(w_{i,j,k-\frac{1}{2}} + w_{i+1,j,k-\frac{1}{2}}\big)\Big) + \frac{1}{Re}\Bigg(\frac{u_{i-\frac{1}{2},j,k} - 2u_{i+\frac{1}{2},j,k}}{} \\
&\frac{+u_{i+\frac{3}{2},j,k}}{\delta x^2} + \frac{u_{i+\frac{1}{2},j-1,k} - 2u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j+1,k}}{\delta y^2} + \frac{u_{i+\frac{1}{2},j,k-1} - 2u_{i+\frac{1}{2},j,k}}{} \\
&\frac{+u_{i+\frac{1}{2},j,k+1}}{\delta z^2}\Bigg)\Bigg] + \frac{1}{F_r^2}g_x \qquad (16)
\end{aligned}
$$

$$\tilde{v}_{i,j+\frac{1}{2},k} = v_{i,j+\frac{1}{2},k} - \delta t \Bigg[v_{i,j+\frac{1}{2},k} \frac{v_{i,j+\frac{3}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y} + \frac{\tilde{p}_{i,j+1,k} - \tilde{p}_{i,j,k}}{\delta y} +$$
$$\frac{1}{4\delta x} \Big((u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j+1,k})(v_{i,j+\frac{1}{2},k} + v_{i+1,j+\frac{1}{2},k}) - (u_{i-\frac{1}{2},j,k} + u_{i-\frac{1}{2},j+1,k})$$
$$(v_{i,j+\frac{1}{2},k} + v_{i-1,j+\frac{1}{2},k}) \Big) + \frac{1}{4\delta z} \Big((v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{1}{2},k+1})(w_{i,j,k+\frac{1}{2}} + w_{i,j+1,k+\frac{1}{2}})$$
$$-(v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{1}{2},k-1})(w_{i,j,k-\frac{1}{2}} + w_{i+1,j+1,k-\frac{1}{2}}) \Big) + \frac{1}{Re} \Bigg(\frac{v_{i-1,j+\frac{1}{2},k} - 2v_{i,j+\frac{1}{2},k} + v_{i+1,j+\frac{1}{2},k}}{\delta x^2} + \frac{v_{i,j-\frac{1}{2},k} - 2v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{3}{2},k}}{\delta y^2} + \frac{v_{i,j+\frac{1}{2},k-1} - 2v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{1}{2},k+1}}{\delta z^2} \Bigg) \Bigg] + \frac{1}{F_r^2} g_y \tag{17}$$

$$\tilde{w}_{i,j,k+\frac{1}{2}} = w_{i,j,k+\frac{1}{2}} - \delta t \Bigg[w_{i,j,k+\frac{1}{2}} \frac{w_{i,j,k+\frac{3}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z} + \frac{\tilde{p}_{i,j,k+1} - \tilde{p}_{i,j,k}}{\delta z} +$$
$$\frac{1}{4\delta x} \Big((u_{i+\frac{1}{2},j,k} + u_{i+\frac{1}{2},j,k+1})(w_{i,j,k+\frac{1}{2}} + w_{i+1,j,k+\frac{1}{2}}) - (u_{i-\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k+1})$$
$$(w_{i,j,k+\frac{1}{2}} + w_{i-1,j,k+\frac{1}{2}}) \Big) + \frac{1}{4\delta y} \Big((v_{i,j+\frac{1}{2},k} + v_{i,j+\frac{1}{2},k+1})(w_{i,j,k+\frac{1}{2}} + w_{i,j+1,k+\frac{1}{2}})$$
$$-(v_{i,j-\frac{1}{2},k} + v_{i,j-\frac{1}{2},k+1})(w_{i,j,k+\frac{1}{2}} + w_{i,j-1,k+\frac{1}{2}}) \Big) + \frac{1}{Re} \Bigg(\frac{w_{i-1,j,k+\frac{1}{2}} - 2w_{i,j,k+\frac{1}{2}} + w_{i+1,j,k+\frac{1}{2}}}{\delta x^2} + \frac{w_{i,j-1,k+\frac{1}{2}} - 2w_{i,j,k+\frac{1}{2}} + w_{i,j+1,k+\frac{1}{2}}}{\delta y^2} + \frac{w_{i,j,k-\frac{1}{2}} - 2w_{i,j,k+\frac{1}{2}} + w_{i,j,k+\frac{3}{2}}}{\delta z^2} \Bigg) \Bigg] + \frac{1}{F_r^2} g_z \tag{18}$$

The Poisson equation (10) is discretized at cell centres and the discretization can be written as

$$\frac{\psi_{i+1,j,k} - 2\psi_{i,j,k} + \psi_{i-1,j,k}}{\delta x^2} + \frac{\psi_{i,j+1,k} - 2\psi_{i,j,k} + \psi_{i,j-1,k}}{\delta y^2} + \frac{\psi_{i,j,k+1} - 2\psi_{i,j,k} + \psi_{i,j,k-1}}{\delta z^2} = \tilde{D}_{i,j,k} \tag{19}$$

where

$$\tilde{D}_{i,j,k} = \frac{\tilde{u}_{i+\frac{1}{2},j,k} - \tilde{u}_{i-\frac{1}{2},j,k}}{\delta x} + \frac{\tilde{v}_{i,j+\frac{1}{2},k} - \tilde{v}_{i,j-\frac{1}{2},k}}{\delta y} + \frac{\tilde{w}_{i,j,k+\frac{1}{2}} - \tilde{w}_{i,j,k-\frac{1}{2}}}{\delta z} ,$$

6.1. Approximate Free Surface Stress Conditions

The stress conditions (13) -(15) derived in Section 4 are approximated by local finite differences considering various surface orientations as follows:

6.1.1. Planar surface parallel to a coordinate axis: A planar surface will be defined to be one in which the normal vector is pointing to one of the coordinate directions i.e. $\mathbf{n} = (n_x, 0, 0)$ or $\mathbf{n} = (0, n_y, 0)$ or $\mathbf{n} = (0, 0, n_z)$. These surfaces are identified by surfaces cells having only one face contiguous with an empty cell (see figure 2).

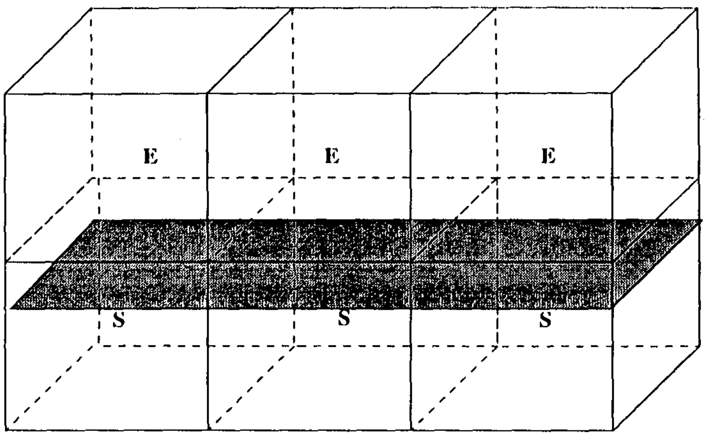

Fig. 2. Surface cells with only the $(k+\frac{1}{2})$–face contiguous with empty cells.

In order to approximate (13) (15) by finite differences we consider the following orientations:

a) **Surface cells with only the $(k+\frac{1}{2})$–face contiguous with an empty cell (see figure 3).**
For these cells we assume that the outward normal vector is pointing to the E-cell in which case we take $\mathbf{n} = (0,0,1)$, $\mathbf{m1} = (0,1,0)$ and $\mathbf{m2} = (1,0,0)$.
Equations (13)-(15) then reduce to

$$p = \frac{2}{Re}\left(\frac{\partial w}{\partial z}\right) , \tag{20}$$

$$\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 , \tag{21}$$

$$\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0 , \tag{22}$$

respectively.

When computing the tilde velocities by (16)-(18) the velocities at empty cells and the pressure at the surface cell are required (see figure 3).

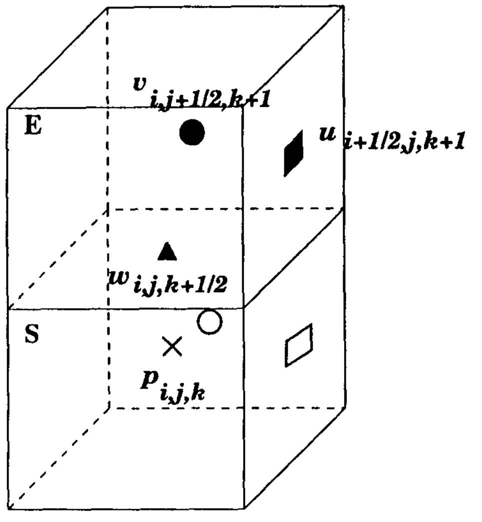

Fig. 3. Surface cell with the $(k+\frac{1}{2})$–face contiguous with an empty cell.

These can be obtained as follows. By discretizing (21) at position $(i+\frac{1}{2}, j, k+\frac{1}{2})$ we have

$$\frac{u_{i+\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k}}{\delta z} + \frac{w_{i+1,j,k+\frac{1}{2}} - w_{i,j,k+\frac{1}{2}}}{\delta x} = 0 \tag{23}$$

and applying a similar discretization to (22) at position $(i, j+\frac{1}{2}, k+\frac{1}{2})$ we get

$$\frac{v_{i,j+\frac{1}{2},k+1} - v_{i,j+\frac{1}{2},k}}{\delta z} + \frac{w_{i,j+1,k+\frac{1}{2}} - w_{i,j,k+\frac{1}{2}}}{\delta y} = 0\ . \tag{24}$$

Now, by requiring mass conservation (4) for the surface cell we have

$$\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z} = 0\ . \tag{25}$$

Equations (23)-(25) provide 3 equations for the unknowns $u_{i+\frac{1}{2},j,k+1}$, $v_{i,j+\frac{1}{2},k+1}$ and $w_{i,j,k+\frac{1}{2}}$ which can be solved explicitly, namely, from (25) we compute

$$w_{i,j,k+\frac{1}{2}} = w_{i,j,k-\frac{1}{2}} - \frac{\delta z}{\delta x}(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}) - \frac{\delta z}{\delta y}(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}) \tag{26}$$

and (23)-(24) give

$$u_{i+\frac{1}{2},j,k+1} = u_{i+\frac{1}{2},j,k} + \frac{\delta z}{\delta x}(w_{i+1,j,k+\frac{1}{2}} - w_{i,j,k+\frac{1}{2}})\ , \tag{27}$$

$$v_{i,j+\frac{1}{2},k+1} = v_{i,j+\frac{1}{2},k} + \frac{\delta z}{\delta y}(w_{i,j+1,k+\frac{1}{2}} - w_{i,j,k+\frac{1}{2}}) \tag{28}$$

respectively.

The pressure $\tilde{p}_{i,j,k}$ for the surface cell is then computed using (20) which gives

$$\tilde{p}_{i,j,k} = \frac{2}{Re}\left(\frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z}\right) \tag{29}$$

b) Surface cells with only the $(k-1/2)$–face contiguous with empty cells (see figure 4).

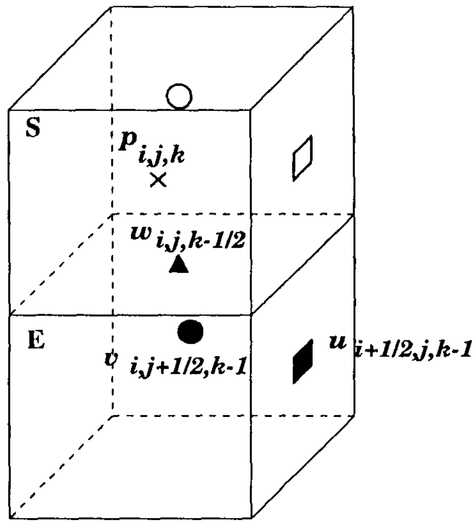

Fig. 4. Surface cell with the $(k-1/2)$–face contiguous with an empty cell.

Here we assume that the normal and tangential unit vectors to the surface take the form $\mathbf{n} = (0,0,-1)$, $\mathbf{m1} = (0,1,0)$, $\mathbf{m2} = (1,0,0)$. The values of the velocities at the empty cell faces and the pressure at the surface cell (see figure 4.) are calculated as in a) above and are given by

$$w_{i,j,k-\frac{1}{2}} = w_{i,j,k+\frac{1}{2}} + \frac{\delta z}{\delta x}(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}) + \frac{\delta z}{\delta y}(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}) \ , \tag{30}$$

$$u_{i+\frac{1}{2},j,k-1} = u_{i+\frac{1}{2},j,k} + \frac{\delta z}{\delta x}(w_{i+1,j,k-\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}) \ , \tag{31}$$

$$v_{i,j+\frac{1}{2},k-1} = v_{i,j+\frac{1}{2},k} + \frac{\delta z}{\delta y}(w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}) \ , \tag{32}$$

$$\tilde{p}_{i,j,k} = \frac{2}{Re}\left(\frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z}\right) \tag{33}$$

respectively.

c) **Surface cell with only the $(i + 1/2)$–face contiguous with an empty cell (see figure 5).**

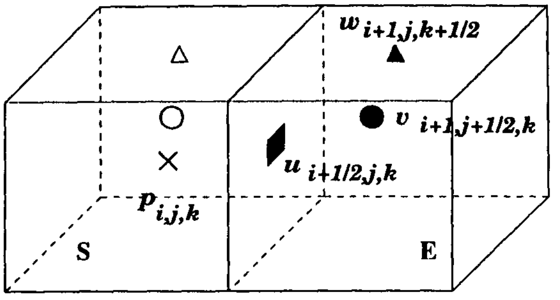

Fig. 5. Surface cell with the $(i + 1/2)$–face contiguous with an empty cell.

For these cells we assume the unit vectors take the form $\mathbf{n} = (1,0,0)$, $\mathbf{m1} = (0,1,0)$, $\mathbf{m2} = (0,0,1)$. In this case, the stress conditions (13)–(15) reduce to

$$p = \frac{2}{Re}\left(\frac{\partial u}{\partial x}\right) , \tag{34}$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 , \tag{35}$$

$$\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0 \tag{36}$$

respectively.

Similarly as in a), the velocities at the empty cell are calculated from applying the mass equation (4) at the centre of the surface cell and eqs. (35) and (36) at positions $(i + 1/2, j, k + 1/2)$ and $(i + 1/2, j + 1/2, k)$ respectively. Then it can be easily verified that the velocities and the pressure at the surface cell are given by

$$u_{i+\frac{1}{2},j,k} = u_{i-\frac{1}{2},j,k} + \frac{\delta x}{\delta y}(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}) + \frac{\delta x}{\delta z}(w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}) , \tag{37}$$

$$v_{i+1,j+1/2,k} = v_{i,j+1/2,k} - \frac{\delta x}{\delta y}(u_{i+\frac{1}{2},j+1,k} + u_{i+\frac{1}{2},j,k}) , \tag{38}$$

$$w_{i+1,j,k+1/2} = w_{i,j,k+1/2} - \frac{\delta x}{\delta z}(u_{i+\frac{1}{2},j,k+1} + u_{i+\frac{1}{2},j,k}) , \tag{39}$$

$$\tilde{p}_{i,j,k} = \frac{2}{Re}\left(\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x}\right) , \tag{40}$$

respectively.

d) **Surface cells with only the $(i-1/2)$–face contiguous with an empty cell (see figure 6).**

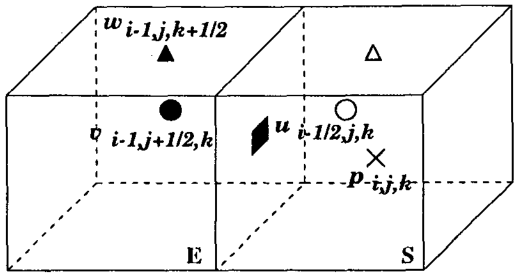

Fig. 6. Surface cell with the $(i-1/2)$–face contiguous with an empty cell.

Here we take $\mathbf{n} = (-1, 0, 0)$, $\mathbf{m1} = (0, 1, 0)$, $\mathbf{m2} = (0, 0, 1)$. In this case, the stress conditions reduce to eqs. (34)-(36) above. The velocities at the empty cell faces and the pressure at the surface cell (see figure 6.) are calculated similarly as in c) above and are given by

$$u_{i-\frac{1}{2},j,k} = u_{i+\frac{1}{2},j,k} + \frac{\delta x}{\delta y}(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}) + \frac{\delta x}{\delta z}(w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}) \;, \quad (41)$$

$$v_{i-1,j+1/2,k} = v_{i,j+1/2,k} + \frac{\delta x}{\delta y}(u_{i-\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j,k}) \;, \quad (42)$$

$$w_{i-1,j,k+1/2} = w_{i,j,k+1/2} - \frac{\delta x}{\delta z}(u_{i-\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k}) \;, \quad (43)$$

$$\tilde{p}_{i,j,k} = \frac{2}{Re}\left(\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x}\right) \;. \quad (44)$$

e) **Surface cells with only the $(j+1/2)$–face contiguous with an empty cell (see figure 7).**

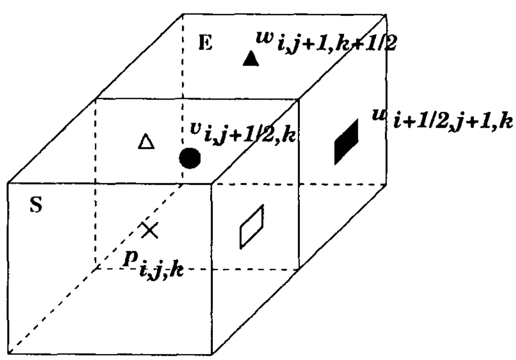

Fig. 7. Surface cell with the $(j+1/2)$–face contiguous with an empty cell.

For these cells we assume the normal and tangential vectors take the form $\mathbf{n} = (0,1,0)$, $\mathbf{m1} = (1,0,0)$, $\mathbf{m2} = (0,0,1)$. Equations (13)–(15) reduce to

$$p = \frac{2}{Re}\left(\frac{\partial v}{\partial y}\right) , \tag{45}$$

$$\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0 , \tag{46}$$

$$\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 , \tag{47}$$

respectively. The velocities at the empty cell faces and the pressure at the surface cell centre are computed similarly as in a), namely, applying the mass conservation equation (4) at the centre of surface cell and eqs. (46)–(47) at positions $(i+1/2, j+1/2, k)$ and $(i, j+1/2, k+1/2)$ respectively. It is easily verified that the velocities at the empty cell are given by

$$v_{i,j+\frac{1}{2},k} = v_{i,j-\frac{1}{2},k} - \frac{\delta y}{\delta x}\left(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}\right) - \frac{\delta y}{\delta z}\left(w_{i,j,k+\frac{1}{2}} - v_{i,j,k-\frac{1}{2}}\right) \tag{48}$$

$$u_{i+\frac{1}{2},j+1,k} = u_{i+\frac{1}{2},j,k} - \frac{\delta y}{\delta x}\left(v_{i+1,j+\frac{1}{2},k} - v_{i,j+\frac{1}{2},k}\right) \tag{49}$$

$$w_{i,j+1,k+\frac{1}{2}} = w_{i,j,k+\frac{1}{2}} - \frac{\delta y}{\delta z}\left(v_{i,j+\frac{1}{2},k+1} - v_{i,j+\frac{1}{2},k}\right) \tag{50}$$

and the pressure $\tilde{p}_{i,j,k}$ is computed by

$$\tilde{p}_{i,j,k} = \frac{2}{Re}\left(\frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y}\right) . \tag{51}$$

f) **Surface cells with only the $(j-1/2)$–face contiguous with an empty cell (see figure 8).**

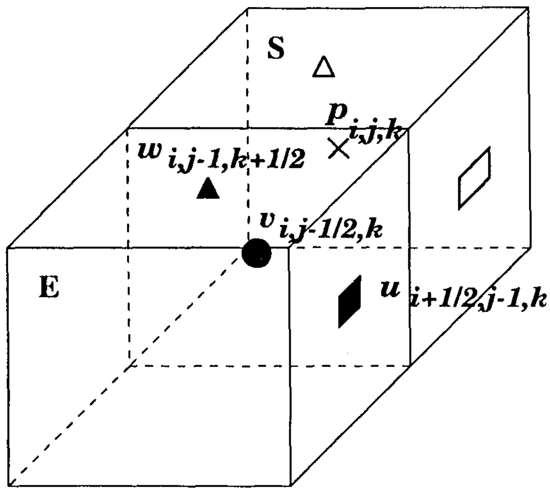

Fig. 8. Surface cell with the $(j-1/2)$–face contiguous with an empty cell.

For these cells the unit vectors take the form $\mathbf{n} = (0,-1,0)$, $\mathbf{m1} = (1,0,0)$, $\mathbf{m2} = (0,0,1)$ and eqs. (13)–(15) reduce to (45)–(47) respectively. The velocities at the empty cell faces and the pressure at surface cell centre are computed in the same manner as in e) above and are given by

$$v_{i,j-\frac{1}{2},k} = v_{i,j+\frac{1}{2},k} + \frac{\delta y}{\delta x}\left(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}\right) + \frac{\delta y}{\delta z}\left(w_{i,j,k+\frac{1}{2}} - v_{i,j,k-\frac{1}{2}}\right) \quad (52)$$

$$u_{i+\frac{1}{2},j-1,k} = u_{i+\frac{1}{2},j,k} + \frac{\delta y}{\delta x}\left(v_{i+1,j-\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) , \quad (53)$$

$$w_{i,j-1,k+\frac{1}{2}} = w_{i,j,k+\frac{1}{2}} + \frac{\delta y}{\delta z}\left(v_{i,j-\frac{1}{2},k+1} - v_{i,j-\frac{1}{2},k}\right) \quad (54)$$

respectively. The pressure $\tilde{p}_{i,j,k}$ is computed by

$$\tilde{p}_{i,j,k} = \frac{2}{Re}\left(\frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y}\right) . \quad (55)$$

6.1.2. 45^0–sloped planar surface: These surfaces are identified by surface cells having two faces contiguous with empty cells. In these cells we assume that the normal vector is pointing at a direction which makes 45^0 with the two other axes, e.g. x and y or x and z or y and z. Thus the following approximations are employed:

a) **Surface cells with the $(k+\frac{1}{2})$ and $(i+\frac{1}{2})$–faces contiguous with empty cells (see figure 9).**
For these cells we assume that the unit normal takes the form $\mathbf{n} = (\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$ in which case the tangential vectors are taken to be:

$$\mathbf{m1} = (\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2}) \quad \text{and} \quad \mathbf{m2} = (0,1,0) \ .$$

Introducing these vectors into equations (13)–(14) we have

$$p = \frac{1}{Re}\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} + \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \ , \tag{56}$$

$$\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z} = 0 \tag{57}$$

respectively.

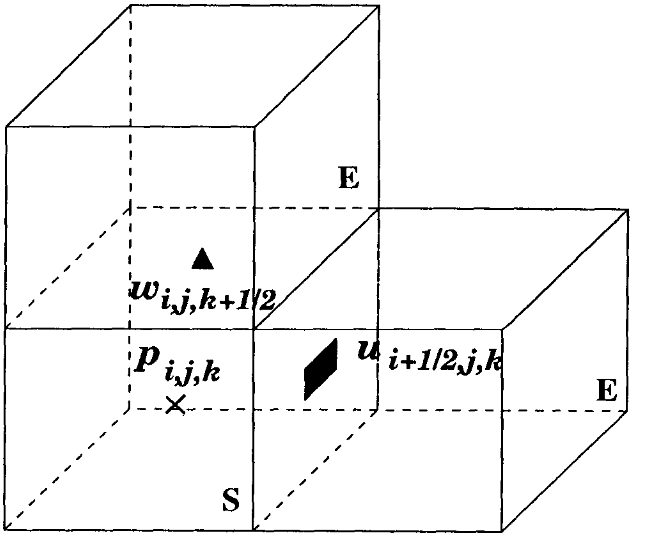

Fig. 9. Surface cell with $(k+\frac{1}{2})$ and $(i+\frac{1}{2})$–faces contiguous with E–cells.

As we can see in figure 9, the values of $u_{i+\frac{1}{2},j,k}$ and $w_{i,j,k+\frac{1}{2}}$ at empty-cell faces are required. These are obtained by applying (57) and the mass conservation equation (4) at the surface cell centre in which case we get

$$\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} - \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z} = 0 \tag{58}$$

$$\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z} = -\left(\frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y}\right) \tag{59}$$

respectively. Solving (58) and (59) for $u_{i+\frac{1}{2},j,k}$ and $w_{i,j,k+\frac{1}{2}}$ we obtain

$$u_{i+\frac{1}{2},j,k} = u_{i-\frac{1}{2},j,k} - \frac{1}{2}\frac{\delta x}{\delta y}\left(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) \tag{60}$$

and

$$w_{i,j,k+\frac{1}{2}} = w_{i,j,k-\frac{1}{2}} - \frac{1}{2}\frac{\delta z}{\delta y}\left(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) \quad . \tag{61}$$

Once the velocities at the empty-cell faces have been computed the pressure at the surface cell centre is calculated by (56), namely

$$\begin{aligned} \tilde{p}_{i,j,k} \quad &= \frac{1}{Re}\left[\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z}\right. \\ &+\frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} - u_{i-\frac{1}{2},j,k-1}}{\delta z}\right. \\ &+ \left.\left.\frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}}{\delta x}\right)\right] \quad . \end{aligned} \tag{62}$$

b) **Surface cells with the $(k+\frac{1}{2})$ and $(i-\frac{1}{2})$–faces contiguous with empty cells (see figure 10).**

Here we assume that $\mathbf{n} = (-\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$ $\mathbf{m1} = (\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$ and $\mathbf{m2} = (0, 1, 0)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} - \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right] \tag{63}$$

$$-\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{64}$$

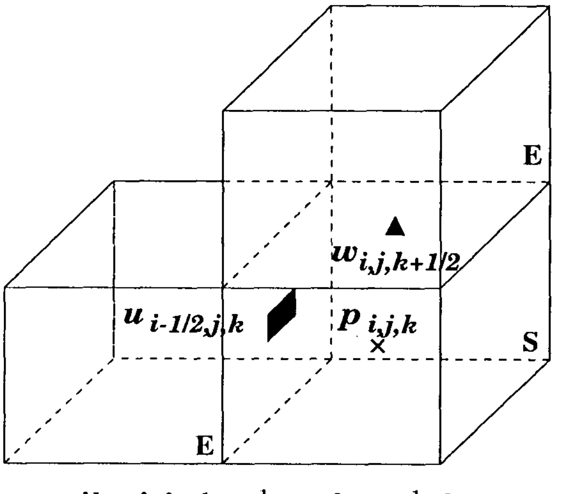

Fig. 10. Surface cell with $k+\frac{1}{2}$ and $i-\frac{1}{2}$ faces contiguous with E-cells.

The values of $u_{i-\frac{1}{2},j,k}$ and $w_{i,j,k+\frac{1}{2}}$ are computed as in **a)** above, namely, by applying (4) and (64) at the surface cell centre. It can be verified that they are given by

$$u_{i-\frac{1}{2},j,k} = u_{i+\frac{1}{2},j,k} + \frac{1}{2}\frac{\delta x}{\delta y}\left(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) \tag{65}$$

and

$$w_{i,j,k+\frac{1}{2}} = w_{i,j,k-\frac{1}{2}} - \frac{1}{2}\frac{\delta z}{\delta y}\left(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) \quad . \tag{66}$$

The pressure at the surface cell centre is then calculated from

$$\begin{aligned}\tilde{p}_{i,j,k} \quad &= \frac{1}{Re}\left[\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z}\right. \\ &-\frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} - u_{i-\frac{1}{2},j,k-1}}{\delta z}\right. \\ &+\left.\left.\frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}}{\delta x}\right)\right] \quad .\end{aligned} \tag{67}$$

c) **Surface cells having the $(k+\frac{1}{2})$ and $(j+\frac{1}{2})$ faces contiguous with empty cells (see figure 11).**
For these cells we take $\mathbf{n} = (0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$, $\mathbf{m1} = (0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ and $\mathbf{m2} = (1, 0, 0)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right)\right] \tag{68}$$

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{69}$$

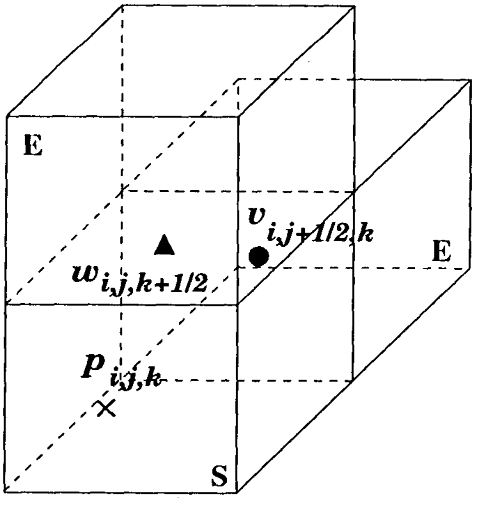

Fig. 11. Surface cell with the $(k+\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with E–cells.

The velocities at empty cell faces are obtained by applying (69) and the conservation of mass equation (4) at the surface cell centre giving

$$\frac{v_{i,j+\frac{1}{2},k}-v_{i,j-\frac{1}{2},k}}{\delta y}-\frac{w_{i,j,k+\frac{1}{2}}-w_{i,j,k-\frac{1}{2}}}{\delta z}=0\ , \tag{70}$$

$$\frac{v_{i,j+\frac{1}{2},k}-v_{i,j-\frac{1}{2},k}}{\delta y}+\frac{w_{i,j,k+\frac{1}{2}}-w_{i,j,k-\frac{1}{2}}}{\delta z}=-\frac{u_{i+\frac{1}{2},j,k}-u_{i-\frac{1}{2},j,k}}{\delta x} \tag{71}$$

respectively. Solving these two equations for $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ we obtain

$$v_{i,j+\frac{1}{2},k}=v_{i,j-\frac{1}{2},k}-\frac{1}{2}\frac{\delta y}{\delta x}\left(u_{i+\frac{1}{2},j,k}-u_{i-\frac{1}{2},j,k}\right)\ , \tag{72}$$

$$w_{i,j,k+\frac{1}{2}}=w_{i,j,k-\frac{1}{2}}-\frac{1}{2}\frac{\delta z}{\delta x}\left(u_{i+\frac{1}{2},j,k}-u_{i-\frac{1}{2},j,k}\right)\ , \tag{73}$$

respectively.

The pressure is computed from (68) applied at the surface cell centre, namely,

$$\begin{aligned}\tilde{p}_{i,j,k} &= \frac{1}{Re}\left[\frac{v_{i,j+\frac{1}{2},k}-v_{i,j-\frac{1}{2},k}}{\delta y}+\frac{w_{i,j,k+\frac{1}{2}}-w_{i,j,k-\frac{1}{2}}}{\delta z}\right.\\ &+\frac{1}{2}\left(\frac{v_{i,j+\frac{1}{2},k}+v_{i,j-\frac{1}{2},k}-v_{i,j+\frac{1}{2},k-1}-v_{i,j-\frac{1}{2},k-1}}{\delta z}\right.\\ &+\left.\left.\frac{w_{i,j,k+\frac{1}{2}}+w_{i,j,k-\frac{1}{2}}-w_{i,j-1,k+\frac{1}{2}}-w_{i,j-1,k-\frac{1}{2}}}{\delta y}\right)\right]\ .\end{aligned} \tag{74}$$

d) **Surface cells having the $(k+\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with empty cells (see figure 12).** For these cells we take $\mathbf{n}=(0,-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2})$, $\mathbf{m1}=(0,\frac{\sqrt{2}}{2},+\frac{\sqrt{2}}{2})$ and $\mathbf{m2}=(1,0,0)$. In this case equations (13)–(14) take the form

$$p=\frac{1}{Re}\left[\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}-\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right] \tag{75}$$

$$-\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \tag{76}$$

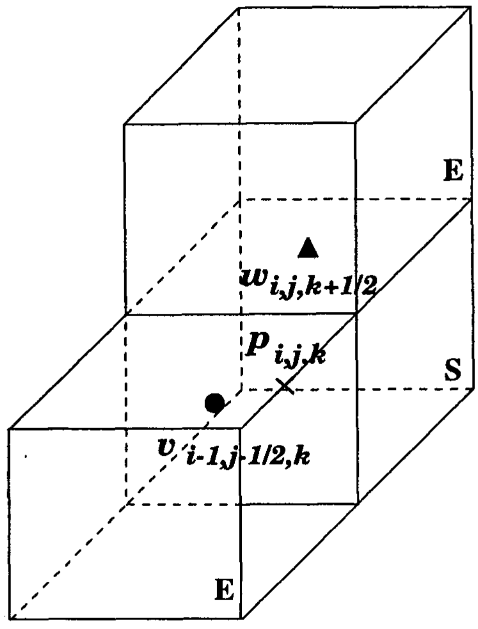

Fig. 12. Surface cell with the $(k+\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with E–cells.

Here we procceed as in **c)** above for the calculation of $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$. It can be verified that theses values are given by

$$v_{i,j-\frac{1}{2},k} = v_{i,j+\frac{1}{2},k} + \frac{1}{2}\frac{\delta y}{\delta x}\left(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}\right) , \tag{77}$$

$$w_{i,j,k+\frac{1}{2}} = w_{i,j,k-\frac{1}{2}} - \frac{1}{2}\frac{\delta y}{\delta x}\left(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}\right) \tag{78}$$

respectively. Once $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ have been computed the pressure follows from (75) applyied at the surface cell centre, giving

$$\begin{aligned}\tilde{p}_{i,j,k} &= \frac{1}{Re}\left[\frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z}\right.\\ &+\frac{1}{2}\left(\frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}}{\delta z}\right.\\ &+\left.\left.\frac{w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta y}\right)\right] .\end{aligned} \tag{79}$$

e) Surface cells having the $(k-\frac{1}{2})$ and $(i+\frac{1}{2})$–faces contiguous with empty cells (see figure 13). For these cells we take $\mathbf{n} = (\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2})$, $\mathbf{m1} = (\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2})$ and $\mathbf{m2} = (0,1,0)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right] \tag{80}$$

$$\frac{\partial u}{\partial x} - \frac{\partial w}{\partial z} = 0 \tag{81}$$

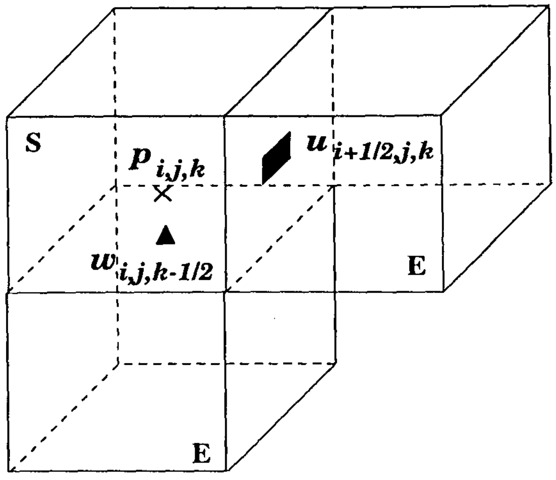

Fig. 13. Surface cell with the $(k - \frac{1}{2})$ and $(i + \frac{1}{2})$–faces contiguous with E–cells.

For these cells the values of $u_{i+\frac{1}{2},j,k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained by applying (81) and (4) at the surface cell centre. It can be verified that they are given by

$$u_{i+\frac{1}{2},j,k} = u_{i-\frac{1}{2},j,k} - \frac{1}{2}\frac{\delta x}{\delta y}\left(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) \tag{82}$$

$$w_{i,j,k-\frac{1}{2}} = w_{i,j,k+\frac{1}{2}} + \frac{1}{2}\frac{\delta z}{\delta y}\left(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) \tag{83}$$

The pressure at the surface cell centre is obtained from (80), giving

$$\begin{aligned}\tilde{p}_{i,j,k} &= \frac{1}{Re}\left[\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z}\right.\\ &+\frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta z}\right.\\ &+\left.\left.\frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}}{\delta x}\right)\right] .\end{aligned} \tag{84}$$

f) **Surface cells having the $(k - \frac{1}{2})$ and $(i - \frac{1}{2})$–faces contiguous with empty cells (see figure 14).** For these cells we take $\mathbf{n} = (-\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2})$, $\mathbf{m1} = (\frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2})$ and $\mathbf{m2} = (0, 1, 0)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)\right] \tag{85}$$

$$-\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{86}$$

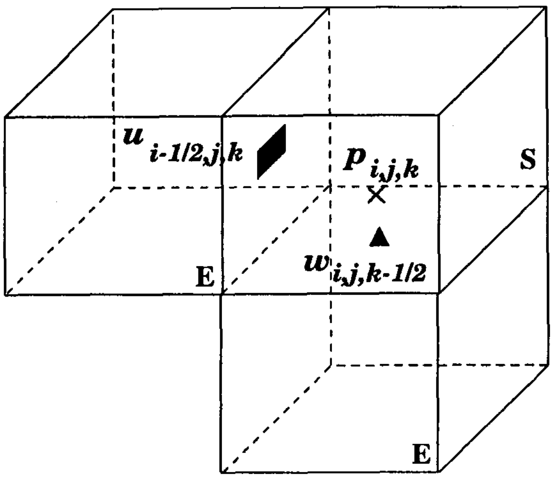

Fig. 14. Surface cell with the $(k-\frac{1}{2})$ and $(i-\frac{1}{2})$–faces contiguous with E–cells.

Here, the values of $u_{i-\frac{1}{2},j,k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as in **e)** above. It can be verified they are given by

$$u_{i-\frac{1}{2},j,k} = u_{i+\frac{1}{2},j,k} + \frac{1}{2}\frac{\delta x}{\delta y}\left(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) \quad (87)$$

$$w_{i,j,k-\frac{1}{2}} = w_{i,j,k+\frac{1}{2}} + \frac{1}{2}\frac{\delta z}{\delta y}\left(v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}\right) \quad (88)$$

The pressure $\tilde{p}_{i,j,k}$ is computed from (85) discretized at the surface cell centre, namely

$$\begin{aligned}\tilde{p}_{i,j,k} &= \frac{1}{Re}\left[\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z}\right. \\ &+\frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta z}\right. \\ &+ \left.\left.\frac{w_{i+1,j,k+\frac{1}{2}} + w_{i+1,j,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta x}\right)\right] . \end{aligned} \quad (89)$$

g) **Surface cells having the $(k-\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with empty cells (see figure 15).** For these cells we take $\mathbf{n} = (0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$, $\mathbf{m1} = (0, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$ and $\mathbf{m2} = (1, 0, 0)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} - \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right] \quad (90)$$

$$\frac{\partial v}{\partial y} - \frac{\partial w}{\partial z} = 0 \quad (91)$$

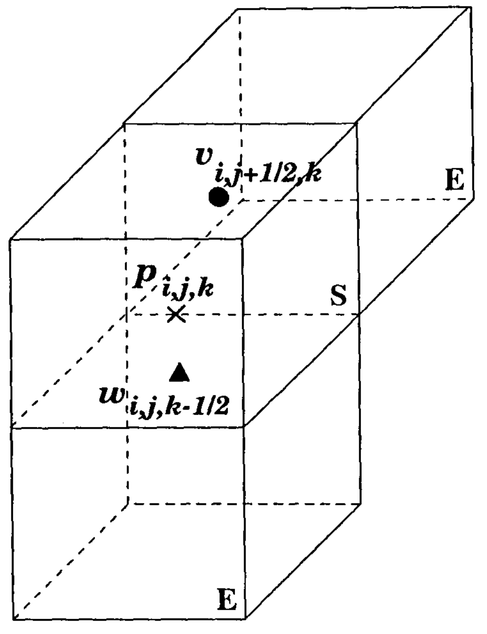

Fig. 15. Surface cell with the $(k-\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with E–cells.

For these cells the values of $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are required. They can be obtained by applying (91) and (4) at the surface cell centres yielding

$$v_{i,j+\frac{1}{2},k} = v_{i,j-\frac{1}{2},k} - \frac{1}{2}\frac{\delta y}{\delta x}\left(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}\right) \tag{92}$$

$$w_{i,j,k-\frac{1}{2}} = w_{i,j,k+\frac{1}{2}} + \frac{1}{2}\frac{\delta z}{\delta x}\left(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}\right) \tag{93}$$

For these cells, the pressure is then computed by

$$\begin{aligned}\tilde{p}_{i,j,k} = \frac{1}{Re}\Bigg[& \frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z} \\ & -\frac{1}{2}\left(\frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}}{\delta z}\right. \\ & \left.+ \frac{w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta y}\right)\Bigg] .\end{aligned} \tag{94}$$

h) **Surface cells having the $(k-\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with empty cells (see figure 16).** For these cells we take $\mathbf{n} = (0, -\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$, $\mathbf{m1} = (0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2})$ and $\mathbf{m2} = (1, 0, 0)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right] \tag{95}$$

$$-\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{96}$$

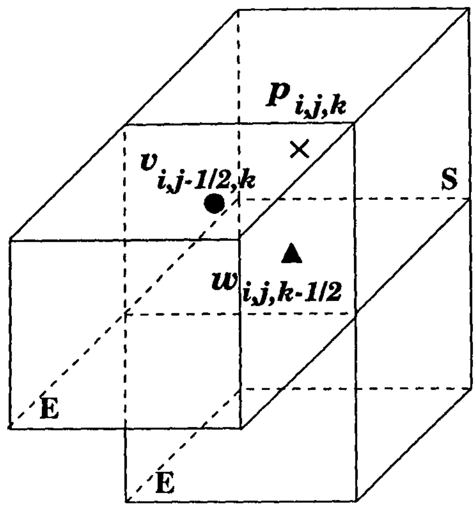

Fig. 16. Surface cell with the $(k - \frac{1}{2})$ and $(j - \frac{1}{2})$–faces contiguous with E–cells.

Here $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ take a form similar to **g)** above and are given by

$$v_{i,j-\frac{1}{2},k} = v_{i,j+\frac{1}{2},k} + \frac{1}{2}\frac{\delta y}{\delta x}\left(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}\right) \quad (97)$$

$$w_{i,j,k-\frac{1}{2}} = w_{i,j,k+\frac{1}{2}} + \frac{1}{2}\frac{\delta z}{\delta x}\left(u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}\right) \quad (98)$$

For these cells, the pressure is then computed by

$$\begin{aligned}\tilde{p}_{i,j,k} &= \frac{1}{Re}\left[\frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y} + \frac{w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta z}\right.\\ &+\frac{1}{2}\left(\frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}}{\delta z}\right.\\ &+\left.\left.\frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}}{\delta y}\right)\right] . \end{aligned} \quad (99)$$

i) **Surface cells having the $(i + \frac{1}{2})$ and $(j + \frac{1}{2})$–faces contiguous with empty cells (see figure 17).** For these cells we take $\mathbf{n} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$, $\mathbf{m1} = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right)$ and $\mathbf{m2} = (0, 0, 1)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \quad (100)$$

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0 \quad (101)$$

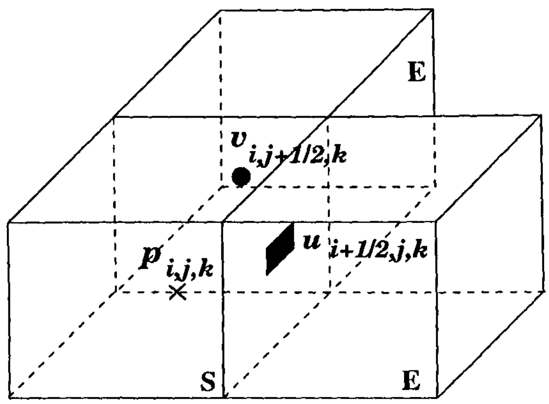

Fig. 17. Surface cell with the $(i+\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with E–cells.

Here the velocities at the empty cell faces, $u_{i+\frac{1}{2},j,k}$ and $v_{i,j+\frac{1}{2},k}$ are obtained by applying (101) and (4) at the surface cell centre. In this case we have

$$\frac{u_{i+\frac{1}{2},j,k}-u_{i-\frac{1}{2},j,k}}{\delta x}-\frac{v_{i,j+\frac{1}{2},k}-v_{i,j-\frac{1}{2},k}}{\delta y} = 0 \ , \quad (102)$$

$$\frac{u_{i+\frac{1}{2},j,k}-u_{i-\frac{1}{2},j,k}}{\delta x}+\frac{v_{i,j+\frac{1}{2},k}-v_{i,j-\frac{1}{2},k}}{\delta y}+\frac{w_{i,j,k+\frac{1}{2}}-w_{i,j,k-\frac{1}{2}}}{\delta z} = 0 \ . \quad (103)$$

Solving (102) and (103) for $u_{i+\frac{1}{2},j,k}$ and $v_{i,j+\frac{1}{2},k}$ we obtain

$$u_{i+\frac{1}{2},j,k} = u_{i-\frac{1}{2},j,k}-\frac{1}{2}\frac{\delta x}{\delta z}\left(w_{i,j,k+\frac{1}{2}}-w_{i,j,k-\frac{1}{2}}\right) , \quad (104)$$

$$v_{i,j+\frac{1}{2},k} = v_{i,j-\frac{1}{2},k}-\frac{1}{2}\frac{\delta y}{\delta z}\left(w_{i,j,k+\frac{1}{2}}-w_{i,j,k-\frac{1}{2}}\right) \quad (105)$$

respectively.

The pressure is obtained from (100) discretized at the surface cell centre which gives

$$\begin{aligned}\tilde{p}_{i,j,k} &= \frac{1}{Re}\left[\frac{u_{i+\frac{1}{2},j,k}-u_{i-\frac{1}{2},j,k}}{\delta x}+\frac{v_{i,j+\frac{1}{2},k}-v_{i,j-\frac{1}{2},k}}{\delta y}\right.\\ &+ \frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j,k}+u_{i-\frac{1}{2},j,k}-u_{i+\frac{1}{2},j-1,k}-u_{i-\frac{1}{2},j-1,k}}{\delta y}\right.\\ &+ \left.\left.\frac{v_{i,j+\frac{1}{2},k}+v_{i,j-\frac{1}{2},k}-v_{i-1,j+\frac{1}{2},k}-v_{i-1,j-\frac{1}{2},k}}{\delta x}\right)\right] . \quad (106)\end{aligned}$$

j) **Surface cells having the $(i+\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with empty cells (see figure 18).** For these cells we take $\mathbf{n} = \left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2},0\right)$,

$\mathbf{m1} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$ and $\mathbf{m2} = (0,0,1)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \tag{107}$$

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0 \tag{108}$$

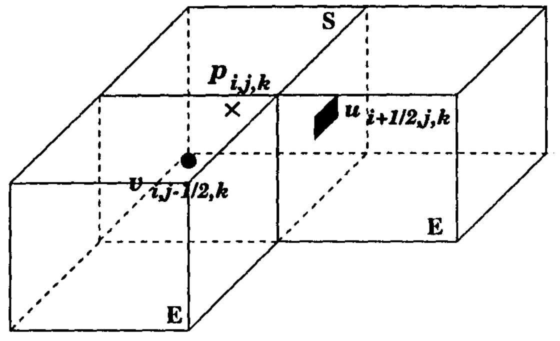

Fig. 18. Surface cell with the $(i+\frac{1}{2})$ and $(j-\frac{1}{2})$ faces contiguous with E–cells.

Here it can be shown that the velocities at the empty cell faces, $u_{i+\frac{1}{2},j,k}$ and $v_{i,j-\frac{1}{2},k}$, are given by

$$u_{i+\frac{1}{2},j,k} = u_{i-\frac{1}{2},j,k} - \frac{1}{2}\frac{\delta x}{\delta z}\left(w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}\right), \tag{109}$$

$$v_{i,j-\frac{1}{2},k} = v_{i,j+\frac{1}{2},k} + \frac{1}{2}\frac{\delta y}{\delta z}\left(w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}\right) \tag{110}$$

respectively. The pressure is then calculated by

$$\begin{aligned}\tilde{p}_{i,j,k} = & \frac{1}{Re}\left[\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y}\right. \\ & - \frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta y}\right. \\ & \left.\left. + \frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i-1,j+\frac{1}{2},k} - v_{i-1,j-\frac{1}{2},k}}{\delta x}\right)\right].\end{aligned} \tag{111}$$

k) **Surface cells having the $(i-\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with empty cells (see figure 19).** For these cells we take $\mathbf{n} = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$, $\mathbf{m1} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$ and $\mathbf{m2} = (0,0,1)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} - \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \tag{112}$$

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0 \tag{113}$$

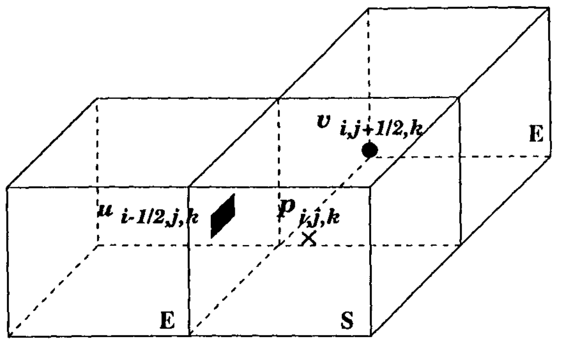

Fig. 19. Surface cell with the $(i-\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with E–cells.

For these cells, the velocities at the empty cell faces and the pressure at the surface cell centre are computed in the same manner as above and are given by

$$u_{i-\frac{1}{2},j,k} = u_{i+\frac{1}{2},j,k} + \frac{1}{2}\frac{\delta x}{\delta z}\left(w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}\right) , \tag{114}$$

$$v_{i,j+\frac{1}{2},k} = v_{i,j-\frac{1}{2},k} - \frac{1}{2}\frac{\delta y}{\delta z}\left(w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}\right) , \tag{115}$$

$$\begin{aligned}\tilde{p}_{i,j,k} = \frac{1}{Re}\Bigg[&\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y} \\ &- \frac{1}{2}\Bigg(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j-1,k} - u_{i-\frac{1}{2},j-1,k}}{\delta y} \\ &+ \frac{v_{i+1,j+\frac{1}{2},k} + v_{i+1,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta x}\Bigg)\Bigg] .\end{aligned} \tag{116}$$

respectively.

l) **Surface cells having the $(i-\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with empty cells (see figure 20).** For these cells we take $\mathbf{n} = \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right)$, $\mathbf{m1} = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right)$ and $\mathbf{m2} = (0,0,1)$. In this case equations (13)–(14) take the form

$$p = \frac{1}{Re}\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \tag{117}$$

$$\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0 \tag{118}$$

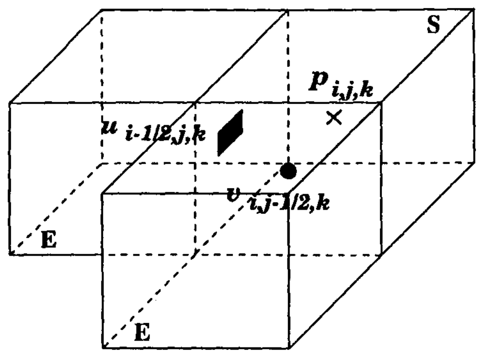

Fig. 20. Surface cell with the $(i-\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with E–cells.

Here it is found that $u_{i-\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $\tilde{p}_{i,j,k}$ are given by

$$u_{i-\frac{1}{2},j,k} = u_{i+\frac{1}{2},j,k} + \frac{1}{2}\frac{\delta x}{\delta z}\left(w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}\right) , \tag{119}$$

$$v_{i,j-\frac{1}{2},k} = v_{i,j-\frac{1}{2},k} + \frac{1}{2}\frac{\delta y}{\delta z}\left(w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}\right) , \tag{120}$$

$$\begin{aligned} \tilde{p}_{i,j,k} &= \frac{1}{Re}\left[\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x} + \frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y}\right. \\ &+ \frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta y}\right. \\ &+ \left.\left.\frac{v_{i+1,j+\frac{1}{2},k} + v_{i+1,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta x}\right)\right] \end{aligned} \tag{121}$$

respectively.

6.1.3. Planar surface cutting three adjacent faces. These surfaces are identified by surface cells having 3 adjacent faces contiguous with empty cells (see figure 21). In order to apply the stress conditions we consider the following approximations:

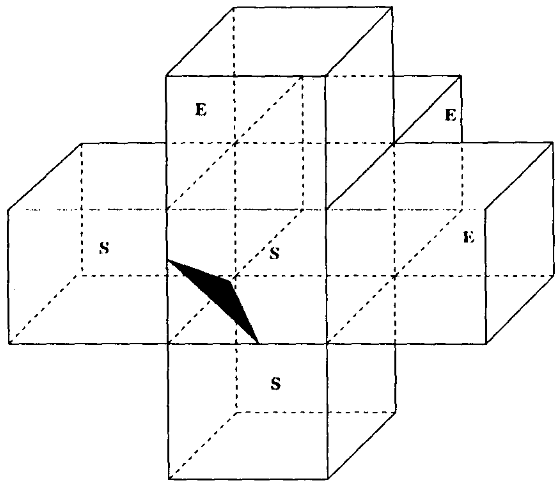

Fig. 21. Surface cell cut by 3 adjacent sides.

a) **Surface cells (S) having the $(i+\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with empty cells (see figure 22).** For these cells we assume the local unit vectors take the form:

$$\mathbf{n} = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right) \;, \quad \mathbf{m1} = \left(0, \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \;, \quad \mathbf{m2} = \left(-2\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}\right) .$$

Introducing **n**, **m1** and **m2** into (13)–(15) we have

$$p - \frac{2}{3Re}\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right] = 0 \;,(122)$$

$$2\frac{\partial v}{\partial y} - 2\frac{\partial w}{\partial z} + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0 \;,(123)$$

$$-4\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial y} + 2\frac{\partial w}{\partial z} - \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + 2\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 \;,(124)$$

respectively. Adding (123) and (124) yields

$$-4\frac{\partial u}{\partial x} + 4\frac{\partial v}{\partial y} - 2\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + 2\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 \;. \qquad (125)$$

Mass conservation for these cells also requires

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \;. \qquad (126)$$

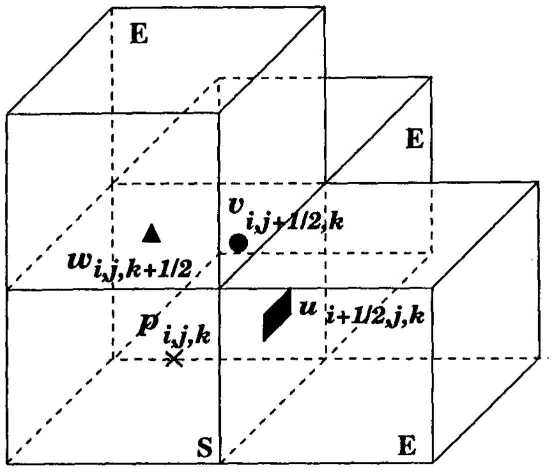

Fig. 22. S–cell with the $(i+\frac{1}{2})$ and $(j+\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with E–cells.

It can be seen that when calculating the tilde velocities through (16)–(18) the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are required. They can be obtained by applying finite differences to (123), (125) and (126) as follows. First, applying (126) at the surface cell centre we have

$$\frac{u_{i+\frac{1}{2},j,k}-u_{i-\frac{1}{2},j,k}}{\delta x}+\frac{v_{i,j+\frac{1}{2},k}-v_{i,j-\frac{1}{2},k}}{\delta y}+\frac{w_{i,j,k+\frac{1}{2}}-w_{i,j,k-\frac{1}{2}}}{\delta z}=0$$

which can be written as

$$v_{i,j+\frac{1}{2},k}+\frac{\delta y}{\delta x}u_{i+\frac{1}{2},j,k}+\frac{\delta y}{\delta z}w_{i,j,k+\frac{1}{2}}=b_3 \tag{127}$$

where

$$b_3=v_{i,j-\frac{1}{2},k}+\frac{\delta y}{\delta x}u_{i-\frac{1}{2},j,k}+\frac{\delta y}{\delta z}w_{i,j,k-\frac{1}{2}}\ . \tag{128}$$

Now, applying (123) at the surface cell centre gives

$$2\left(\frac{v_{i,j+\frac{1}{2},k}-v_{i,j-\frac{1}{2},k}}{\delta y}\right)-2\left(\frac{w_{i,j,k+\frac{1}{2}}-w_{i,j,k-\frac{1}{2}}}{\delta z}\right)+\frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j,k}+u_{i-\frac{1}{2},j,k}-u_{i+\frac{1}{2},j-1,k}-u_{i-\frac{1}{2},j-1,k}}{\delta y}+\frac{v_{i,j+\frac{1}{2},k}+v_{i,j-\frac{1}{2},k}-v_{i-1,j+\frac{1}{2},k}-v_{i-1,j-\frac{1}{2},k}}{\delta x}\right)$$
$$-\frac{1}{2}\left(\frac{u_{i+\frac{1}{2},j,k}+u_{i-\frac{1}{2},j,k}-u_{i+\frac{1}{2},j,k-1}-u_{i-\frac{1}{2},j,k-1}}{\delta z}+\frac{w_{i,j,k+\frac{1}{2}}+w_{i,j,k-\frac{1}{2}}-w_{i-1,j,k+\frac{1}{2}}-w_{i-1,j,k-\frac{1}{2}}}{\delta x}\right)=0$$

which may be written as (after multiplying by 2)

$$\left(4+\frac{\delta y}{\delta x}\right)v_{i,j+\frac{1}{2},k}+\left(1-\frac{\delta y}{\delta x}\right)u_{i+\frac{1}{2},j,k}-\left(4\frac{\delta y}{\delta z}+\frac{\delta y}{\delta x}\right)w_{i,j,k+\frac{1}{2}}=b_1 \tag{129}$$

where

$$\begin{aligned} b_1 &= u_{i+\frac{1}{2},j-1,k} + u_{i-\frac{1}{2},j-1,k} + \left(\frac{\delta y}{\delta z} - 1\right) u_{i-\frac{1}{2},j,k} - \left(\frac{\delta y}{\delta x}\right) \left(v_{i,j-\frac{1}{2},k}\right. \\ &\left. - v_{i-1,j+\frac{1}{2},k} - v_{i-1,j-\frac{1}{2},k}\right) - \left(\frac{\delta y}{\delta z}\right) \left(u_{i+\frac{1}{2},j,k-1} + u_{i-\frac{1}{2},j,k-1}\right) \\ &+ \left(\frac{\delta y}{\delta x}\right) \left(w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}\right) + 4v_{i,j-\frac{1}{2},k} \end{aligned} \tag{130}$$

Similarly, discretizing (125) at surface cell position (i, j, k), we obtain

$$\begin{aligned} &4\left(\frac{v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta y}\right) - 4\left(\frac{u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta x}\right) - \left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k}}{}\right. \\ &\left.\frac{-u_{i+\frac{1}{2},j,k-1} - u_{i-\frac{1}{2},j,k-1}}{\delta z} + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}}{\delta x}\right) \\ &+ \left(\frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} + v_{i,j-\frac{1}{2},k-1}}{\delta z} + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}}}{}\right. \\ &\left.\frac{-w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}}{\delta y}\right) = 0 \end{aligned}$$

which gives

$$\left(4 + \frac{\delta y}{\delta z}\right) v_{i,j+\frac{1}{2},k} - \left(4 + \frac{\delta y}{\delta x}\right) u_{i+\frac{1}{2},j,k} + \left(1 + \frac{\delta y}{\delta x}\right) w_{i,j,k+\frac{1}{2}} = b_2 \tag{131}$$

where

$$\begin{aligned} b_2 &= 4v_{i,j-\frac{1}{2},k} + \left(-4\frac{\delta y}{\delta x} + \frac{\delta y}{\delta z}\right) u_{i-\frac{1}{2},j,k} - \left(\frac{\delta y}{\delta z}\right) \left(u_{i+\frac{1}{2},j,k-1} + u_{i-\frac{1}{2},j,k-1}\right) \\ &- \left(\frac{\delta y}{\delta z}\right) \left(v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}\right) + \left(\frac{\delta y}{\delta x}\right) \left(w_{i,j,k-\frac{1}{2}}\right. \\ &\left. - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}\right) + w_{i,j-1,k+\frac{1}{2}} + w_{i,j-1,k-\frac{1}{2}} - w_{i,j,k-\frac{1}{2}} \end{aligned} \tag{132}$$

It can be seen that eqs. (127), (129) and (131) provide a linear system for the unknowns $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ which in matrix form is given by

$$\begin{bmatrix} \left(4 + \frac{\delta y}{\delta x}\right) & \left(1 - \frac{\delta y}{\delta x}\right) & -\left(4\frac{\delta y}{\delta z} + \frac{\delta y}{\delta x}\right) \\ \left(4 + \frac{\delta y}{\delta z}\right) & -\left(4 + \frac{\delta y}{\delta x}\right) & \left(1 + \frac{\delta y}{\delta x}\right) \\ 1 & \frac{\delta y}{\delta x} & \frac{\delta y}{\delta z} \end{bmatrix} \begin{bmatrix} v_{i,j+\frac{1}{2},k} \\ u_{i+\frac{1}{2},j,k} \\ w_{i,j,k+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{133}$$

The system (133) can be easily solved by Gaussian elimination. Once the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ have been computed, the pressure follows from (122) applied at the surface cell centre, giving

$$\begin{aligned}\tilde{p}_{i,j,k} = \frac{2}{3Re}\Bigg[&\left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j-1,k} - u_{i-\frac{1}{2},j-1,k}}{\delta y} + \frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i-1,j+\frac{1}{2},k} - v_{i-1,j-\frac{1}{2},k}}{\delta x}\right) + \left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} + u_{i-\frac{1}{2},j,k-1}}{\delta z}\right.\\ &\left. + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}}{\delta x}\right) + \left(\frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}}{\delta z} + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}}{\delta y}\right)\Bigg] \quad (134)\end{aligned}$$

b) **Surface cells having the $(i-\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with empty cells (see figure 23).**

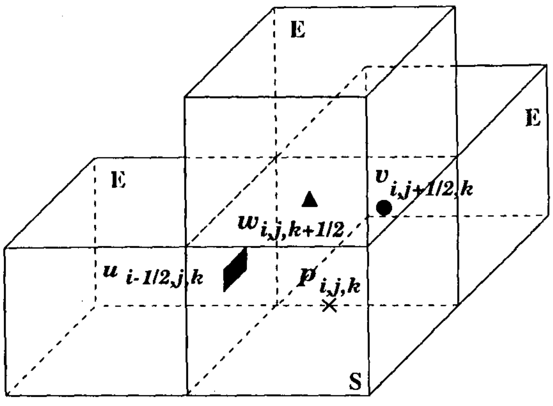

Fig. 23. S–cell with the $(i-\frac{1}{2})$ and $(j+\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with E–cells.

For these cells we assume the local unit vectors take the form:

$$\mathbf{n}=\left(-\frac{\sqrt{3}}{3},\ \frac{\sqrt{3}}{3},\ \frac{\sqrt{3}}{3}\right)\ ,\quad \mathbf{m1}=\left(0,\ \frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2}\right)\ ,\quad \mathbf{m2}=\left(2\frac{\sqrt{6}}{6},\ \frac{\sqrt{6}}{6},\ \frac{\sqrt{6}}{6}\right)\ .$$

Introducing **n**, **m1** and **m2** into (13)-(15) gives

$$p-\frac{2}{3Re}\left[-\left(\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right] = 0\ , \quad (135)$$

$$2\frac{\partial v}{\partial y}-2\frac{\partial w}{\partial z}-\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) = 0\ , \quad (136)$$

$$-4\frac{\partial u}{\partial x}+2\frac{\partial v}{\partial y}+2\frac{\partial w}{\partial z}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+2\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0\ , \quad (137)$$

and adding (136) and (137) we get

$$-4\frac{\partial u}{\partial x}+4\frac{\partial v}{\partial y}+2\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+2\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=0\ . \quad (138)$$

Requiring mass conservation we have

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\ . \quad (139)$$

Again, the values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are required when computing the tilde velocities at nodes adjacents to the surface cell (see figure 23). These can be obtained in a similar manner as in **a)** above, namely by applying (136), (138) and (139) at the surface cell centre. It can be shown that this leads to the following linear system

$$\begin{bmatrix} \left(4+\frac{\delta x}{\delta z}\right) & \left(4\frac{\delta x}{\delta y}+\frac{\delta x}{\delta z}\right) & \left(\frac{\delta x}{\delta y}-1\right) \\ \left(\frac{\delta x}{\delta z}-\frac{\delta x}{\delta y}\right) & \left(4\frac{\delta x}{\delta y}+1\right) & -\left(4\frac{\delta x}{\delta z}+1\right) \\ -1 & \frac{\delta x}{\delta y} & \frac{\delta x}{\delta z} \end{bmatrix} \begin{bmatrix} u_{i-\frac{1}{2},j,k} \\ v_{i,j+\frac{1}{2},k} \\ w_{i,j,k+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{140}$$

where

$$\begin{aligned} b_1 &= 4u_{i+\frac{1}{2},j,k} - \left(\frac{\delta x}{\delta z}\right)\left(u_{i+\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} - u_{i-\frac{1}{2},j,k-1}\right) + 4\left(\frac{\delta x}{\delta y}\right) v_{i,j-\frac{1}{2},k} \\ &- \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}\right) + w_{i,j,k-\frac{1}{2}} - w_{i+1,j,k+\frac{1}{2}} - w_{i+1,j,k-\frac{1}{2}} \\ &- \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j,k-\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}\right) \\ b_2 &= \left(\frac{\delta x}{\delta y}\right)\left(u_{i+\frac{1}{2},j,k} - u_{i+\frac{1}{2},j-1,k} - u_{i-\frac{1}{2},j-1,k}\right) - \left(\frac{\delta x}{\delta z}\right)\left(u_{i+\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1}\right. \\ &- \left. u_{i-\frac{1}{2},j,k-1}\right) + 4\left(\frac{\delta x}{\delta y}\right) v_{i,j-\frac{1}{2},k} + v_{i+1,j+\frac{1}{2},k} + v_{i+1,j-\frac{1}{2},k} - v_{i,j-\frac{1}{2},k} \\ &- 4\left(\frac{\delta x}{\delta z}\right) w_{i,j,k-\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i+1,j,k+\frac{1}{2}} - w_{i+1,j,k-\frac{1}{2}} \\ b_3 &= -u_{i+\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right) v_{i,j-\frac{1}{2},k} + \left(\frac{\delta x}{\delta z}\right) w_{i,j,k-\frac{1}{2}} . \end{aligned}$$

The solution of (140) is obtained by using Gaussian elimination.

After computing $u_{i-\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ from (140), the pressure is calculated using (135) applied at the surface cell centre, namely

$$\begin{aligned} \tilde{p}_{i,j,k} = \frac{2}{3Re}\Bigg[&-\left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j-1,k} - u_{i-\frac{1}{2},j-1,k}}{\delta y} + \frac{v_{i+1,j+\frac{1}{2},k}}{}\right. \\ &\left.\frac{+v_{i+1,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta x}\right) - \left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} + u_{i-\frac{1}{2},j,k-1}}{\delta z}\right. \\ &+ \left.\frac{w_{i+1,j,k+\frac{1}{2}} + w_{i+1,j,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta x}\right) + \left(\frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k}}{}\right. \\ &\left.\frac{-v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}}{\delta z} + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}}{\delta y}\right)\Bigg] \end{aligned} \tag{141}$$

c) **Surface cells having the $(i+\frac{1}{2})$, $(j-\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with empty cells (see figure 24).**

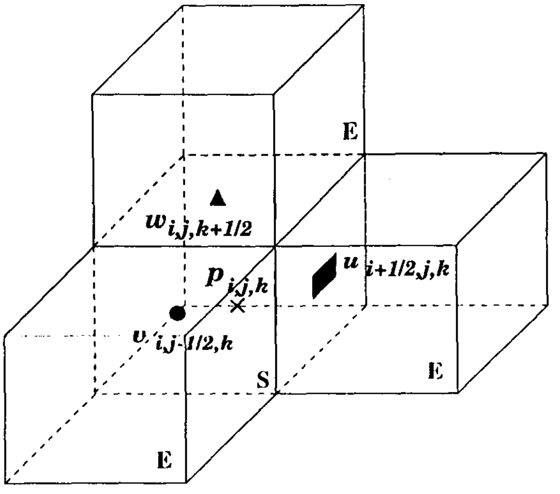

Fig. 24. S–cell with the $(i+\frac{1}{2})$ and $(j-\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with E–cells.

For these cells we assume the local unit vectors take the form:

$$\mathbf{n}=\left(\frac{\sqrt{3}}{3},\ -\frac{\sqrt{3}}{3},\ \frac{\sqrt{3}}{3}\right)\ ,\quad \mathbf{m1}=\left(\frac{\sqrt{2}}{2},\ 0,\ -\frac{\sqrt{2}}{2}\right)\ ,\quad \mathbf{m2}=\left(\frac{\sqrt{6}}{6},\ \frac{2\sqrt{6}}{6},\ \frac{\sqrt{6}}{6}\right)\ .$$

Introducing **n**, **m1** and **m2** into (13)-(15) yields

$$p-\frac{2}{3Re}\left[-\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)-\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right]\ =\ 0\ , \quad (142)$$

$$2\frac{\partial u}{\partial x}-2\frac{\partial w}{\partial z}-\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)\ =\ 0\ , \quad (143)$$

$$2\frac{\partial u}{\partial x}-4\frac{\partial v}{\partial y}+2\frac{\partial w}{\partial z}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+2\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\ =\ 0\ , \quad (144)$$

respectively. Adding (143) and (144) provides

$$4\frac{\partial u}{\partial x}-4\frac{\partial v}{\partial y}+2\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+2\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=0\ . \quad (145)$$

Mass conservation gives

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\ . \quad (146)$$

The velocities at the empty-cell faces contiguous with the surface cell faces are required when computing the tilde velocities. These are obtained by applying eqs. (146), (143) and (145) at the surface cell centre leading to the following linear system:

$$\begin{bmatrix} \left(4+\frac{\delta x}{\delta y}\right) & -\left(\frac{\delta x}{\delta z}+1\right) & -\left(4\frac{\delta x}{\delta z}+\frac{\delta x}{\delta y}\right) \\ \left(4+\frac{\delta x}{\delta z}\right) & \left(4\frac{\delta x}{\delta y}+\frac{\delta x}{\delta z}\right) & 1-\frac{\delta x}{\delta y} \\ 1 & -\frac{\delta x}{\delta y} & \frac{\delta x}{\delta z} \end{bmatrix} \begin{bmatrix} u_{i+\frac{1}{2},j,k} \\ v_{i,j-\frac{1}{2},k} \\ w_{i,j,k+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (147)$$

where

$$\begin{aligned} b_1 &= 4u_{i-\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right)\left(u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i-\frac{1}{2},j,k}\right) \\ &- \left(\frac{\delta x}{\delta y}\right)\left(v_{i,j+\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}\right) + v_{i,j+\frac{1}{2},k} - v_{i-1,j+\frac{1}{2},k} - v_{i-1,j-\frac{1}{2},k} \\ &- 4\left(\frac{\delta x}{\delta z}\right) w_{i,j,k-\frac{1}{2}} + \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j,k-\frac{1}{2}} - w_{i,j+1,k+\frac{1}{2}} - w_{i,j+1,k-\frac{1}{2}}\right) \\ b_2 &= 4u_{i-\frac{1}{2},j,k} - \left(\frac{\delta x}{\delta z}\right)\left(u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} - u_{i-\frac{1}{2},j,k-1}\right) \\ &+ 4\left(\frac{\delta x}{\delta y}\right) v_{i,j+\frac{1}{2},k} - \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j+\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}\right) \\ &- \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}\right) - w_{i,j,k-\frac{1}{2}} + w_{i-1,j,k+\frac{1}{2}} + w_{i-1,j,k-\frac{1}{2}} \\ b_3 &= u_{i-\frac{1}{2},j,k} - \left(\frac{\delta x}{\delta y}\right) v_{i,j+\frac{1}{2},k} + \left(\frac{\delta x}{\delta z}\right) w_{i,j,k-\frac{1}{2}} \, . \end{aligned}$$

The solution of (147) is obtained by Gaussian elimination. Once the velocities at the empty cell faces have been computed the pressure is calculated by (142) applied at the surface cell centre yielding

$$\begin{aligned} \tilde{p}_{i,j,k} = \frac{2}{3Re}\Bigg[&-\left(\frac{u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta y} + \frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i-1,j+\frac{1}{2},k} - v_{i-1,j-\frac{1}{2},k}}{\delta x}\right) \\ &+ \left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} + u_{i-\frac{1}{2},j,k-1}}{\delta z} + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}}{\delta x}\right) \\ &- \left(\frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}}{\delta z} + \frac{w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta y}\right)\Bigg] \quad (148) \end{aligned}$$

d) Surface cells having the $(i-\frac{1}{2})$, $(j-\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with empty cells (see figure 25).

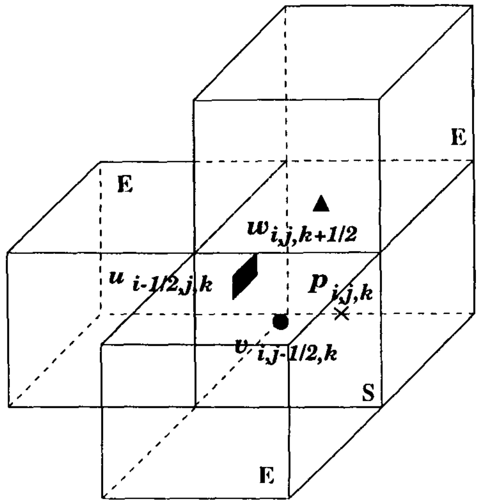

Fig. 25. S–cell with the $(i-\frac{1}{2})$ and $(j-\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with E–cells.

For these cells we assume the local unit vectors take the form:

$$\mathbf{n} = \left(-\frac{\sqrt{3}}{3},\ -\frac{\sqrt{3}}{3},\ \frac{\sqrt{3}}{3}\right)\ ,\ \mathbf{m1} = \left(\frac{\sqrt{2}}{2},\ 0,\ \frac{\sqrt{2}}{2}\right)\ ,\ \mathbf{m2} = \left(\frac{-2\sqrt{6}}{6},\ \frac{\sqrt{6}}{6},\ \frac{\sqrt{6}}{6}\right)\ .$$

Introducing $\mathbf{n}$, $\mathbf{m1}$ and $\mathbf{m2}$ into (13)-(15) gives

$$p - \frac{2}{3Re}\left[\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)-\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right] = 0\ , \quad (149)$$

$$2\frac{\partial u}{\partial x} - 2\frac{\partial w}{\partial z} + \left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) = 0\ , \quad (150)$$

$$2\frac{\partial u}{\partial x} - 4\frac{\partial v}{\partial y} + 2\frac{\partial w}{\partial z} - \left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) - 2\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right) + \left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0\ , \quad (151)$$

respectively. Adding (150) and (151) yields

$$4\frac{\partial u}{\partial x} - 4\frac{\partial v}{\partial y} - 2\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) + 2\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0\ . \quad (152)$$

Mass conservation gives

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} = 0\ . \quad (153)$$

The values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained in a similar way as in **c)** above. By applying eqs. (153), (150) and (152) in the surface cell centre we obtain the following linear system:

$$
\begin{bmatrix} -\left(4+\frac{\delta x}{\delta z}\right) & 4\left(\frac{\delta x}{\delta y}\right)+\frac{\delta x}{\delta z} & 1-\left(\frac{\delta x}{\delta y}\right) \\ \left(-4+\frac{\delta x}{\delta y}\right) & -1+\left(\frac{\delta x}{\delta z}\right) & -4\left(\frac{\delta x}{\delta z}\right)-\frac{\delta x}{\delta y} \\ -1 & -\frac{\delta x}{\delta y} & \frac{\delta x}{\delta z} \end{bmatrix} \begin{bmatrix} u_{i-\frac{1}{2},j,k} \\ v_{i,j-\frac{1}{2},k} \\ w_{i,j,k+\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{154}
$$

where

$$
\begin{aligned}
b_1 &= -4u_{i+\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right)\left(u_{i+\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} - u_{i-\frac{1}{2},j,k-1}\right) + \left(\frac{\delta x}{\delta y}\right) v_{i,j+\frac{1}{2},k} \\
&- \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j+\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}\right) + \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j,k-\frac{1}{2}}\right. \\
&\quad \left. - w_{i,j+1,k+\frac{1}{2}} - w_{i,j+1,k-\frac{1}{2}}\right) + w_{i+1,j,k+\frac{1}{2}} + w_{i+1,j,k-\frac{1}{2}} - w_{i,j,k-\frac{1}{2}} \\
b_2 &= -4u_{i+\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right)\left(u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i+\frac{1}{2},j,k}\right) \\
&- \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j+\frac{1}{2},k} - v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}\right) - v_{i,j+\frac{1}{2},k} - v_{i+1,j+\frac{1}{2},k} \\
&- v_{i+1,j-\frac{1}{2},k} + \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j,k+\frac{1}{2}} - w_{i,j+1,k+\frac{1}{2}} - w_{i,j+1,k-\frac{1}{2}}\right) \\
b_3 &= -u_{i+\frac{1}{2},j,k} - \left(\frac{\delta x}{\delta y}\right) v_{i,j+\frac{1}{2},k} + \left(\frac{\delta x}{\delta z}\right) w_{i,j,k-\frac{1}{2}} .
\end{aligned}
$$

Once $u_{i-\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ have been computed the pressure at the surface cell centre is given by

$$
\begin{aligned}
\tilde{p}_{i,j,k} = \frac{2}{3Re} \Bigg[\Bigg(& \frac{u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta y} + \frac{v_{i+1,j+\frac{1}{2},k}}{} \\
& \frac{+v_{i+1,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta x} \Bigg) - \Bigg(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j,k-1} + u_{i-\frac{1}{2},j,k-1}}{\delta z} \\
& + \frac{w_{i+1,j,k+\frac{1}{2}} + w_{i+1,j,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta x} \Bigg) - \Bigg(\frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k}}{} \\
& \frac{-v_{i,j+\frac{1}{2},k-1} - v_{i,j-\frac{1}{2},k-1}}{\delta z} + \frac{w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta y} \Bigg) \Bigg]
\end{aligned} \tag{155}
$$

e) **Surface cells having the $(i+\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with empty cells (see figure 26).**

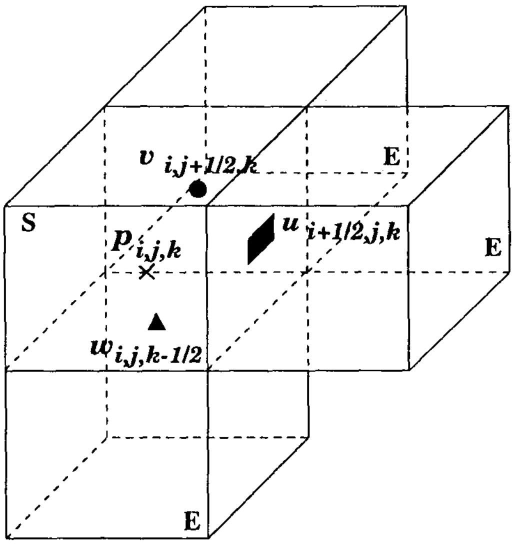

Fig. 26. S–cell with the $(i+\frac{1}{2})$ and $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with E–cells faces.

For these cells we assume the local unit vectors take the form:

$$\mathbf{n} = \left(\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\right) \ , \quad \mathbf{m1} = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}, 0\right) \ , \quad \mathbf{m2} = \left(\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{2\sqrt{6}}{6}\right) .$$

Introducing **n**, **m1** and **m2** into (13)–(15) we have

$$p - \frac{2}{3Re}\left[\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) - \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)\right] = 0 \ , \quad (156)$$

$$2\frac{\partial u}{\partial x} - 2\frac{\partial v}{\partial y} - \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 \ , \quad (157)$$

$$2\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial y} - 4\frac{\partial w}{\partial z} + 2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 \ , \quad (158)$$

respectively. Adding (157) and (158) it yields

$$4\frac{\partial u}{\partial x} - 4\frac{\partial w}{\partial z} + 2\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) + 2\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0 . \quad (159)$$

Mass conservation gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 . \quad (160)$$

Similarly as above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are required. They are obtained by applying eqs. (160), (157) and (159) at the surface cell centre which leads to the following linear system

$$\begin{bmatrix} \left(4+\frac{\delta x}{\delta z}\right) & -\left(4\frac{\delta x}{\delta y}+\frac{\delta x}{\delta z}\right) & \left(\frac{\delta x}{\delta y}-1\right) \\ \left(4+\frac{\delta x}{\delta y}\right) & \left(1-\frac{\delta x}{\delta z}\right) & \left(4\frac{\delta x}{\delta z}+\frac{\delta x}{\delta y}\right) \\ 1 & \frac{\delta x}{\delta y} & -\frac{\delta x}{\delta z} \end{bmatrix} \begin{bmatrix} u_{i+\frac{1}{2},j,k} \\ v_{i,j+\frac{1}{2},k} \\ w_{i,j,k-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{161}$$

where

$$\begin{aligned} b_1 &= 4u_{i-\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta z}\right)\left(u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i-\frac{1}{2},j,k}\right) - 4\left(\frac{\delta x}{\delta y}\right) v_{i,j-\frac{1}{2},k} \\ &+ \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k+1} - v_{i,j-\frac{1}{2},k+1}\right) + w_{i,j,k+\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} \\ &- w_{i-1,j,k-\frac{1}{2}} - \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j,k+\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}\right) \\ b_2 &= 4u_{i-\frac{1}{2},j,k} - \left(\frac{\delta x}{\delta y}\right)\left(u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j-1,k} - u_{i-\frac{1}{2},j-1,k}\right) + \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j-\frac{1}{2},k}\right. \\ &- \left. v_{i,j+\frac{1}{2},k+1} - v_{i,j-\frac{1}{2},k+1}\right) - v_{i,j-\frac{1}{2},k} + v_{i-1,j+\frac{1}{2},k} + v_{i-1,j-\frac{1}{2},k} \\ &- \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j,k+\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}\right) + 4\left(\frac{\delta x}{\delta z}\right) w_{i,j,k+\frac{1}{2}} \\ b_3 &= u_{i-\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right) v_{i,j-\frac{1}{2},k} - \left(\frac{\delta x}{\delta z}\right) w_{i,j,k+\frac{1}{2}} \,. \end{aligned}$$

After calculating the velocities at the empty cell faces, the pressure at the surface cell centre is computed by

$$\begin{aligned} \tilde{p}_{i,j,k} = \frac{2}{3Re}\Bigg[\Bigg(&\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j-1,k} - u_{i-\frac{1}{2},j-1,k}}{\delta y} + \frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k}}{} \\ &\frac{-v_{i-1,j+\frac{1}{2},k} - v_{i-1,j-\frac{1}{2},k}}{\delta x}\Bigg) - \Bigg(\frac{u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k}}{\delta z} \\ &+ \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}}{\delta x}\Bigg) - \Bigg(\frac{v_{i,j+\frac{1}{2},k+1} + v_{i,j-\frac{1}{2},k+1}}{} \\ &\frac{-v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta z} + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}}{\delta y}\Bigg)\Bigg] \end{aligned} \tag{162}$$

f) Surface cells having the $(i-\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with empty cells (see figure 27).

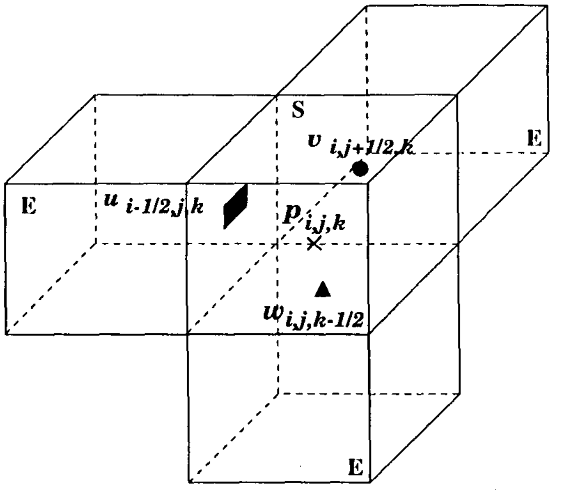

Fig. 27. S cell with the $(i-\frac{1}{2})$ and $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with E–cells faces.

For these cells we assume the local unit vectors take the form:

$$\mathbf{n} = \left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\right) \ , \ \mathbf{m1} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right) \ , \ \mathbf{m2} = \left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}, \frac{2\sqrt{6}}{6}\right) .$$

Introducing **n**, **m1** and **m2** into (13)-(15) yields

$$p - \frac{2}{3Re}\left[-\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)-\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right] = 0 \ , \quad (163)$$

$$2\frac{\partial u}{\partial x}-2\frac{\partial v}{\partial y}+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0 \ , \quad (164)$$

$$2\frac{\partial u}{\partial x}+2\frac{\partial v}{\partial y}-4\frac{\partial w}{\partial z}-2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0 \ , \quad (165)$$

respectively. Adding (164) and (165) yields

$$4\frac{\partial u}{\partial x}-4\frac{\partial w}{\partial z}-2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+2\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=0 \ . \quad (166)$$

Mass conservation gives

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \ . \quad (167)$$

The values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are found as above. Namely, by applying eqs. (167), (164) and (166) at the surface cell centre it leads to the following linear system

$$\begin{bmatrix} -\left(4+\frac{\delta x}{\delta z}\right) & -\left(4\frac{\delta x}{\delta y}+\frac{\delta x}{\delta z}\right) & \frac{\delta x}{\delta y}-1 \\ -\left(4+\frac{\delta x}{\delta y}\right) & 1-\frac{\delta x}{\delta z} & \left(4\frac{\delta x}{\delta z}+\frac{\delta x}{\delta y}\right) \\ -1 & \frac{\delta x}{\delta y} & -\frac{\delta x}{\delta z} \end{bmatrix} \begin{bmatrix} u_{i-\frac{1}{2},j,k} \\ v_{i,j+\frac{1}{2},k} \\ w_{i,j,k-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{168}$$

where

$$\begin{aligned} b_2 &= -4u_{i+\frac{1}{2},j,k} - \left(\frac{\delta x}{\delta z}\right)\left(u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k}\right) + 4\left(\frac{\delta x}{\delta y}\right) v_{i,j-\frac{1}{2},k} \\ &- \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j+\frac{1}{2},k+1} + v_{i,j-\frac{1}{2},k+1} - v_{i,j-\frac{1}{2},k}\right) - \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j,k+\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}}\right. \\ &- \left. w_{i,j-1,k-\frac{1}{2}}\right) + w_{i,j,k+\frac{1}{2}} - w_{i+1,j,k+\frac{1}{2}} - w_{i+1,j,k-\frac{1}{2}} \\ b_2 &= -4u_{i+\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right)\left(u_{i+\frac{1}{2},j,k} - u_{i+\frac{1}{2},j-1,k} - u_{i-\frac{1}{2},j-1,k}\right) \\ &+ \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k+1} - v_{i,j-\frac{1}{2},k+1}\right) + v_{i+1,j+\frac{1}{2},k} + v_{i+1,j-\frac{1}{2},k} - v_{i,j-\frac{1}{2},k} \\ &- \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j,k+\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}\right) + 4\left(\frac{\delta x}{\delta z}\right) w_{i,j,k+\frac{1}{2}} \\ b_3 &= -u_{i+\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right) v_{i,j-\frac{1}{2},k} - \left(\frac{\delta x}{\delta z}\right) w_{i,j,k+\frac{1}{2}} . \end{aligned}$$

After solving (168) by Gaussian elimination the pressure at the surface cell centre is computed by

$$\begin{aligned} \tilde{p}_{i,j,k} = \frac{2}{3Re}\Bigg[&- \left(\frac{u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k} - u_{i+\frac{1}{2},j-1,k} - u_{i-\frac{1}{2},j-1,k}}{\delta y} + \frac{v_{i+1,j+\frac{1}{2},k}}{} \right. \\ &\left. \frac{+v_{i+1,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta x} \right) + \left(\frac{u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k}}{\delta z} \right. \\ &\left. + \frac{w_{i+1,j,k+\frac{1}{2}} + w_{i+1,j,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta x} \right) - \left(\frac{v_{i,j+\frac{1}{2},k+1} + v_{i,j-\frac{1}{2},k+1}}{} \right. \\ &\left. \frac{-v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta z} + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i,j-1,k+\frac{1}{2}} - w_{i,j-1,k-\frac{1}{2}}}{\delta y} \right) \Bigg] \end{aligned} \tag{169}$$

g) **Surface cells having the $(i+\frac{1}{2})$, $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with empty cells (see figure 28).**

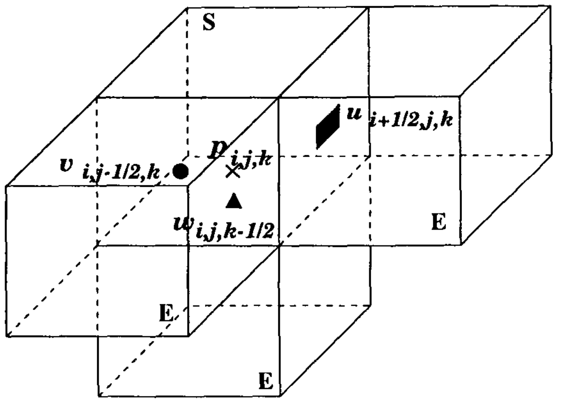

Fig. 28. S–cell with the $(i+\frac{1}{2})$ and $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with E–cell faces.

For these cells we assume the local unit vectors can be approximated by

$$\mathbf{n} = \left(\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}, -\frac{\sqrt{3}}{3}\right) , \quad \mathbf{m1} = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right) , \quad \mathbf{m2} = \left(\frac{\sqrt{6}}{6}, -\frac{\sqrt{6}}{6}, \frac{2\sqrt{6}}{6}\right) .$$

Introducing **n**, **m1** and **m2** into (13)–(15) yields

$$p - \frac{2}{3Re}\left[-\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) - \left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) + \left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right] = 0 \, , \tag{170}$$

$$2\frac{\partial u}{\partial x} - 2\frac{\partial v}{\partial y} - \left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right) - \left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0 \, , \tag{171}$$

$$2\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial y} - 4\frac{\partial w}{\partial z} - 2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) + \left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right) - \left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0 \, , \tag{172}$$

respectively. Adding (171) and (172) we have

$$4\frac{\partial u}{\partial x} - 4\frac{\partial w}{\partial z} - 2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right) - 2\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0 \, . \tag{173}$$

Mass conservation gives

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \, . \tag{174}$$

The values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are required when computing the tilde velocities. They are obtained by applying eqs. (174), (171) and (173) at the surface cell centre. It can be shown that the resulting linear system is given by

$$\begin{bmatrix} \left(4+\frac{\delta x}{\delta z}\right) & \left(4\frac{\delta x}{\delta y}+\frac{\delta x}{\delta z}\right) & \left(\frac{\delta x}{\delta y}-1\right) \\ \left(4+\frac{\delta x}{\delta y}\right) & \left(\frac{\delta x}{\delta z}-1\right) & \left(4\frac{\delta x}{\delta z}+\frac{\delta x}{\delta y}\right) \\ 1 & -\frac{\delta x}{\delta y} & -\frac{\delta x}{\delta z} \end{bmatrix} \begin{bmatrix} u_{i+\frac{1}{2},j,k} \\ v_{i,j-\frac{1}{2},k} \\ w_{i,j,k-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{175}$$

where

$$\begin{aligned} b_1 &= 4u_{i+\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta z}\right)\left(u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k}\right) + 4\left(\frac{\delta x}{\delta y}\right) v_{i,j+\frac{1}{2},k} \\ &+ \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j+\frac{1}{2},k+1} + v_{i,j-\frac{1}{2},k+1} - v_{i,j+\frac{1}{2},k}\right) + \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}}\right. \\ &- \left. w_{i,j,k+\frac{1}{2}}\right) + w_{i,j,k+\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}} \\ b_2 &= 4u_{i-\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right)\left(u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i-\frac{1}{2},j,k}\right) + \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j+\frac{1}{2},k+1}\right. \\ &+ \left. v_{i,j-\frac{1}{2},k+1} - v_{i,j+\frac{1}{2},k}\right) + v_{i,j+\frac{1}{2},k} - v_{i-1,j+\frac{1}{2},k} - v_{i-1,j-\frac{1}{2},k} \\ &+ 4\left(\frac{\delta x}{\delta z}\right) w_{i,j,k+\frac{1}{2}} + \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}}\right) \\ b_3 &= u_{i-\frac{1}{2},j,k} - \left(\frac{\delta x}{\delta y}\right) v_{i,j+\frac{1}{2},k} - \left(\frac{\delta x}{\delta z}\right) w_{i,j,k+\frac{1}{2}} . \end{aligned}$$

After solving (175), the pressure at the surface cell centre is given by (170) applied at position (i, j, k) which gives

$$\begin{aligned} \tilde{p}_{i,j,k} = \frac{2}{3Re}\Bigg[&-\Bigg(\frac{u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta y} + \frac{v_{i,j+\frac{1}{2},k} + v_{i,j-\frac{1}{2},k} - v_{i-1,j+\frac{1}{2},k} - v_{i-1,j-\frac{1}{2},k}}{\delta x}\Bigg) \\ &- \Bigg(\frac{u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k}}{\delta z} + \frac{w_{i,j,k+\frac{1}{2}} + w_{i,j,k-\frac{1}{2}} - w_{i-1,j,k+\frac{1}{2}} - w_{i-1,j,k-\frac{1}{2}}}{\delta x}\Bigg) \\ &+ \Bigg(\frac{v_{i,j+\frac{1}{2},k+1} + v_{i,j-\frac{1}{2},k+1} - v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta z} + \frac{w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta y}\Bigg)\Bigg] \end{aligned} \tag{176}$$

h) **Surface cells having the $(i-\frac{1}{2})$, $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with empty cells (see figure 29).**

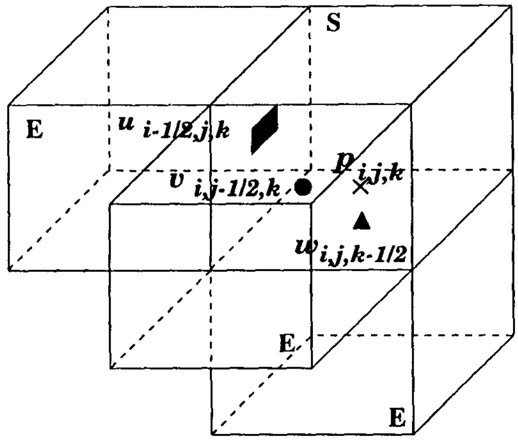

Fig. 29. S–cell with the $(i-\frac{1}{2})$ and $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with E–cell faces.

For these cells we assume the local unit vectors can be approximated by

$$\mathbf{n} = \left(-\frac{\sqrt{3}}{3},\ -\frac{\sqrt{3}}{3},\ -\frac{\sqrt{3}}{3}\right)\ ,\ \mathbf{m1} = \left(\frac{\sqrt{2}}{2},\ -\frac{\sqrt{2}}{2},\ 0\right)\ ,\ \mathbf{m2} = \left(\frac{\sqrt{6}}{6},\ \frac{\sqrt{6}}{6},\ \frac{-2\sqrt{6}}{6}\right) .$$

Introducing **n**, **m1** and **m2** into (13)–(15) yields

$$p - \frac{2}{3Re}\left[-\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)\right] = 0\ , \quad (177)$$

$$2\frac{\partial u}{\partial x}-2\frac{\partial v}{\partial y}-\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)-\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0\ , \quad (178)$$

$$2\frac{\partial u}{\partial x}+2\frac{\partial v}{\partial y}-4\frac{\partial w}{\partial z}-2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)+\left(\frac{\partial u}{\partial z}-\frac{\partial w}{\partial x}\right)-\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right) = 0\ , \quad (179)$$

respectively. Adding (178) and (179) we have

$$4\frac{\partial u}{\partial x}-4\frac{\partial w}{\partial z}-2\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-2\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)=0\ . \quad (180)$$

Mass conservation gives

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (181)$$

The values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are required when computing the tilde velocities. They can be obtained by applying eqs. (181), (178) and (180) at the surface cell centre which leads the following linear system

$$
\begin{bmatrix} -\left(4+\frac{\delta x}{\delta y}\right) & \left(\frac{\delta x}{\delta y}-1\right) & \left(4\frac{\delta x}{\delta z}+\frac{\delta x}{\delta y}\right) \\ -\left(4+\frac{\delta x}{\delta z}\right) & \left(4\frac{\delta x}{\delta y}+\frac{\delta x}{\delta z}\right) & \left(\frac{\delta x}{\delta y}-1\right) \\ -1 & \frac{\delta x}{\delta y} & \frac{\delta x}{\delta z} \end{bmatrix} \begin{bmatrix} u_{i-\frac{1}{2},j,k} \\ v_{i,j-\frac{1}{2},k} \\ w_{i,j,k-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad (182)
$$

where

$$
\begin{aligned}
b_1 &= -4u_{i+\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right)\left(u_{i+\frac{1}{2},j,k} - u_{i+\frac{1}{2},j+1,k} - u_{i-\frac{1}{2},j+1,k}\right) - \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j+\frac{1}{2},k}\right. \\
&- \left. v_{i,j+\frac{1}{2},k+1} - v_{i,j-\frac{1}{2},k+1}\right) + v_{i,j+\frac{1}{2},k} - v_{i+1,j+\frac{1}{2},k} - v_{i+1,j-\frac{1}{2},k} \\
&+ 4\left(\frac{\delta x}{\delta z}\right) w_{i,j,k+\frac{1}{2}} + \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}}\right) \\
b_2 &= -4u_{i+\frac{1}{2},j,k} - \left(\frac{\delta x}{\delta z}\right)\left(u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k}\right) + 4\left(\frac{\delta x}{\delta y}\right) v_{i,j+\frac{1}{2},k} \\
&+ \left(\frac{\delta x}{\delta z}\right)\left(v_{i,j+\frac{1}{2},k+1} + v_{i,j-\frac{1}{2},k+1} - v_{i,j+\frac{1}{2},k}\right) + w_{i,j,k+\frac{1}{2}} - w_{i+1,j,k+\frac{1}{2}} \\
&- w_{i+1,j,k-\frac{1}{2}} + \left(\frac{\delta x}{\delta y}\right)\left(w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}}\right) \\
b_3 &= u_{i+\frac{1}{2},j,k} + \left(\frac{\delta x}{\delta y}\right) v_{i,j+\frac{1}{2},k} + \left(\frac{\delta x}{\delta z}\right) w_{i,j,k+\frac{1}{2}} \,.
\end{aligned}
$$

Once $u_{i-\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ have been computed, the pressure at the surface cell is obtained by (177) applied at position (i,j,k) which gives

$$
\begin{aligned}
\tilde{p}_{i,j,k} = \frac{2}{3Re}\Bigg[&-\Bigg(\frac{u_{i+\frac{1}{2},j+1,k} + u_{i-\frac{1}{2},j+1,k} - u_{i+\frac{1}{2},j,k} - u_{i-\frac{1}{2},j,k}}{\delta y} + \frac{v_{i+1,j+\frac{1}{2},k} + v_{i+1,j-\frac{1}{2},k} - v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta x}\Bigg) \\
&- \Bigg(\frac{u_{i+\frac{1}{2},j,k+1} + u_{i-\frac{1}{2},j,k+1} - u_{i+\frac{1}{2},j,k} + u_{i-\frac{1}{2},j,k}}{\delta z} + \frac{w_{i+1,j,k+\frac{1}{2}} + w_{i+1,j,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta x}\Bigg) \\
&+ \Bigg(\frac{v_{i,j+\frac{1}{2},k+1} + v_{i,j-\frac{1}{2},k+1} - v_{i,j+\frac{1}{2},k} - v_{i,j-\frac{1}{2},k}}{\delta z} + \frac{w_{i,j+1,k+\frac{1}{2}} + w_{i,j+1,k-\frac{1}{2}} - w_{i,j,k+\frac{1}{2}} - w_{i,j,k-\frac{1}{2}}}{\delta y}\Bigg)\Bigg] \quad (183)
\end{aligned}
$$

6.2. Boundary Conditions on Curved Surfaces

When the discretized Navier-Stokes equations (16)–(18) are applied at nodes adjacent to a boundary cell (B-cell), the velocities u, v and w on the B–cell faces are required. If no-slip conditions are imposed on the boundary surface these values can be estimated in terms of function values at internal nodes and boundary values by linear interpolation.
It can be seen that the boundary cells can have one, two and three faces contiguous with interior cells. More specificaly, there are 6 possible configurations of B–cells with only one face contiguous with an interior cell, 12 cases of B–cells with two adjacent faces contiguous with interior cells and 8 cases of B–cells having three adjacent faces contiguous with interior cells. In this Section we give the equations for calculating the velocities on the boundary cell faces by considering various B–cell configurations as follows:

6.2.1. B–cells having only one face contiguous with an interior cell.

For these cells we compute the velocities on the B–cell faces in terms of the velocities of the adjacent interior cell and the velocity at the boundary by using linear interpolation. The several cases to consider will be given next:

a) B-Cell with the $(i+\frac{1}{2})$ face contiguous with an interior cell (see figure 30).

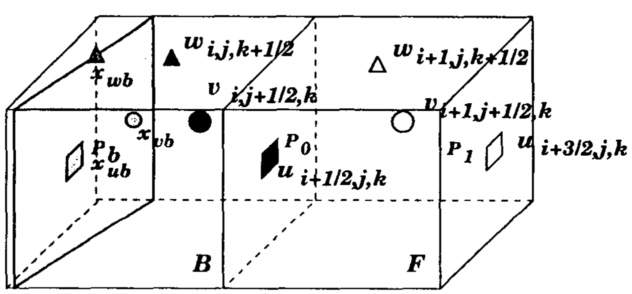

Fig. 30. B–cell with the $(i+\frac{1}{2})$–face contiguous with an interior cell.

As we can see in figure 30, the velocities $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are required when computing the tilde velocities through (16)–(18). These can be obtained by employing linear interpolation between the velocity on the interior cell and the boundary velocity as follows: Consider figure 30 for the calculation of $u_{i+\frac{1}{2},j,k}$. Let $\mathbf{P_0} = (x_{i+\frac{1}{2}}, y_j, z_k)$, $\mathbf{P_1} = (x_{i+\frac{3}{2}}, y_j, z_k)$ and $\mathbf{P_b} = (x_{ub}, y_j, z_k)$ where x_{ub} denotes the intersection point between the line defined by $\mathbf{P_0}$ and $\mathbf{P_1}$ and the boundary surface, namely, x_{ub} is calculated from

$$f(x_{ub}, y_j, z_k) = 0$$

where $f(x, y, z)$ is the equation describing the local boundary. Thus, linear interpolation between $\mathbf{P_b}$ and $\mathbf{P_1}$ gives

$$u(x) = \frac{x - x_{i+\frac{3}{2}}}{x_{ub} - x_{i+\frac{3}{2}}} u_b + \frac{x - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k}$$

so that an approximation for $u_{i+\frac{1}{2},j,k}$ is obtained by

$$u_{i+\frac{1}{2},j,k} = \frac{x_{i+\frac{1}{2}} - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k} - \frac{\delta x}{x_{ub} - x_{i+\frac{3}{2}}} u_b \tag{184}$$

where u_b is the boundary velocity in the x-direction. The other two velocities on the B-cell faces are obtained similarly and are given by

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j+\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b \tag{185}$$

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k+\frac{1}{2}} - \frac{\delta x}{x_{wb} - x_{i+1}} w_b \tag{186}$$

where x_{vb} and x_{wb} are obtained from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ ,$$

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ ,$$

respectively and v_b and w_b are the boundary velocities in the y and z directions respectively.

b) B-Cell with the $(i-\frac{1}{2})$–face contiguous with an interior cell (see figure 31).

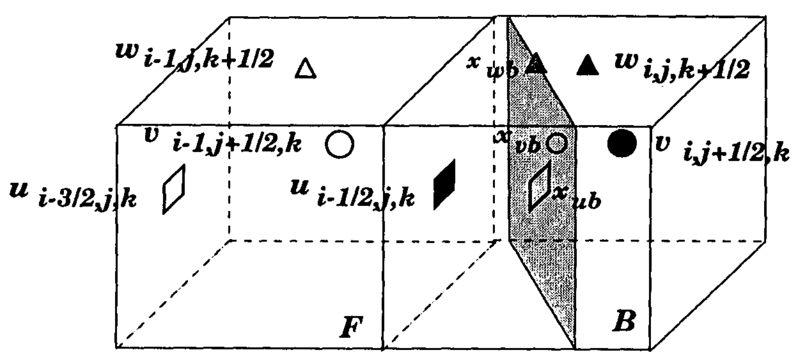

Fig. 31. B–cell with the $(i - \frac{1}{2})$–face contiguous with an interior cell.

Here we procceed as in **a)** above and it can be shown that the velocities at the boundary cell faces are given by

$$u_{i-\frac{1}{2},j,k} = \frac{x_{i-\frac{1}{2}} - x_{ub}}{x_{i-\frac{3}{2}} - x_{ub}} u_{i-\frac{3}{2},j,k} + \frac{\delta x}{x_{ub} - x_{i-\frac{3}{2}}} u_b \tag{187}$$

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j+\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b \tag{188}$$

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k+\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b \tag{189}$$

where x_{ub}, x_{vb} and x_{wb} are calculated from

$$f(x_{ub}, y_j, z_k) = 0 \ ,$$

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ ,$$

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ ,$$

respectively.

c) B–Cell with the $(j + \frac{1}{2})$–face contiguous with an interior cell.

Similarly as in **a)** above, the velocities on the B-cell faces are computed by employing linear interpolation between the velocity of the interior cell and the velocity on the boundary (see figure 32). It can be verified that in this case the velocities $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are given by

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j+1} - y_{ub}} u_{i+\frac{1}{2},j+1,k} - \frac{\delta y}{y_{ub} - y_{j+1}} u_b \tag{190}$$

$$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b \tag{191}$$

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k+\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b \tag{192}$$

where y_{ub}, y_{vb} and y_{wb} are calculated from

$$f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0 \ ,$$

$$f(x_i, y_{vb}, z_k) = 0 \ ,$$

$$f(x_i, y_{wb}, z_{k+\frac{1}{2}}) = 0 \ ,$$

respectively.

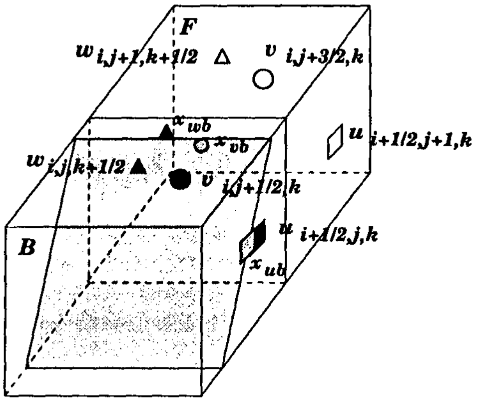

Fig. 32. B–cell with the $(j+\frac{1}{2})$–face contiguous with an interior cell.

d) B–Cell with the $(j-\frac{1}{2})$–face contiguous with an interior cell (see figure 33).

As in **c)**, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are approximated by

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j-1} - y_{ub}} u_{i+\frac{1}{2},j-1,k} + \frac{\delta y}{y_{ub} - y_{j-1}} u_b \tag{193}$$

$$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} + \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b \tag{194}$$

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j-1} - y_{wb}} w_{i,j-1,k+\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j-1}} w_b \tag{195}$$

where y_{ub}, y_{vb} and y_{wb} are calculated from

$$f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0 \ ,$$

$$f(x_i, y_{vb}, z_k) = 0 \ ,$$

$$f(x_i, y_{wb}, z_{k+\frac{1}{2}}) = 0 \ ,$$

respectively.

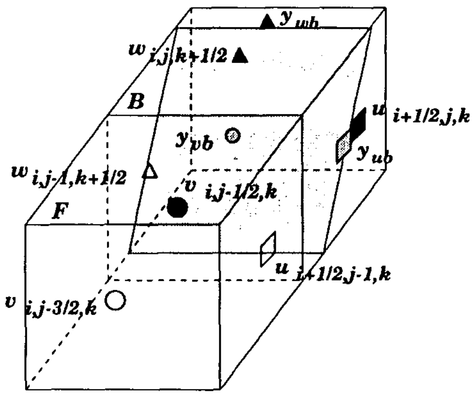

Fig. 33. B–cell with the $(j-\frac{1}{2})$–face contiguous with an interior cell.

e) B–Cell with the $(k+\frac{1}{2})$–face contiguous with an interior cell (see figure 34).

Again, interpolation between the velocity of the interior cell and the velocity on the boundary surface gives the following approximations for $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k+1} - z_{ub}} u_{i+\frac{1}{2},j,k+1} - \frac{\delta z}{z_{ub} - z_{k+1}} u_b \tag{196}$$

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j+\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b \tag{197}$$

$$w_{i,j,k+\frac{1}{2}} = \frac{z_k - z_{wb}}{z_{k+1} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+1}} w_b \tag{198}$$

where y_{ub}, y_{vb} and y_{wb} are calculated from

$$f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0 \ ,$$

$$f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0 \ ,$$

$$f(x_i, y_i, z_{wb}) = 0 \ ,$$

respectively.

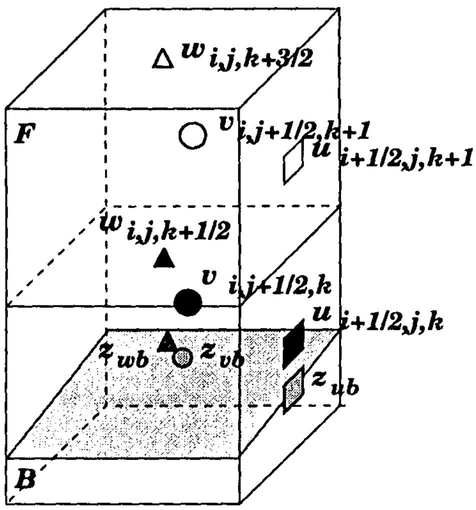

Fig. 34. B–cell with the $k+\frac{1}{2}$–face contiguous with an interior cell.

f) **B–Cell with the $(k-\frac{1}{2})$–face contiguous with an interior cell (see figure 35).**

Similarly as in **e)** above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are given by

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k-1} - z_{ub}} u_{i+\frac{1}{2},j,k-1} + \frac{\delta z}{z_{ub} - z_{k-1}} u_b \tag{199}$$

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j+\frac{1}{2},k-1} + \frac{\delta z}{z_{vb} - z_{k-\frac{3}{2}}} v_b \tag{200}$$

$$w_{i,j,k+\frac{1}{2}} = \frac{z_k - z_{wb}}{z_{k-1} - z_{wb}} w_{i,j,k-\frac{3}{2}} + \frac{\delta z}{z_{wb} - z_{k-1}} w_b \tag{201}$$

where y_{ub}, y_{vb} and y_{wb} are calculated from

$$f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0 \ ,$$

$$f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0 \ ,$$

$$f(x_i, y_i, z_{wb}) = 0 \ ,$$

respectively.

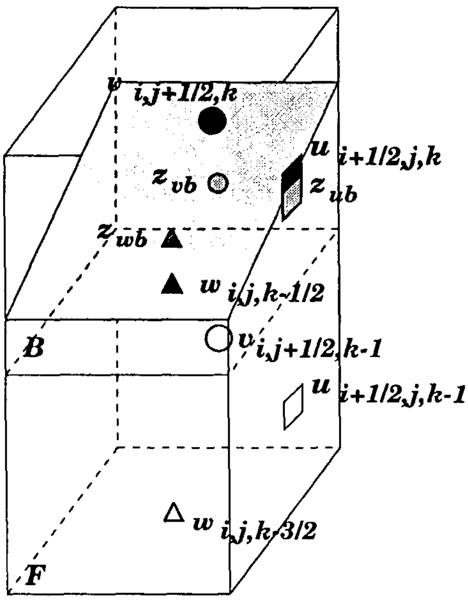

Fig. 35. B–cell with the $(k-\frac{1}{2})$–face contiguous with an interior cell.

6.2.2. B–cells having two adjacent faces contiguous with interior cells.

The twelve configurations of B–cells having two adjacent faces contiguous with interior cells will be treated in a similiar way as for B–cells having only one face contiguous with an interior cell. Each of these cases will be reduced to linear interpolation in one direction and then the equations derived in Section 6.2.1 will be used to obtain approximations for the velocities on the B–cell faces. The twelve cases will be considered next:

a) **B–cell with the $(i+\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with interior cells (see figure 36).**

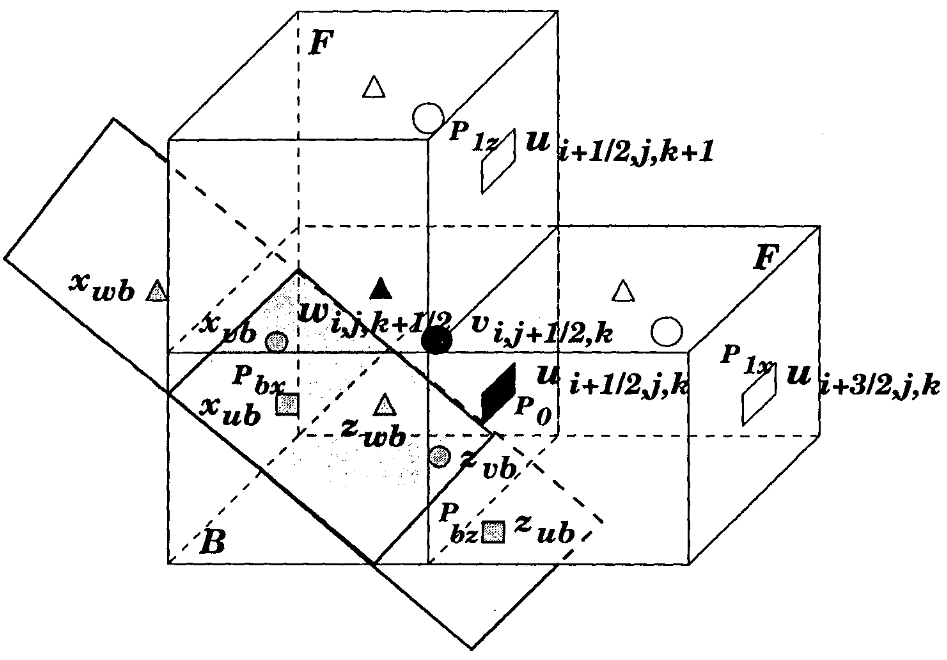

Fig. 36. B–cell with the $(i+\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with interior cells.

As we can see from figure 36, in order to obtain an approximation for $u_{i+\frac{1}{2},j,k}$ one may employ linear interpolation in the x-direction by using $u_{i+\frac{3}{2},j,k}$ and u_b or interpolate in the z-direction by using $u_{i+\frac{1}{2},j,k+1}$ and u_b. To choose which direction is the more appropriate to perform the interpolation we procceed as follows.

Consider figure 36 for the calculation of $u_{i+\frac{1}{2},j,k}$. Let $\mathbf{P_0} = (x_{i+\frac{1}{2}}, y_j, z_k)$, $\mathbf{P_{1x}} = (x_{i+\frac{3}{2}}, y_j, z_k)$, $\mathbf{P_{bx}} = (x_{ub}, y_j, z_k)$, $\mathbf{P_{1z}} = (x_{i+\frac{1}{2}}, y_j, z_{k+1})$ and $\mathbf{P_{bz}} = (x_{i+\frac{1}{2}}, y_j, z_{ub})$, where x_{ub} is the intersection point between the line defined by $\mathbf{P_0}$ and $\mathbf{P_{1x}}$ and the boundary surface; z_{ub} is the intersection point of the line defined by $\mathbf{P_0}$ and $\mathbf{P_{1z}}$ and the boundary surface. The values of x_{ub} and z_{ub} can be computed from

$$f(x_{ub}, y_j, z_k) = 0$$

and

$$f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

respectively. Once x_{ub} and z_{ub} have been obtained we can calculate the distances

$$d_{xu} = \left|x_{ub} - x_{i+\frac{1}{2}}\right| \quad \text{and} \quad d_{zu} = |z_{ub} - z_k| \ .$$

To choose the direction for interpolation we take the closest point to $\mathbf{P_0}$. For instance, if $d_{xu} < d_{zu}$ we take $\mathbf{P_{bx}}$ and interpolate between $\mathbf{P_{bx}}$ and $\mathbf{P_{1x}}$. In this case, it can be easily verified that $u_{i+\frac{1}{2},j,k}$ is given by

$$u_{i+\frac{1}{2},j,k} = \frac{x_{i+\frac{1}{2}} - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k} - \frac{\delta x}{x_{ub} - x_{i+\frac{3}{2}}} u_b \ . \tag{202}$$

On the other hand, if $d_{xu} > d_{zu}$ then we choose $\mathbf{P_{bz}}$ and interpolate between $\mathbf{P_{bz}}$ and $\mathbf{P_{1z}}$ which gives

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k+1} - z_{ub}} u_{i+\frac{1}{2},j,k+1} - \frac{\delta z}{z_{ub} - z_{k+1}} u_b \ . \tag{203}$$

For the other two velocities, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$, the same criteria are applied. For instance, to obtain an approximation for $v_{i,j+\frac{1}{2},k}$ we compute the intersection points x_{vb} and z_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0$$

and

$$f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0$$

and calculate the distances

$$d_{xv} = |x_{vb} - x_i| \quad \text{and} \quad d_{zv} = |z_{vb} - z_k| \ .$$

Finally, we check which is the smallest distance and compute $v_{i,j+\frac{1}{2},k}$, that is, if $d_{xv} < d_{zv}$ we interpolate between $v_{i+1,j+\frac{1}{2},k}$ and v_b which gives

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j+\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b \ . \tag{204}$$

Otherwise, we interpolate in the z-direction which yields

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j+\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b \ . \tag{205}$$

Similarly, in order to calculate $w_{i,j,k+\frac{1}{2}}$ we first compute intersection points x_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0$$

and

$$f(x_i, y_j, z_{wb}) = 0$$

and calculate the distances

$$d_{xw} = |x_{wb} - x_i| \quad \text{and} \quad d_{zw} = |z_{wb} - z_k| \ .$$

If $d_{xw} < d_{zw}$ then interpolating in the x-direction leads to

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k+\frac{1}{2}} - \frac{\delta x}{x_{wb} - x_{i+1}} w_b; \tag{206}$$

otherwise we interpolate in the z-direction which gives

$$w_{i,j,k+\frac{1}{2}} = \frac{z_k - z_{wb}}{z_{k+1} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+1}} w_b \ . \tag{207}$$

b) B–cell with the $(i - \frac{1}{2})$ and $(k + \frac{1}{2})$–faces contiguous with interior cells (see figure 37).

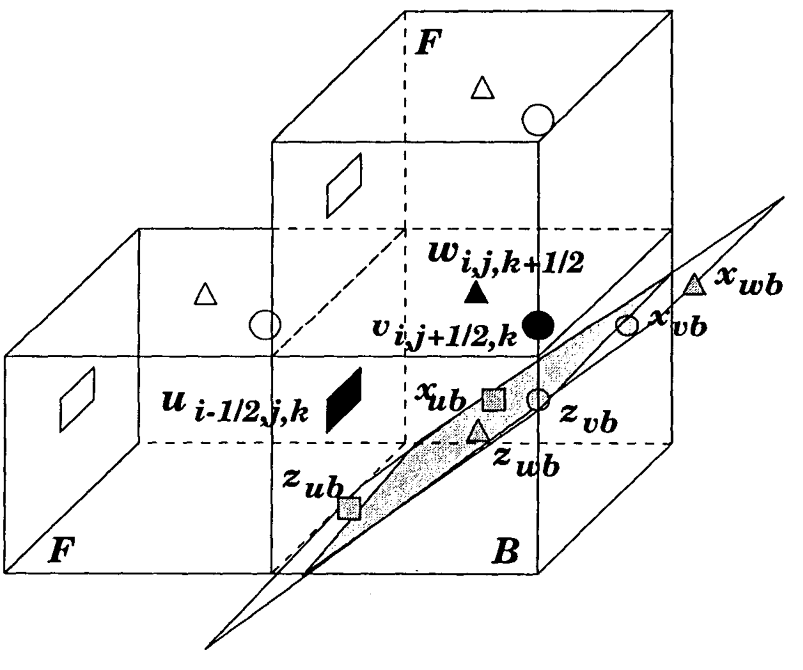

Fig. 37. B–cell with the $(i - \frac{1}{2})$ and $(k + \frac{1}{2})$–faces contiguous with interior cells.

Here we follow the same ideas used in **a)** above. The values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

- Computation of $u_{i-\frac{1}{2},j,k}$
  1. Compute x_{ub} and z_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad f(x_{i-\frac{1}{2}}, y_j, z_{ub}) = 0$$

  2. Calculate the distances d_{xu} and d_{zu}

$$d_{xu} = \left| x_{ub} - x_{i-\frac{1}{2}} \right| \ , \qquad d_{zu} = |z_{ub} - z_k|$$

  3. Compute $u_{i-\frac{1}{2},j,k}$
     If$(d_{xu} < d_{zu})$ Then

$$u_{i-\frac{1}{2},j,k} = \frac{x_{i-\frac{1}{2}} - x_{ub}}{x_{i-\frac{3}{2}} - x_{ub}} u_{i-\frac{3}{2},j,k} + \frac{\delta x}{x_{ub} - x_{i-\frac{3}{2}}} u_b$$

     Else

$$u_{i-\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k+1} - z_{ub}} u_{i-\frac{1}{2},j,k+1} - \frac{\delta z}{z_{ub} - z_{k+1}} u_b$$

- Computation of $v_{i,j+\frac{1}{2},k}$
  1. Compute x_{vb} and z_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0$$

  2. Calculate the distances d_{xv} and d_{zv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{zv} = |z_{vb} - z_k|$$

  3. Compute $v_{i,j+\frac{1}{2},k}$
     If$(d_{xv} < d_{zv})$ Then

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j+\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b$$

     Else

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j+\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b$$

- Computation of $w_{i,j,k+\frac{1}{2}}$
  1. Compute x_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0$$

  2. Calculate the distances d_{xw} and d_{zw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{zw} = \left| z_{wb} - z_{k+\frac{1}{2}} \right|$$

  3. Compute $w_{i,j,k+\frac{1}{2}}$
     If$(d_{xw} < d_{zw})$ Then

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k+\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b$$

     Else

$$w_{i,j,k+\frac{1}{2}} = \frac{z_k - z_{wb}}{z_{k+1} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+1}} w_b$$

c) **B–cell with the $(j+\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with interior cells (see figure 38).**

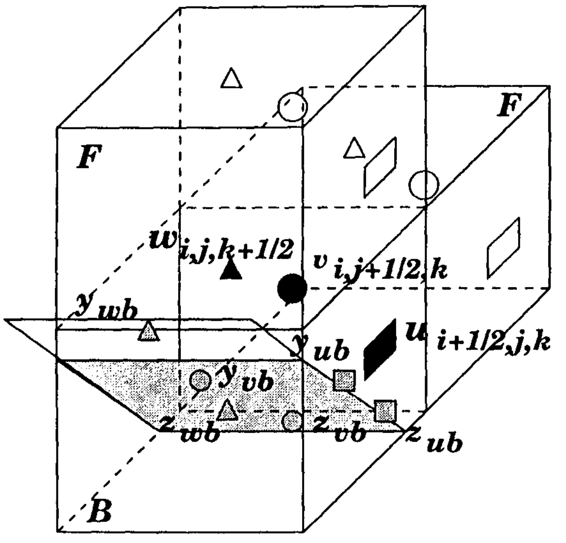

Fig. 38. B–cell with the $(j+\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute y_{ub} and z_{ub} from

$$f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

2. Calculate the distances d_{yu} and d_{zu}

$$d_{yu} = |y_{ub} - y_j| \ , \qquad d_{zu} = |z_{ub} - z_k|$$

3. Compute $u_{i+\frac{1}{2},j,k}$
If$(d_{yu} < d_{zu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j+1} - y_{ub}} u_{i+\frac{1}{2},j+1,k} - \frac{\delta y}{y_{ub} - y_{j+1}} u_b$$

Else

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k+1} - z_{ub}} u_{i+\frac{1}{2},j,k+1} - \frac{\delta z}{z_{ub} - z_{k+1}} u_b$$

– Computation of $v_{i,j+\frac{1}{2},k}$

1. Compute y_{vb} and z_{vb} from

$$f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0$$

2. Calculate the distances d_{yv} and d_{zv}

$$d_{yv} = \left| y_{vb} - y_{j+\frac{1}{2}} \right| \ , \qquad d_{zv} = |z_{vb} - z_k|$$

3. Compute $v_{i,j+\frac{1}{2},k}$
If$(d_{yv} < d_{zv})$ Then

$$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b$$

Else

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j+\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b$$

- Computation of $w_{i,j,k+\frac{1}{2}}$

1. Compute y_{wb} and z_{wb} from

$$f(x_i, y_{wb}, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0$$

2. Calculate the distances d_{yw} and d_{zw}

$$d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left| z_{wb} - z_{k+\frac{1}{2}} \right|$$

3. Compute $w_{i,j,k+\frac{1}{2}}$
If$(d_{yw} < d_{zw})$ Then

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k+\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b$$

Else

$$w_{i,j,k+\frac{1}{2}} = \frac{z_k - z_{wb}}{z_{k+1} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+1}} w_b$$

d) **B–cell with the $(j-\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with interior cells (see figure 39).**

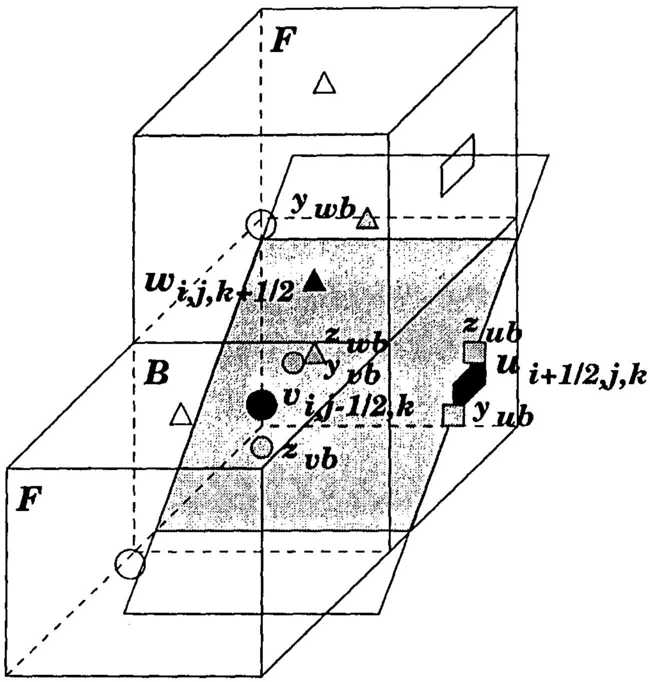

Fig. 39. B–cell with the $(j-\frac{1}{2})$ and $(k+\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute y_{ub} and z_{ub} from

$$f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

2. Calculate the distances d_{yu} and d_{zu}

$$d_{yu} = |y_{ub} - y_j| \ , \qquad d_{zu} = |z_{ub} - z_k|$$

3. Compute $u_{i+\frac{1}{2},j,k}$
   If$(d_{yu} < d_{zu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j-1} - y_{ub}} u_{i+\frac{1}{2},j-1,k} + \frac{\delta y}{y_{ub} - y_{j-1}} u_b$$

   Else

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k+1} - z_{ub}} u_{i+\frac{1}{2},j,k+1} - \frac{\delta z}{z_{ub} - z_{k+1}} u_b$$

- Computation of $v_{i,j-\frac{1}{2},k}$

1. Compute y_{vb} and z_{vb} from

$$f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j-\frac{1}{2}}, z_{vb}) = 0$$

2. Calculate the distances d_{yv} and d_{zv}

$$d_{yv} = \left|y_{vb} - y_{j-\frac{1}{2}}\right| \ , \qquad d_{zv} = |z_{vb} - z_k|$$

3. Compute $v_{i,j-\frac{1}{2},k}$
   If$(d_{yv} < d_{zv})$ Then

$$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b$$

   Else

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j+\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b$$

- Computation of $w_{i,j,k+\frac{1}{2}}$

1. Compute y_{wb} and z_{wb} from

$$f(x_i, y_{wb}, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0$$

2. Calculate the distances d_{yw} and d_{zw}

$$d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left|z_{wb} - z_{k+\frac{1}{2}}\right|$$

3. Compute $w_{i,j,k+\frac{1}{2}}$
   If$(d_{yw} < d_{zw})$ Then

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k+\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b$$

   Else

$$w_{i,j,k+\frac{1}{2}} = \frac{z_k - z_{wb}}{z_{k+1} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+1}} w_b$$

e) **B–cell with the $(i+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells (see figure 40).**

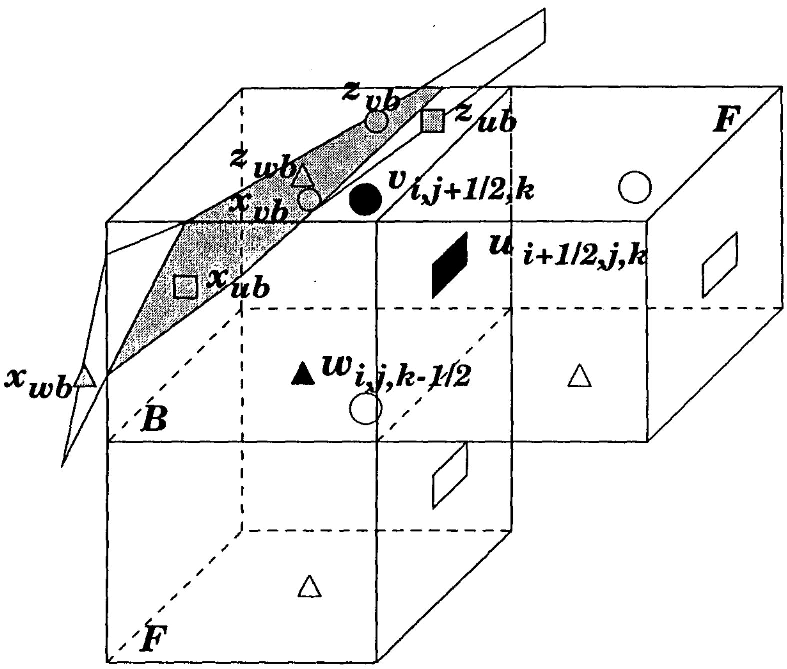

Fig. 40. B–cell with the $(i+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute x_{ub} and z_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad\qquad f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

2. Calculate the distances d_{xu} and d_{zu}

$$d_{xu} = \left| x_{ub} - x_{i+\frac{1}{2}} \right| \ , \qquad\qquad d_{zu} = |z_{ub} - z_k|$$

3. Compute $u_{i+\frac{1}{2},j,k}$
If$(d_{xu} < d_{zu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{x_{i=\frac{1}{2}} - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k} - \frac{\delta x}{x_{ub} - x_{i+\frac{3}{2}}} u_b$$

Else

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k-1} - z_{ub}} u_{i+\frac{1}{2},j,k-1} + \frac{\delta z}{z_{ub} - z_{k-1}} u_b$$

- Computation of $v_{i,j+\frac{1}{2},k}$

1. Compute x_{vb} and z_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0$$

2. Calculate the distances d_{xv} and d_{zv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{zv} = |z_{vb} - z_k|$$

3. Compute $v_{i,j+\frac{1}{2},k}$
If$(d_{xv} < d_{zv})$ Then

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j+\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b$$

Else

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j+\frac{1}{2},k-1} + \frac{\delta z}{z_{vb} - z_{k-\frac{3}{2}}} v_b$$

- Computation of $w_{i,j,k-\frac{1}{2}}$

1. Compute x_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0$$

2. Calculate the distances d_{xw} and d_{zw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{zw} = \left|z_{wb} - z_{k-\frac{1}{2}}\right|$$

3. Compute $w_{i,j,k-\frac{1}{2}}$
If$(d_{xw} < d_{zw})$ Then

$$w_{i,j,k-\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k-\frac{1}{2}} - \frac{\delta x}{x_{wb} - x_{i+1}} w_b$$

Else

$$w_{i,j,k-\frac{1}{2}} = \frac{z_{k-\frac{1}{2}} - z_{wb}}{z_{k-\frac{3}{2}} - z_{wb}} w_{i,j,k-\frac{3}{2}} + \frac{\delta z}{z_{wb} - z_{k-\frac{3}{2}}} w_b$$

f) **B–cell with the $(i-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells (see figure 41).**

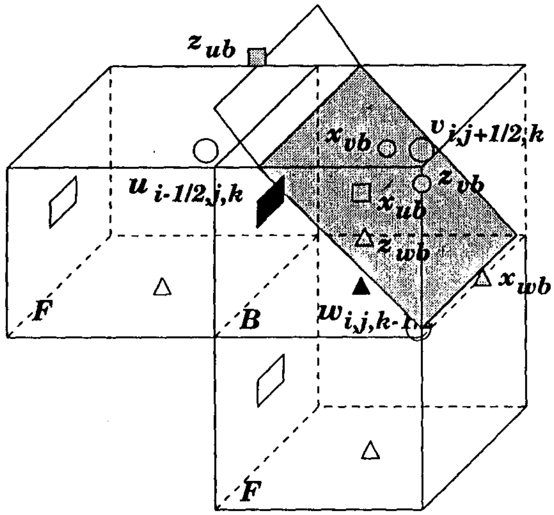

Fig. 41. B–cell with the $(i-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as follows:

- Computation of $u_{i-\frac{1}{2},j,k}$
  1. Compute x_{ub} and z_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad f(x_{i-\frac{1}{2}}, y_j, z_{ub}) = 0$$

  2. Calculate the distances d_{xu} and d_{zu}

$$d_{xu} = \left| x_{ub} - x_{i-\frac{1}{2}} \right| \ , \qquad d_{zu} = |z_{ub} - z_k|$$

  3. Compute $u_{i-\frac{1}{2},j,k}$
     If$(d_{xu} < d_{zu})$ Then

$$u_{i-\frac{1}{2},j,k} = \frac{x_{i-\frac{1}{2}} - x_{ub}}{x_{i-\frac{3}{2}} - x_{ub}} u_{i-\frac{3}{2},j,k} + \frac{\delta x}{x_{ub} - x_{i-\frac{3}{2}}} u_b$$

     Else

$$u_{i-\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k-1} - z_{ub}} u_{i-\frac{1}{2},j,k-1} + \frac{\delta z}{z_{ub} - z_{k-1}} u_b$$

- Computation of $v_{i,j+\frac{1}{2},k}$
  1. Compute x_{vb} and z_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0$$

2. Calculate the distances d_{xv} and d_{zv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{zv} = |z_{vb} - z_k|$$

3. Compute $v_{i,j+\frac{1}{2},k}$
If$(d_{xv} < d_{zv})$ Then

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j+\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b$$

Else

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j+\frac{1}{2},k-1} + \frac{\delta z}{z_{vb} - z_{k-\frac{3}{2}}} v_b$$

– Computation of $w_{i,j,k-\frac{1}{2}}$

1. Compute x_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0$$

2. Calculate the distances d_{xw} and d_{zw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{zw} = \left|z_{wb} - z_{k-\frac{1}{2}}\right|$$

3. Compute $w_{i,j,k-\frac{1}{2}}$
If$(d_{xw} < d_{zw})$ Then

$$w_{i,j,k-\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k-\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b$$

Else

$$w_{i,j,k-\frac{1}{2}} = \frac{z_{k-\frac{1}{2}} - z_{wb}}{z_{k-\frac{3}{2}} - z_{wb}} w_{i,j,k-\frac{3}{2}} + \frac{\delta z}{z_{wb} - z_{k-\frac{3}{2}}} w_b$$

g) **B–cell with the $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells (see figure 42).**

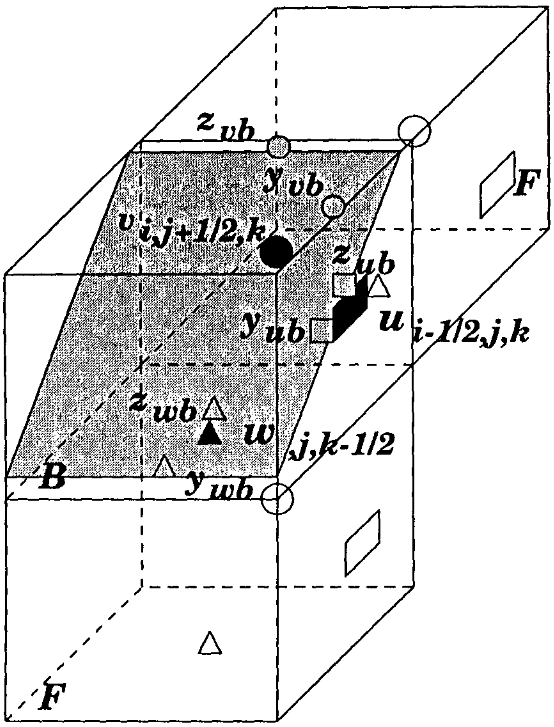

Fig. 42. B–cell with the $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute y_{ub} and z_{ub} from

$$f(x_{i+\frac{1}{2}},y_{ub},z_k)=0\ , \qquad f(x_{i+\frac{1}{2}},y_j,z_{ub})=0$$

2. Calculate the distances d_{yu} and d_{zu}

$$d_{yu}=|y_{ub}-y_j|\ , \qquad d_{zu}=|z_{ub}-z_k|$$

3. Compute $u_{i+\frac{1}{2},j,k}$
If$(d_{yu}<d_{zu})$ Then

$$u_{i+\frac{1}{2},j,k}=\frac{y_j-y_{ub}}{y_{j+1}-y_{ub}}u_{i+\frac{1}{2},j+1,k}-\frac{\delta y}{y_{ub}-y_{j+1}}u_b$$

Else

$$u_{i+\frac{1}{2},j,k}=\frac{z_k-z_{ub}}{z_{k-1}-z_{ub}}u_{i+\frac{1}{2},j,k-1}+\frac{\delta z}{z_{ub}-z_{k-1}}u_b$$

- Computation of $v_{i,j+\frac{1}{2},k}$

1. Compute y_{vb} and z_{vb} from

$$f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0$$

2. Calculate the distances d_{yv} and d_{zv}

$$d_{yv} = \left| y_{vb} - y_{j+\frac{1}{2}} \right| \ , \qquad d_{zv} = |z_{vb} - z_k|$$

3. Compute $v_{i,j+\frac{1}{2},k}$
If$(d_{yv} < d_{zv})$ Then

$$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b$$

Else

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j+\frac{1}{2},k-1} + \frac{\delta z}{z_{vb} - z_{k-1}} v_b$$

- Computation of $w_{i,j,k-\frac{1}{2}}$

1. Compute y_{wb} and z_{wb} from

$$f(x_i, y_{wb}, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0$$

2. Calculate the distances d_{yw} and d_{zw}

$$d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left| z_{wb} - z_{k-\frac{1}{2}} \right|$$

3. Compute $w_{i,j,k-\frac{1}{2}}$
If$(d_{yw} < d_{zw})$ Then

$$w_{i,j,k-\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k-\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b$$

Else

$$w_{i,j,k-\frac{1}{2}} = \frac{z_k - z_{wb}}{z_{k-1} - z_{wb}} w_{i,j,k-\frac{3}{2}} + \frac{\delta z}{z_{wb} - z_{k-1}} w_b$$

h) **B–cell with the $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells (see figure 43).**

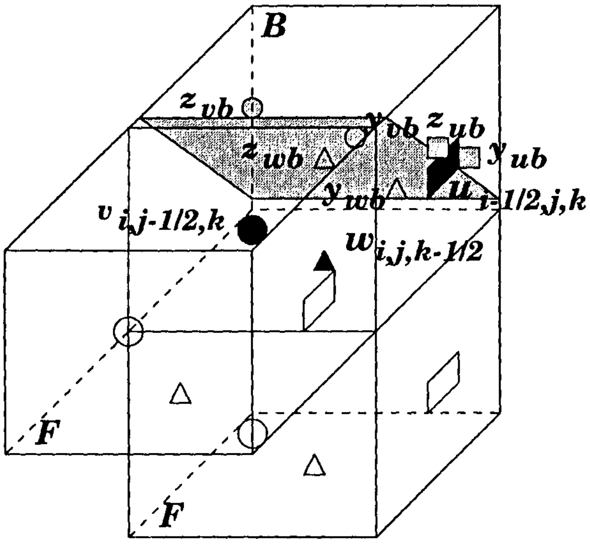

Fig. 43. B–cell with the $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute y_{ub} and z_{ub} from

$$f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

2. Calculate the distances d_{yu} and d_{zu}

$$d_{yu} = |y_{ub} - y_j| \ , \qquad d_{zu} = |z_{ub} - z_k|$$

3. Compute $u_{i+\frac{1}{2},j,k}$
If$(d_{yu} < d_{zu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j-1} - y_{ub}} u_{i+\frac{1}{2},j-1,k} + \frac{\delta y}{y_{ub} - y_{j-1}} u_b$$

Else

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k-1} - z_{ub}} u_{i+\frac{1}{2},j,k-1} + \frac{\delta z}{z_{ub} - z_{k-1}} u_b$$

– Computation of $v_{i,j-\frac{1}{2},k}$

1. Compute y_{vb} and z_{vb} from

$$f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j-\frac{1}{2}}, z_{vb}) = 0$$

2. Calculate the distances d_{yv} and d_{zv}

$$d_{yv} = \left| y_{vb} - y_{j-\frac{1}{2}} \right| \ , \qquad d_{zv} = |z_{vb} - z_k|$$

3. Compute $v_{i,j-\frac{1}{2},k}$
If($d_{yv} < d_{zv}$) Then

$$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} + \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b$$

Else

$$v_{i,j-\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j-\frac{1}{2},k-1} - \frac{\delta z}{z_{vb} - z_{k-1}} v_b$$

– Computation of $w_{i,j,k-\frac{1}{2}}$

1. Compute y_{wb} and z_{wb} from

$$f(x_i, y_{wb}, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0$$

2. Calculate the distances d_{yw} and d_{zw}

$$d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left| z_{wb} - z_{k-\frac{1}{2}} \right|$$

3. Compute $w_{i,j,k-\frac{1}{2}}$
If($d_{yw} < d_{zw}$) Then

$$w_{i,j,k-\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j-1} - y_{wb}} w_{i,j-1,k-\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j-1}} w_b$$

Else

$$w_{i,j,k-\frac{1}{2}} = \frac{z_k - z_{wb}}{z_{k-1} - z_{wb}} w_{i,j,k-\frac{3}{2}} + \frac{\delta z}{z_{wb} - z_{k-1}} w_b$$

i) **B–cell with the $(i+\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with interior cells (see figure 44).**

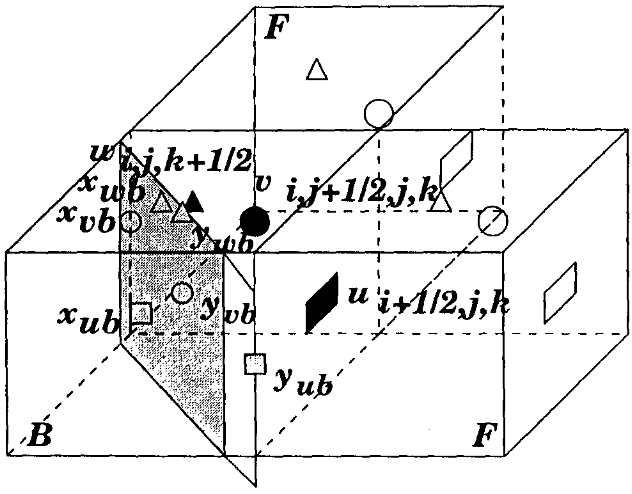

Fig. 44. B–cell with the $(i+\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute x_{ub} and y_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0$$

2. Calculate the distances d_{xu} and d_{yu}

$$d_{xu} = \left| x_{ub} - x_{i+\frac{1}{2}} \right| \ , \qquad d_{yu} = |y_{ub} - y_j|$$

3. Compute $u_{i+\frac{1}{2},j,k}$
If$(d_{xu} < d_{yu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{x_{i+\frac{1}{2}} - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k} - \frac{\delta x}{x_{ub} - x_{i+\frac{3}{2}}} u_b$$

Else

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j+1} - y_{ub}} u_{i+\frac{1}{2},j+1,k} - \frac{\delta y}{y_{ub} - y_{j+1}} u_b$$

– Computation of $v_{i,j+\frac{1}{2},k}$

1. Compute x_{vb} and y_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_{vb}) = 0$$

2. Calculate the distances d_{xv} and d_{yv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j+\frac{1}{2}}\right|$$

3. Compute $v_{i,j+\frac{1}{2},k}$
If$(d_{xv} < d_{yv})$ Then

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j+\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b$$

Else

$$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b$$

– Computation of $w_{i,j,k+\frac{1}{2}}$

1. Compute x_{wb} and y_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_k) = 0$$

2. Calculate the distances d_{xw} and d_{yw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j|$$

3. Compute $w_{i,j,k+\frac{1}{2}}$
If$(d_{xw} < d_{yw})$ Then

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k+\frac{1}{2}} - \frac{\delta x}{x_{wb} - x_{i+1}} w_b$$

Else

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k+\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b$$

j) **B–cell with the $(i-\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with interior cells (see figure 45).**

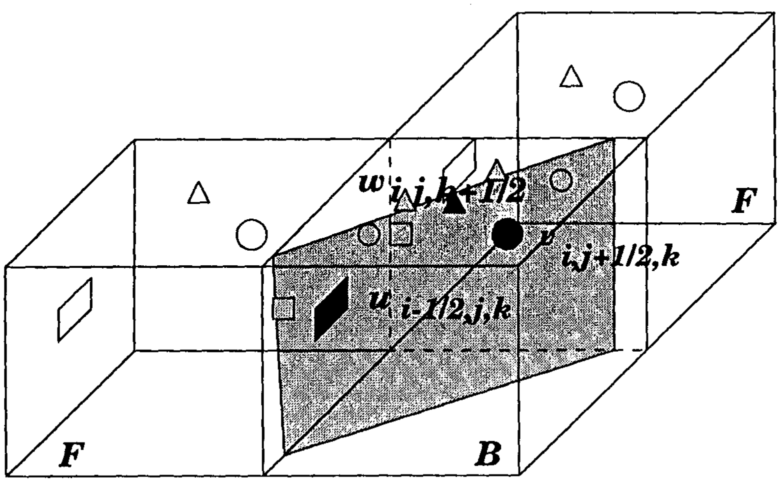

Fig. 45. B–cell with the $(i-\frac{1}{2})$ and $(j+\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

- Computation of $u_{i-\frac{1}{2},j,k}$
  1. Compute x_{ub} and y_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad f(x_{i-\frac{1}{2}}, y_{ub}, z_k) = 0$$

  2. Calculate the distances d_{xu} and d_{yu}

$$d_{xu} = \left| x_{ub} - x_{i-\frac{1}{2}} \right| \ , \qquad d_{yu} = |y_{ub} - y_j|$$

  3. Compute $u_{i-\frac{1}{2},j,k}$
     If$(d_{xu} < d_{yu})$ Then

$$u_{i-\frac{1}{2},j,k} = \frac{x_{i-\frac{1}{2}} - x_{ub}}{x_{i-\frac{3}{2}} - x_{ub}} u_{i-\frac{3}{2},j,k} + \frac{\delta x}{x_{ub} - x_{i-\frac{3}{2}}} u_b$$

     Else

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j+1} - y_{ub}} u_{i+\frac{1}{2},j+1,k} - \frac{\delta y}{y_{ub} - y_{j+1}} u_b$$

- Computation of $v_{i,j+\frac{1}{2},k}$
  1. Compute x_{vb} and y_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_{vb}) = 0$$

2. Calculate the distances d_{xv} and d_{yv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j+\frac{1}{2}}\right|$$

3. Compute $v_{i,j+\frac{1}{2},k}$
If($d_{xv} < d_{yv}$) Then

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j+\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b$$

Else

$$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b$$

– Computation of $w_{i,j,k+\frac{1}{2}}$

1. Compute x_{wb} and y_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_k) = 0$$

2. Calculate the distances d_{xw} and d_{yw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j|$$

3. Compute $w_{i,j,k+\frac{1}{2}}$
If($d_{xw} < d_{yw}$) Then

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k+\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b$$

Else

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k+\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b$$

k) **B–cell with the $(i+\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with interior cells (see figure 46).**

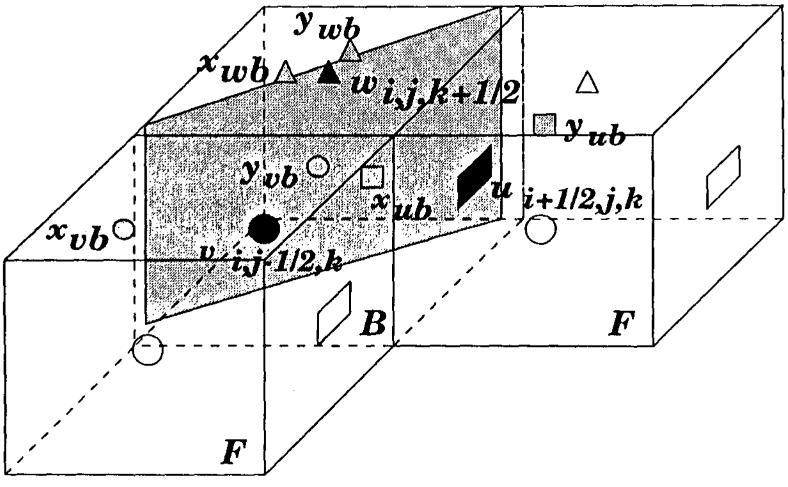

Fig. 46. B–cell with the $(i+\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute x_{ub} and y_{ub} from

$$f(x_{ub},y_j,z_k)=0\ , \qquad f(x_{i+\frac{1}{2}},y_{ub},z_k)=0$$

2. Calculate the distances d_{xu} and d_{yu}

$$d_{xu}=\left|x_{ub}-x_{i+\frac{1}{2}}\right|\ , \qquad d_{yu}=|y_{ub}-y_j|$$

3. Compute $u_{i+\frac{1}{2},j,k}$
If$(d_{xu}<d_{yu})$ Then

$$u_{i+\frac{1}{2},j,k}=\frac{x_{i+\frac{1}{2}}-x_{ub}}{x_{i+\frac{3}{2}}-x_{ub}}u_{i+\frac{3}{2},j,k}-\frac{\delta x}{x_{ub}-x_{i+\frac{3}{2}}}u_b$$

Else

$$u_{i+\frac{1}{2},j,k}=\frac{y_j-y_{ub}}{y_{j-1}-y_{ub}}u_{i+\frac{1}{2},j-1,k}+\frac{\delta y}{y_{ub}-y_{j-1}}u_b$$

– Computation of $v_{i,j-\frac{1}{2},k}$

1. Compute x_{vb} and y_{vb} from

$$f(x_{vb},y_{j-\frac{1}{2}},z_k)=0\ , \qquad f(x_i,y_{vb},z_{vb})=0$$

2. Calculate the distances d_{xv} and d_{yv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j-\frac{1}{2}}\right|$$

3. Compute $v_{i,j-\frac{1}{2},k}$
If($d_{xv} < d_{yv}$) Then

$$v_{i,j-\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j-\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b$$

Else

$$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} + \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b$$

- Computation of $w_{i,j,k+\frac{1}{2}}$
  1. Compute x_{wb} and y_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_k) = 0$$

  2. Calculate the distances d_{xw} and d_{yw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j|$$

  3. Compute $w_{i,j,k+\frac{1}{2}}$
  If($d_{xw} < d_{yw}$) Then

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k+\frac{1}{2}} - \frac{\delta x}{x_{wb} - x_{i+1}} w_b$$

  Else

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j-1} - y_{wb}} w_{i,j-1,k+\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j-1}} w_b$$

l) **B–cell with the $(i-\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with interior cells (see figure 47).**

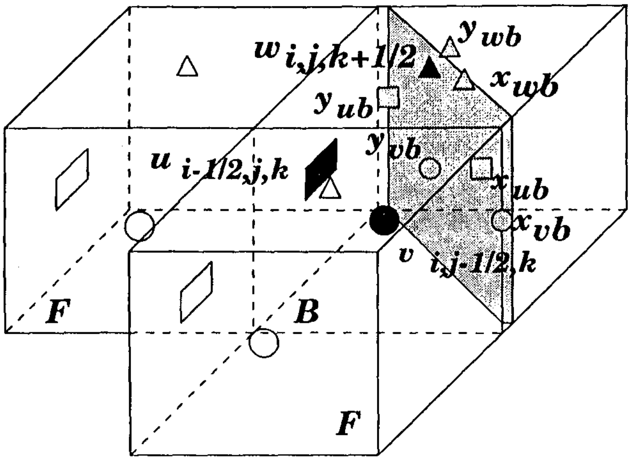

Fig. 47. B–cell with the $(i-\frac{1}{2})$ and $(j-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i-\frac{1}{2},j,k}$

1. Compute x_{ub} and y_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad f(x_{i-\frac{1}{2}}, y_{ub}, z_k) = 0$$

2. Calculate the distances d_{xu} and d_{yu}

$$d_{xu} = \left| x_{ub} - x_{i-\frac{1}{2}} \right| \ , \qquad d_{yu} = |y_{ub} - y_j|$$

3. Compute $u_{i-\frac{1}{2},j,k}$
If$(d_{xu} < d_{yu})$ Then

$$u_{i-\frac{1}{2},j,k} = \frac{x_{i-\frac{1}{2}} - x_{ub}}{x_{i-\frac{3}{2}} - x_{ub}} u_{i-\frac{3}{2},j,k} + \frac{\delta x}{x_{ub} - x_{i-\frac{3}{2}}} u_b$$

Else

$$u_{i-\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j-1} - y_{ub}} u_{i-\frac{1}{2},j-1,k} + \frac{\delta y}{y_{ub} - y_{j-1}} u_b$$

– Computation of $v_{i,j-\frac{1}{2},k}$

1. Compute x_{vb} and y_{vb} from

$$f(x_{vb}, y_{j-\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_{vb}) = 0$$

2. Calculate the distances d_{xv} and d_{yv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j-\frac{1}{2}}\right|$$

3. Compute $v_{i,j-\frac{1}{2},k}$
If$(d_{xv} < d_{yv})$ Then

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j+\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b$$

Else

$$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} + \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b$$

– Computation of $w_{i,j,k+\frac{1}{2}}$

1. Compute x_{wb} and y_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_k) = 0$$

2. Calculate the distances d_{xw} and d_{yw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j|$$

3. Compute $w_{i,j,k+\frac{1}{2}}$
If$(d_{xw} < d_{yw})$ Then

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k+\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b$$

Else

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j-1} - y_{wb}} w_{i,j-1,k+\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j-1}} w_b$$

6.2.3. B-cells having three adjacent faces contiguous with interior cells.

Here we have 8 different configurations of B-cells having three adjacent faces contiguous with interior cells. Each case will be treated in a similar manner as that employed for B-cells with two adjacent faces contiguous with interior cells as follows:

a) B-cell with the $(i+\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k+\frac{1}{2})$-faces contiguous with interior cells (see figure 48).

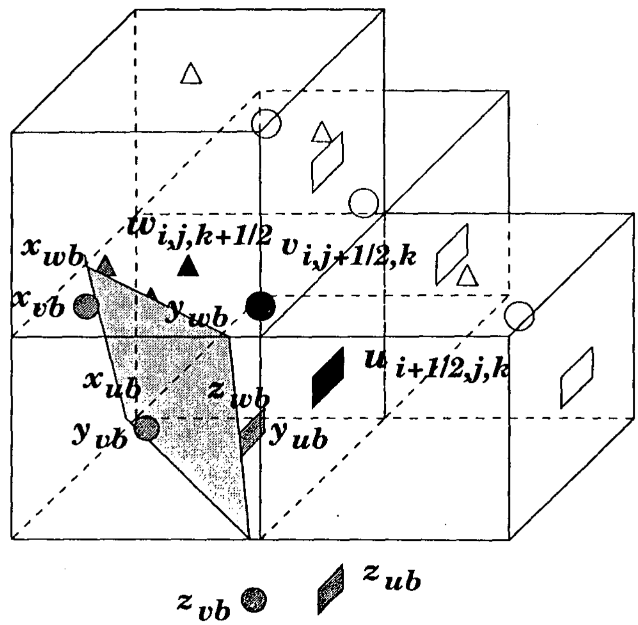

Fig. 48. B-cell with the $(i+\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k+\frac{1}{2})$-faces contiguous with interior cells.

As we can see in figure 48, an approximation for $u_{i+\frac{1}{2},j,k}$ may be obtained by employing linear interpolation in the x-direction using $u_{i+\frac{3}{2},j,k}$ and u_b or interpolate in the y-direction by using $u_{i+\frac{1}{2},j+1,k}$ and u_b or interpolate in the z-direction using $u_{i+\frac{1}{2},j,k+1}$ and u_b. To select which direction is most acceptable to perform the interpolation we adopt the same ideas used in Section **6.2.2**.

Let us consider figure 48 for the calculation of $u_{i+\frac{1}{2},j,k}$. Let $\mathbf{P_0} = (x_{i+\frac{1}{2}}, y_j, z_k)$, $\mathbf{P_{1x}} = (x_{i+\frac{3}{2}}, y_j, z_k)$, $\mathbf{P_{bx}} = (x_{ub}, y_j, z_k)$, $\mathbf{P_{1y}} = (x_{i+\frac{1}{2}}, y_{j+1}, z_k)$ and $\mathbf{P_{by}} = (x_{i+\frac{3}{2}}, y_{ub}, z_k)$, $\mathbf{P_{1z}} = (x_{i+\frac{1}{2}}, y_j, z_{k+1})$ and $\mathbf{P_{bz}} = (x_{i+\frac{1}{2}}, y_j, z_{ub})$, where x_{ub} is the intersection point between the line defined by $\mathbf{P_0}$ and $\mathbf{P_{1x}}$ and the boundary surface; y_{ub} is the intersection point of the line defined by $\mathbf{P_0}$ and $\mathbf{P_{1y}}$ and the boundary surface and z_{ub} is the intersection point of the line defined by $\mathbf{P_0}$ and $\mathbf{P_{1z}}$ and the boundary surface. The values of x_{ub}, y_{ub} and z_{ub} can be computed from

$$f(x_{ub}, y_j, z_k) = 0$$

and

$$f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

$$f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0$$

respectively. Once x_{ub}, y_{ub} and z_{ub} have been obtained we can calculate the distances

$$d_{xu} = \left|x_{ub} - x_{i+\frac{1}{2}}\right| \ , \quad d_{yu} = |y_{ub} - y_j| \ , \qquad d_{zu} = |z_{ub} - z_k| \ .$$

To choose the direction for interpolation we take the closest point to $\mathbf{P_0}$, namely, let

$$d_{min} = min\,(d_{xu}, d_{yu}, d_{zu}) \ .$$

Thus, if $d_{min} = d_{xu}$ then we interpolate between $\mathbf{P_{bx}}$ and $\mathbf{P_{1x}}$. In this case, $u_{i+\frac{1}{2},j,k}$ is given by

$$u_{i+\frac{1}{2},j,k} = \frac{x_{i+\frac{1}{2}} - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k} - \frac{\delta x}{x_{ub} - x_{i+\frac{3}{2}}} u_b \ .$$

On the other hand, if $d_{min} = d_{yu}$ then we interpolate between $\mathbf{P_{by}}$ and $\mathbf{P_{1y}}$ which gives

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j+1} - y_{ub}} u_{i+\frac{1}{2},j+1,k} - \frac{\delta y}{y_{ub} - y_{j+1}} u_b \ .$$

Otherwise, $d_{min} = d_{zu}$ in which case we interpolate between $\mathbf{P_{bz}}$ and $\mathbf{P_{1z}}$, giving

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k+1} - z_{ub}} u_{i+\frac{1}{2},j,k+1} - \frac{\delta z}{z_{ub} - z_{k+1}} u_b \ .$$

The other two velocities, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$, are obtained similarly. For instance, to obtain an approximation for $v_{i,j+\frac{1}{2},k}$ we compute the intersection points x_{vb}, y_{vb} and z_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ ,$$

$$f(x_i, y_{vb}, z_k) = 0 \ ,$$

$$f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0$$

respectively and calculate the distances

$$d_{xv} = |x_{vb} - x_i| \ , \quad d_{yv} = |y_{vb} - y_j| \ , \quad d_{zv} = |z_{vb} - z_k| \ .$$

Finally, we check which is the smallest distance and compute $v_{i,j+\frac{1}{2},k}$, namely, if $d_{min} = d_{xv}$ we interpolate between $v_{i+1,j+\frac{1}{2},k}$ and v_b which gives

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j+\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b \ .$$

Otherwise, if $d_{min} = d_{yv}$ we interpolate in the y-direction which yields

$$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b \ .$$

Otherwise, we interpolate in the z-direction yielding

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j+\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b \ .$$

Similarly, for calculating $w_{i,j,k+\frac{1}{2}}$ we first compute intersection points x_{wb}, y_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0$$

$$f(x_i, y_{wb}, z_{k+\frac{1}{2}}) = 0$$

$$f(x_i, y_j, z_{wb}) = 0$$

and compute the distances

$$d_{xw} = |x_{wb} - x_i| \ , \quad d_{yw} = |y_{wb} - y_j| \ , \quad d_{zw} = |z_{wb} - z_k|$$

and calculate $d_{min} = min\,(d_{xu}, d_{yu}, d_{zu})$. If $d_{min} = d_{xw}$ then interpolating in the x-direction leads to

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k+\frac{1}{2}} - \frac{\delta x}{x_{wb} - x_{i+1}} w_b \ .$$

If $d_{min} = d_{yw}$ then interpolating in the y-direction gives

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k+\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b \ ,$$

otherwise we interpolate in the z-direction which yields

$$w_{i,j,k+\frac{1}{2}} = \frac{z_{k+\frac{1}{2}} - z_{wb}}{z_{k+\frac{3}{2}} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+\frac{3}{2}}} w_b$$

b) **B–cell with the $(i-\frac{1}{2})$, $(j+\frac{1}{2})$ $(k+\frac{1}{2})$-faces contiguous with interior cells (see figure 49).**

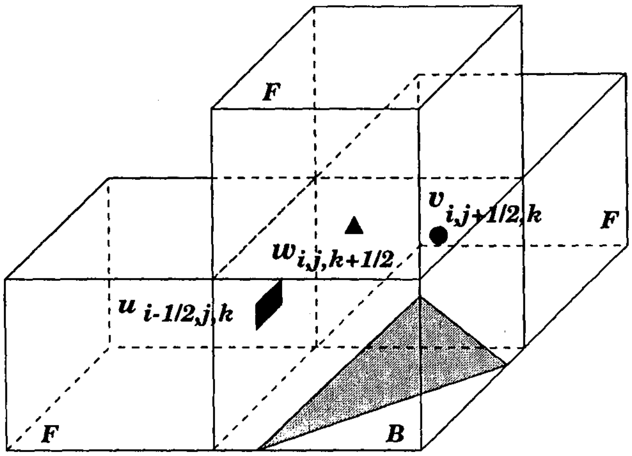

Fig. 49. B–cell with the $(i-\frac{1}{2})$, $(j+\frac{1}{2})$ $(k+\frac{1}{2})$–faces contiguous with interior cells.

Here we follow the same ideas used in **a)** above. The values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i-\frac{1}{2},j,k}$

1. Compute x_{ub}, y_{ub} and z_{ub} from

$$f(x_{ub},y_j,z_k)=0\ ,\qquad f(x_{i-\frac{1}{2}},y_{ub},z_k)=0\ ,\qquad f(x_{i-\frac{1}{2}},y_j,z_{ub})=0$$

2. Calculate the distances d_{xu}, d_{yu} and d_{zu}

$$d_{xu}=\left|x_{ub}-x_{i-\frac{1}{2}}\right|\ ,\qquad d_{yu}=|y_{ub}-y_j|\ ,\qquad d_{zu}=|z_{ub}-z_k|$$

3. Compute d_{min}

$$d_{min}=min\,(d_{xu},d_{yu},d_{zu})$$

4. Compute $u_{i-\frac{1}{2},j,k}$
If$(d_{min}=d_{xu})$ Then

$$u_{i-\frac{1}{2},j,k}=\frac{x_{i-\frac{1}{2}}-x_{ub}}{x_{i-\frac{3}{2}}-x_{ub}}u_{i-\frac{3}{2},j,k}+\frac{\delta x}{x_{ub}-x_{i-\frac{3}{2}}}u_b\ .$$

If$(d_{min}=d_{yu})$ Then

$$u_{i-\frac{1}{2},j,k}=\frac{y_j-y_{ub}}{y_{j+1}-y_{ub}}u_{i-\frac{1}{2},j+1,k}-\frac{\delta y}{y_{ub}-y_{j+1}}u_b\ .$$

If$(d_{min}=d_{zu})$ Then

$$u_{i-\frac{1}{2},j,k}=\frac{z_k-z_{ub}}{z_{k+1}-z_{ub}}u_{i-\frac{1}{2},j,k+1}-\frac{\delta z}{z_{ub}-z_{k+1}}u_b\ .$$

- Computation of $v_{i,j+\frac{1}{2},k}$
  1. Compute x_{vb}, y_{vb} and z_{vb} from
  $$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0 \ .$$
  2. Calculate the distances d_{xv}, d_{yv} and d_{zv}
  $$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j+\frac{1}{2}}\right| \ , \qquad d_{zv} = |z_{vb} - z_k| \ .$$
  3. Compute d_{min}
  $$d_{min} = min\,(d_{xv}, d_{yv}, d_{zv})$$
  4. Compute $v_{i,j+\frac{1}{2},k}$
  If$(d_{min} = d_{xv})$ Then
  $$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j+\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b \ .$$
  If$(d_{min} = d_{yv})$ Then
  $$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b \ .$$
  If$(d_{min} = d_{zv})$ Then
  $$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j+\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b \ .$$
- Computation of $w_{i,j,k+\frac{1}{2}}$
  1. Compute x_{wb}, y_{wb} and z_{wb} from
  $$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0 \ .$$
  2. Calculate the distances d_{xw}, d_{yw} and d_{zw}
  $$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left|z_{wb} - z_{k+\frac{1}{2}}\right|$$
  3. Compute d_{min}
  $$d_{min} = min\,(d_{xw}, d_{yw}, d_{zw})$$
  4. Compute $w_{i,j,k+\frac{1}{2}}$
  If$(d_{min} = d_{xw})$ Then
  $$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k+\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b \ .$$
  If$(d_{min} = d_{yw})$ Then
  $$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k+\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b \ .$$
  If$(d_{min} = d_{zw})$ Then
  $$w_{i,j,k+\frac{1}{2}} = \frac{z_{k+\frac{1}{2}} - z_{wb}}{z_{k+\frac{3}{2}} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+\frac{3}{2}}} w_b \ .$$

c) **B–cell with the $(i+\frac{1}{2})$, $(j-\frac{1}{2})$ $(k+\frac{1}{2})$–faces contiguous with interior cells (see figure 50).**

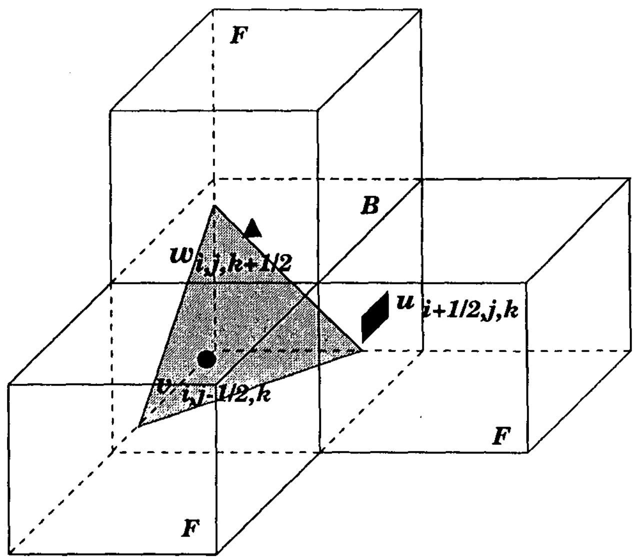

Fig. 50. B–cell with the $(i+\frac{1}{2})$, $(j-\frac{1}{2})$ $(k+\frac{1}{2})$—faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

- Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute x_{ub}, y_{ub} and z_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

2. Calculate the distances d_{xu}, d_{yu} and d_{zu}

$$d_{xu} = \left|x_{ub} - x_{i+\frac{1}{2}}\right| \ , \qquad d_{yu} = |y_{ub} - y_j| \ , \qquad d_{zu} = |z_{ub} - z_k|$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xu}, d_{yu}, d_{zu})$$

4. Compute $u_{i+\frac{1}{2},j,k}$
If($d_{min} = d_{xu}$) Then

$$u_{i+\frac{1}{2},j,k} = \frac{x_{i+\frac{1}{2}} - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k} - \frac{\delta x}{x_{ub} - x_{i+\frac{3}{2}}} u_b \ .$$

If($d_{min} = d_{yu}$) Then

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j-1} - y_{ub}} u_{i+\frac{1}{2},j-1,k} + \frac{\delta y}{y_{ub} - y_{j-1}} u_b \ .$$

If$(d_{min} = d_{zu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k+1} - z_{ub}} u_{i+\frac{1}{2},j,k+1} - \frac{\delta z}{z_{ub} - z_{k+1}} u_b \ .$$

- Computation of $v_{i,j-\frac{1}{2},k}$

  1. Compute x_{vb}, y_{vb} and z_{vb} from

$$f(x_{vb}, y_{j-\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j-\frac{1}{2}}, z_{vb}) = 0 \ .$$

  2. Calculate the distances d_{xv}, d_{yv} and d_{zv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j-\frac{1}{2}}\right| \ , \qquad d_{zv} = |z_{vb} - z_k| \ .$$

  3. Compute d_{min}

$$d_{min} = min\,(d_{xv}, d_{yv}, d_{zv})$$

  4. Compute $v_{i,j-\frac{1}{2},k}$
     If$(d_{min} = d_{xv})$ Then

$$v_{i,j-\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j-\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b \ .$$

     If$(d_{min} = d_{yv})$ Then

$$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} + \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b \ .$$

     If$(d_{min} = d_{zv})$ Then

$$v_{i,j-\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j-\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b \ .$$

- Computation of $w_{i,j,k+\frac{1}{2}}$

  1. Compute x_{wb}, y_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0 \ .$$

  2. Calculate the distances d_{xw}, d_{yw} and d_{zw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left|z_{wb} - z_{k+\frac{1}{2}}\right|$$

  3. Compute d_{min}

$$d_{min} = min\,(d_{xw}, d_{yw}, d_{zw})$$

4. Compute $w_{i,j,k+\frac{1}{2}}$
If($d_{min} = d_{xw}$) Then

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k+\frac{1}{2}} - \frac{\delta x}{x_{wb} - x_{i+1}} w_b \ .$$

If($d_{min} = d_{yw}$) Then

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j-1} - y_{wb}} w_{i,j-1,k+\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j-1}} w_b \ .$$

If($d_{min} = d_{zw}$) Then

$$w_{i,j,k+\frac{1}{2}} = \frac{z_{k+\frac{1}{2}} - z_{wb}}{z_{k+\frac{3}{2}} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+\frac{3}{2}}} w_b \ .$$

d) B–cell with the $(i-\frac{1}{2})$, $(j-\frac{1}{2})$ $(k+\frac{1}{2})$–faces contiguous with interior cells (see figure 51).

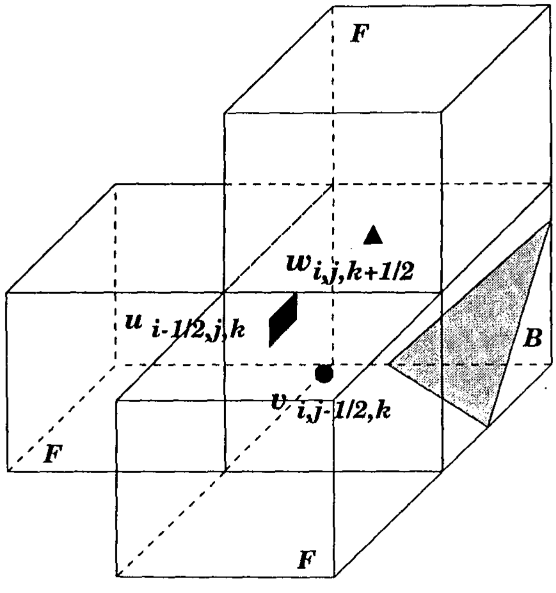

Fig. 51. B–cell with the $(i-\frac{1}{2})$, $(j-\frac{1}{2})$ $(k+\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k+\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i-\frac{1}{2},j,k}$

1. Compute x_{ub}, y_{ub} and z_{ub} from

$$f(x_{ub},y_j,z_k)=0\ ,\qquad f(x_{i-\frac{1}{2}},y_{ub},z_k)=0\ ,\qquad f(x_{i-\frac{1}{2}},y_j,z_{ub})=0$$

2. Calculate the distances d_{xu}, d_{yu} and d_{zu}

$$d_{xu}=\left|x_{ub}-x_{i-\frac{1}{2}}\right|\ ,\qquad d_{yu}=|y_{ub}-y_j|\ ,\qquad d_{zu}=|z_{ub}-z_k|$$

3. Compute d_{min}

$$d_{min}=min\,(d_{xu},d_{yu},d_{zu})$$

4. Compute $u_{i-\frac{1}{2},j,k}$
If$(d_{min}=d_{xu})$ Then

$$u_{i-\frac{1}{2},j,k}=\frac{x_{i-\frac{1}{2}}-x_{ub}}{x_{i-\frac{3}{2}}-x_{ub}}u_{i-\frac{3}{2},j,k}+\frac{\delta x}{x_{ub}-x_{i-\frac{3}{2}}}u_b\ .$$

If$(d_{min}=d_{yu})$ Then

$$u_{i-\frac{1}{2},j,k}=\frac{y_j-y_{ub}}{y_{j-1}-y_{ub}}u_{i-\frac{1}{2},j-1,k}+\frac{\delta y}{y_{ub}-y_{j-1}}u_b\ .$$

If$(d_{min} = d_{zu})$ Then

$$u_{i-\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k+1} - z_{ub}} u_{i-\frac{1}{2},j,k+1} - \frac{\delta z}{z_{ub} - z_{k+1}} u_b \ .$$

- Computation of $v_{i,j-\frac{1}{2},k}$

1. Compute x_{vb}, y_{vb} and z_{vb} from

$$f(x_{vb}, y_{j-\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j-\frac{1}{2}}, z_{vb}) = 0 \ .$$

2. Calculate the distances d_{xv}, d_{yv} and d_{zv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j-\frac{1}{2}}\right| \ , \qquad d_{zv} = |z_{vb} - z_k| \ .$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xv}, d_{yv}, d_{zv})$$

4. Compute $v_{i,j-\frac{1}{2},k}$
If$(d_{min} = d_{xv})$ Then

$$v_{i,j-\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j-\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b \ .$$

If$(d_{min} = d_{yv})$ Then

$$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} + \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b \ .$$

If$(d_{min} = d_{zv})$ Then

$$v_{i,j-\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k+1} - z_{vb}} v_{i,j-\frac{1}{2},k+1} - \frac{\delta z}{z_{vb} - z_{k+1}} v_b \ .$$

- Computation of $w_{i,j,k+\frac{1}{2}}$

1. Compute x_{wb}, y_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_{k+\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0 \ .$$

2. Calculate the distances d_{xw}, d_{yw} and d_{zw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left|z_{wb} - z_{k+\frac{1}{2}}\right|$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xw}, d_{yw}, d_{zw})$$

4. Compute $w_{i,j,k+\frac{1}{2}}$
If$(d_{min} = d_{xw})$ Then

$$w_{i,j,k+\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k+\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b \ .$$

If$(d_{min} = d_{yw})$ Then

$$w_{i,j,k+\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j-1} - y_{wb}} w_{i,j-1,k+\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j-1}} w_b \ .$$

If$(d_{min} = d_{zw})$ Then

$$w_{i,j,k+\frac{1}{2}} = \frac{z_{k+\frac{1}{2}} - z_{wb}}{z_{k+\frac{3}{2}} - z_{wb}} w_{i,j,k+\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k+\frac{3}{2}}} w_b \ .$$

e) B–cell with the $(i+\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells (see figure 52).

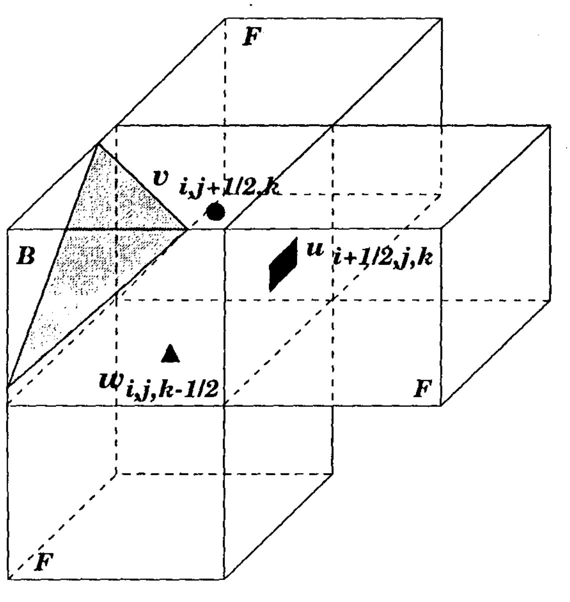

Fig. 52. B–cell with the $(i+\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute x_{ub}, y_{ub} and z_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

2. Calculate the distances d_{xu}, d_{yu} and d_{zu}

$$d_{xu} = \left|x_{ub} - x_{i+\frac{1}{2}}\right| \ , \qquad d_{yu} = |y_{ub} - y_j| \ , \qquad d_{zu} = |z_{ub} - z_k|$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xu}, d_{yu}, d_{zu})$$

4. Compute $u_{i+\frac{1}{2},j,k}$
If($d_{min} = d_{xu}$) Then

$$u_{i+\frac{1}{2},j,k} = \frac{x_{i+\frac{1}{2}} - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k} - \frac{\delta x}{x_{ub} - x_{i+\frac{3}{2}}} u_b \ .$$

If($d_{min} = d_{yu}$) Then

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j+1} - y_{ub}} u_{i+\frac{1}{2},j+1,k} - \frac{\delta y}{y_{ub} - y_{j+1}} u_b \ .$$

If($d_{min} = d_{zu}$) Then

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k-1} - z_{ub}} u_{i+\frac{1}{2},j,k-1} + \frac{\delta z}{z_{ub} - z_{k-1}} u_b \ .$$

– Computation of $v_{i,j+\frac{1}{2},k}$

1. Compute x_{vb}, y_{vb} and z_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0 \ .$$

2. Calculate the distances d_{xv}, d_{yv} and d_{zv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j+\frac{1}{2}}\right| \ , \qquad d_{zv} = |z_{vb} - z_k| \ .$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xv}, d_{yv}, d_{zv})$$

4. Compute $v_{i,j+\frac{1}{2},k}$
If($d_{min} = d_{xv}$) Then

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j+\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b \ .$$

If($d_{min} = d_{yv}$) Then

$$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b \ .$$

If($d_{min} = d_{zv}$) Then

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j+\frac{1}{2},k-1} + \frac{\delta z}{z_{vb} - z_{k-1}} v_b \ .$$

– Computation of $w_{i,j,k-\frac{1}{2}}$

1. Compute x_{wb}, y_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k-\frac{1}{2}}) = 0 \,, \qquad f(x_i, y_{wb}, z_{k-\frac{1}{2}}) = 0 \,, \qquad f(x_i, y_j, z_{wb}) = 0 \,.$$

2. Calculate the distances d_{xw}, d_{yw} and d_{zw}

$$d_{xw} = |x_{wb} - x_i| \,, \qquad d_{yw} = |y_{wb} - y_j| \,, \qquad d_{zw} = \left|z_{wb} - z_{k-\frac{1}{2}}\right|$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xw}, d_{yw}, d_{zw})$$

4. Compute $w_{i,j,k-\frac{1}{2}}$
If($d_{min} = d_{xw}$) Then

$$w_{i,j,k-\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k-\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i+1}} w_b \,.$$

If($d_{min} = d_{yw}$) Then

$$w_{i,j,k-\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k-\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j+1}} w_b \,.$$

If($d_{min} = d_{zw}$) Then

$$w_{i,j,k-\frac{1}{2}} = \frac{z_{k-\frac{1}{2}} - z_{wb}}{z_{k-\frac{3}{2}} - z_{wb}} w_{i,j,k-\frac{3}{2}} - \frac{\delta z}{z_{wb} - z_{k-\frac{3}{2}}} w_b \,.$$

f) **B–cell with the $(i-\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells (see figure 53).**

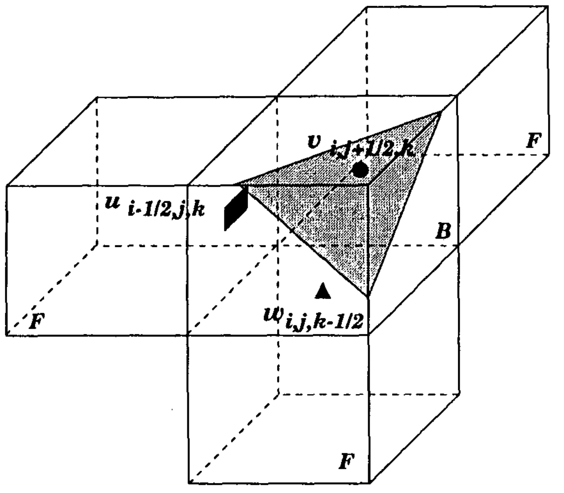

Fig. 53. B–cell with the $(i-\frac{1}{2})$, $(j+\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j+\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute x_{ub}, y_{ub} and z_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \; , \qquad f(x_{i-\frac{1}{2}}, y_{ub}, z_k) = 0 \; , \qquad f(x_{i-\frac{1}{2}}, y_j, z_{ub}) = 0$$

2. Calculate the distances d_{xu}, d_{yu} and d_{zu}

$$d_{xu} = \left|x_{ub} - x_{i-\frac{1}{2}}\right| \; , \qquad d_{yu} = |y_{ub} - y_j| \; , \qquad d_{zu} = |z_{ub} - z_k|$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xu}, d_{yu}, d_{zu})$$

4. Compute $u_{i-\frac{1}{2},j,k}$
If$(d_{min} = d_{xu})$ Then

$$u_{i-\frac{1}{2},j,k} = \frac{x_{i-\frac{1}{2}} - x_{ub}}{x_{i-\frac{3}{2}} - x_{ub}} u_{i-\frac{3}{2},j,k} + \frac{\delta x}{x_{ub} - x_{i-\frac{3}{2}}} u_b \; .$$

If$(d_{min} = d_{yu})$ Then

$$u_{i-\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j+1} - y_{ub}} u_{i-\frac{1}{2},j+1,k} - \frac{\delta y}{y_{ub} - y_{j+1}} u_b \; .$$

If$(d_{min} = d_{zu})$ Then

$$u_{i-\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k-1} - z_{ub}} u_{i-\frac{1}{2},j,k-1} + \frac{\delta z}{z_{ub} - z_{k-1}} u_b \; .$$

– Computation of $v_{i,j+\frac{1}{2},k}$

1. Compute x_{vb}, y_{vb} and z_{vb} from

$$f(x_{vb}, y_{j+\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j+\frac{1}{2}}, z_{vb}) = 0 \ .$$

2. Calculate the distances d_{xv}, d_{yv} and d_{zv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j+\frac{1}{2}}\right| \ , \qquad d_{zv} = |z_{vb} - z_k| \ .$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xv}, d_{yv}, d_{zv})$$

4. Compute $v_{i,j+\frac{1}{2},k}$
If($d_{min} = d_{xv}$) Then

$$v_{i,j+\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j+\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b \ .$$

If($d_{min} = d_{yv}$) Then

$$v_{i,j+\frac{1}{2},k} = \frac{y_{j+\frac{1}{2}} - y_{vb}}{y_{j+\frac{3}{2}} - y_{vb}} v_{i,j+\frac{3}{2},k} - \frac{\delta y}{y_{vb} - y_{j+\frac{3}{2}}} v_b \ .$$

If($d_{min} = d_{zv}$) Then

$$v_{i,j+\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j+\frac{1}{2},k-1} + \frac{\delta z}{z_{vb} - z_{k-1}} v_b \ .$$

– Computation of $w_{i,j,k-\frac{1}{2}}$

1. Compute x_{wb}, y_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0 \ .$$

2. Calculate the distances d_{xw}, d_{yw} and d_{zw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left|z_{wb} - z_{k-\frac{1}{2}}\right|$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xw}, d_{yw}, d_{zw})$$

4. Compute $w_{i,j,k-\frac{1}{2}}$
If($d_{min} = d_{xw}$) Then

$$w_{i,j,k-\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k-\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b \ .$$

If($d_{min} = d_{yw}$) Then

$$w_{i,j,k-\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j+1} - y_{wb}} w_{i,j+1,k-\frac{1}{2}} - \frac{\delta y}{y_{wb} - y_{j+1}} w_b \ .$$

If($d_{min} = d_{zw}$) Then

$$w_{i,j,k-\frac{1}{2}} = \frac{z_{k-\frac{1}{2}} - z_{wb}}{z_{k-\frac{3}{2}} - z_{wb}} w_{i,j,k-\frac{3}{2}} + \frac{\delta z}{z_{wb} - z_{k-\frac{3}{2}}} w_b \ .$$

g) **B–cell with the $(i+\frac{1}{2})$, $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells (see figure 54).**

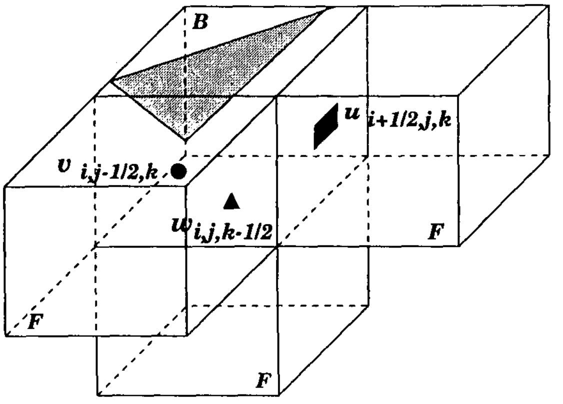

Fig. 54. B–cell with the $(i+\frac{1}{2})$, $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i+\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as follows:

– Computation of $u_{i+\frac{1}{2},j,k}$

1. Compute x_{ub}, y_{ub} and z_{ub} from

$$f(x_{ub}, y_j, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_{ub}, z_k) = 0 \ , \qquad f(x_{i+\frac{1}{2}}, y_j, z_{ub}) = 0$$

2. Calculate the distances d_{xu}, d_{yu} and d_{zu}

$$d_{xu} = \left| x_{ub} - x_{i+\frac{1}{2}} \right| \ , \qquad d_{yu} = |y_{ub} - y_j| \ , \qquad d_{zu} = |z_{ub} - z_k|$$

3. Compute d_{min}

$$d_{min} = min\,(d_{xu}, d_{yu}, d_{zu})$$

4. Compute $u_{i+\frac{1}{2},j,k}$
If$(d_{min} = d_{xu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{x_{i+\frac{1}{2}} - x_{ub}}{x_{i+\frac{3}{2}} - x_{ub}} u_{i+\frac{3}{2},j,k} - \frac{\delta x}{x_{ub} - x_{i+\frac{3}{2}}} u_b \ .$$

If$(d_{min} = d_{yu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{y_j - y_{ub}}{y_{j-1} - y_{ub}} u_{i+\frac{1}{2},j-1,k} + \frac{\delta y}{y_{ub} - y_{j-1}} u_b \ .$$

If$(d_{min} = d_{zu})$ Then

$$u_{i+\frac{1}{2},j,k} = \frac{z_k - z_{ub}}{z_{k-1} - z_{ub}} u_{i+\frac{1}{2},j,k-1} + \frac{\delta z}{z_{ub} - z_{k-1}} u_b \ .$$

- Computation of $v_{i,j-\frac{1}{2},k}$
  1. Compute x_{vb}, y_{vb} and z_{vb} from
     $$f(x_{vb}, y_{j-\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j-\frac{1}{2}}, z_{vb}) = 0 \ .$$
  2. Calculate the distances d_{xv}, d_{yv} and d_{zv}
     $$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j-\frac{1}{2}}\right| \ , \qquad d_{zv} = |z_{vb} - z_k| \ .$$
  3. Compute d_{min}
     $$d_{min} = min\left(d_{xv}, d_{yv}, d_{zv}\right)$$
  4. Compute $v_{i,j-\frac{1}{2},k}$
     If($d_{min} = d_{xv}$) Then
     $$v_{i,j-\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i+1} - x_{vb}} v_{i+1,j-\frac{1}{2},k} - \frac{\delta x}{x_{vb} - x_{i+1}} v_b \ .$$
     If($d_{min} = d_{yv}$) Then
     $$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} + \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b \ .$$
     If($d_{min} = d_{zv}$) Then
     $$v_{i,j-\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j-\frac{1}{2},k-1} + \frac{\delta z}{z_{vb} - z_{k-1}} v_b \ .$$
- Computation of $w_{i,j,k-\frac{1}{2}}$
  1. Compute x_{wb}, y_{wb} and z_{wb} from
     $$f(x_{wb}, y_j, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0 \ .$$
  2. Calculate the distances d_{xw}, d_{yw} and d_{zw}
     $$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left|z_{wb} - z_{k-\frac{1}{2}}\right|$$
  3. Compute d_{min}
     $$d_{min} = min\left(d_{xw}, d_{yw}, d_{zw}\right)$$
  4. Compute $w_{i,j,k-\frac{1}{2}}$
     If($d_{min} = d_{xw}$) Then
     $$w_{i,j,k-\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i+1} - x_{wb}} w_{i+1,j,k-\frac{1}{2}} - \frac{\delta x}{x_{wb} - x_{i+1}} w_b \ .$$
     If($d_{min} = d_{yw}$) Then
     $$w_{i,j,k-\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j-1} - y_{wb}} w_{i,j-1,k-\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j-1}} w_b \ .$$
     If($d_{min} = d_{zw}$) Then
     $$w_{i,j,k-\frac{1}{2}} = \frac{z_{k-\frac{1}{2}} - z_{wb}}{z_{k-\frac{3}{2}} - z_{wb}} w_{i,j,k-\frac{3}{2}} + \frac{\delta z}{z_{wb} - z_{k-\frac{3}{2}}} w_b \ .$$

h) **B–cell with the $(i-\frac{1}{2})$, $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells (see figure 55).**

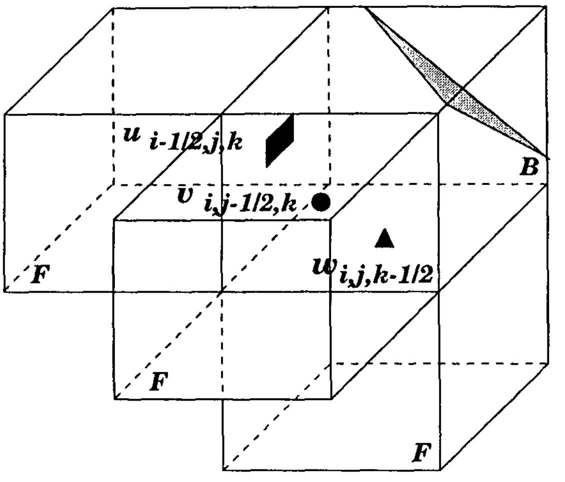

Fig. 55. B–cell with the $(i-\frac{1}{2})$, $(j-\frac{1}{2})$ and $(k-\frac{1}{2})$–faces contiguous with interior cells.

As above, the values of $u_{i-\frac{1}{2},j,k}$, $v_{i,j-\frac{1}{2},k}$ and $w_{i,j,k-\frac{1}{2}}$ are obtained as follows:

- Computation of $u_{i-\frac{1}{2},j,k}$
  1. Compute x_{ub}, y_{ub} and z_{ub} from

$$f(x_{ub},y_j,z_k)=0\ ,\qquad f(x_{i-\frac{1}{2}},y_{ub},z_k)=0\ ,\qquad f(x_{i-\frac{1}{2}},y_j,z_{ub})=0$$

  2. Calculate the distances d_{xu}, d_{yu} and d_{zu}

$$d_{xu}=\left|x_{ub}-x_{i-\frac{1}{2}}\right|\ ,\qquad d_{yu}=|y_{ub}-y_j|\ ,\qquad d_{zu}=|z_{ub}-z_k|$$

  3. Compute d_{min}

$$d_{min}=min\,(d_{xu},d_{yu},d_{zu})$$

  4. Compute $u_{i-\frac{1}{2},j,k}$
     If$(d_{min}=d_{xu})$ Then

$$u_{i-\frac{1}{2},j,k}=\frac{x_{i-\frac{1}{2}}-x_{ub}}{x_{i-\frac{3}{2}}-x_{ub}}u_{i-\frac{3}{2},j,k}+\frac{\delta x}{x_{ub}-x_{i-\frac{3}{2}}}u_b\ .$$

     If$(d_{min}=d_{yu})$ Then

$$u_{i-\frac{1}{2},j,k}=\frac{y_j-y_{ub}}{y_{j-1}-y_{ub}}u_{i-\frac{1}{2},j-1,k}+\frac{\delta y}{y_{ub}-y_{j-1}}u_b\ .$$

     If$(d_{min}=d_{zu})$ Then

$$u_{i-\frac{1}{2},j,k}=\frac{z_k-z_{ub}}{z_{k-1}-z_{ub}}u_{i-\frac{1}{2},j,k-1}+\frac{\delta z}{z_{ub}-z_{k-1}}u_b\ .$$

- Computation of $v_{i,j-\frac{1}{2},k}$
  1. Compute x_{vb}, y_{vb} and z_{vb} from

$$f(x_{vb}, y_{j-\frac{1}{2}}, z_k) = 0 \ , \qquad f(x_i, y_{vb}, z_k) = 0 \ , \qquad f(x_i, y_{j-\frac{1}{2}}, z_{vb}) = 0 \ .$$

  2. Calculate the distances d_{xv}, d_{yv} and d_{zv}

$$d_{xv} = |x_{vb} - x_i| \ , \qquad d_{yv} = \left|y_{vb} - y_{j-\frac{1}{2}}\right| \ , \qquad d_{zv} = |z_{vb} - z_k| \ .$$

  3. Compute d_{min}

$$d_{min} = min\,(d_{xv}, d_{yv}, d_{zv})$$

  4. Compute $v_{i,j-\frac{1}{2},k}$
     If($d_{min} = d_{xv}$) Then

$$v_{i,j-\frac{1}{2},k} = \frac{x_i - x_{vb}}{x_{i-1} - x_{vb}} v_{i-1,j-\frac{1}{2},k} + \frac{\delta x}{x_{vb} - x_{i-1}} v_b \ .$$

     If($d_{min} = d_{yv}$) Then

$$v_{i,j-\frac{1}{2},k} = \frac{y_{j-\frac{1}{2}} - y_{vb}}{y_{j-\frac{3}{2}} - y_{vb}} v_{i,j-\frac{3}{2},k} + \frac{\delta y}{y_{vb} - y_{j-\frac{3}{2}}} v_b \ .$$

     If($d_{min} = d_{zv}$) Then

$$v_{i,j-\frac{1}{2},k} = \frac{z_k - z_{vb}}{z_{k-1} - z_{vb}} v_{i,j-\frac{1}{2},k-1} + \frac{\delta z}{z_{vb} - z_{k-1}} v_b \ .$$

- Computation of $w_{i,j,k-\frac{1}{2}}$
  1. Compute x_{wb}, y_{wb} and z_{wb} from

$$f(x_{wb}, y_j, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_{wb}, z_{k-\frac{1}{2}}) = 0 \ , \qquad f(x_i, y_j, z_{wb}) = 0 \ .$$

  2. Calculate the distances d_{xw}, d_{yw} and d_{zw}

$$d_{xw} = |x_{wb} - x_i| \ , \qquad d_{yw} = |y_{wb} - y_j| \ , \qquad d_{zw} = \left|z_{wb} - z_{k-\frac{1}{2}}\right|$$

  3. Compute d_{min}

$$d_{min} = min\,(d_{xw}, d_{yw}, d_{zw})$$

  4. Compute $w_{i,j,k-\frac{1}{2}}$
     If($d_{min} = d_{xw}$) Then

$$w_{i,j,k-\frac{1}{2}} = \frac{x_i - x_{wb}}{x_{i-1} - x_{wb}} w_{i-1,j,k-\frac{1}{2}} + \frac{\delta x}{x_{wb} - x_{i-1}} w_b \ .$$

If$(d_{min} = d_{yw})$ Then

$$w_{i,j,k-\frac{1}{2}} = \frac{y_j - y_{wb}}{y_{j-1} - y_{wb}} w_{i,j-1,k-\frac{1}{2}} + \frac{\delta y}{y_{wb} - y_{j-1}} w_b \,.$$

If$(d_{min} = d_{zw})$ Then

$$w_{i,j,k-\frac{1}{2}} = \frac{z_{k-\frac{1}{2}} - z_{wb}}{z_{k-\frac{3}{2}} - z_{wb}} w_{i,j,k-\frac{3}{2}} + \frac{\delta z}{z_{wb} - z_{k-\frac{3}{2}}} w_b \,.$$

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