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Abstract

This paper presents and discusses some issues about tutoring systems as an useful tool in the mathematical learning/teaching process. In particular it is proposed a tutoring system called **TEGRAM**, which is based on Tangram - a millenary Chinese game that provides a large number of activities that lead the students to the construction of geometric concepts. It is presented the architecture of the system and it is shown an example of interaction between a student and the system. The learning process in which students and teachers are active participants is emphasized. It is also proposed a cooperative environment so that the student/teacher dialogue is strongly stimulated. The tutoring system does not aim substitute the teacher in his tasks. It is supposed to be a support tool in the teaching/learning process. This is an ongoing project and a next experimental phase, involving potential users, will provide us feedback for adaptations.

1. Introduction

It is well known that the utilization of computers in education is always increasing. It is important, however, to emphasize the computer's task in situations where it is supposed to be a complementary tool in the teaching/learning process - never an automatized tutor that transmits information to the student nor a teacher's substitute. It should be considered as a helpful tool in the reasoning development, favoring the intuitive though of the learner. It must be seen as a tool to provide the formalization of the intuitive concepts acquired by the student. So, the learning process must come to mind through activities *made by the student*, not just by transferring information from the computer and/or the teacher to her/him [8].

The use of computers to help the teaching/learning process is one of the richer ranges in which Artificial Intelligence (AI) application has been strongly developed nowadays. The researches about Intelligent Tutoring Systems (ITS) are examples of the interaction between AI and Education.

The term Intelligent Tutoring Systems was suggested by Sleeman and Brown [13]. ITSs are education-purpose computer programs that incorporate techniques and methods from the AI community in order to provide tutors which know *what* they teach, *who* they teach and *how* to teach it. AI attempts to produce in a computer a kind of behavior which, if performed by a human, would be described as "intelligent"; ITS may similarly be thought of as attempts to produce in a computer a behaviour which, if performed by a human, would be described as "good teaching" [10] [15] .

Although it is really helpful, computers cannot be seen as a panacea of educational problems. By itself it means almost nothing; it needs a strong educational basis to support and give information about relevant points of the teaching/learning process. For example, it is absolutely necessary to know the more frequent learning problems related to the matter as well as the appropriate activities to each student in each moment. Another important point is the profile of the student who is interacting with the system. So, the computer must be seen as an additional element in the educational environment to favor different drills that are consonant with the student's aptitude, previous experiences, rhythm and preferences [9]. According to Epstein and Hillegeist [4, p.13]:

> "We believe that the key to successfully using computers in education is to clarify the roles of the students, the teacher, the computer, and the interactions among them. Our experience tells us that students and teachers must view the computer as a *useful tool*, not as either a threat or an impediment to good instruction. They must feel a sense of control, and they must feel important in this new environment."

The construction of the concepts comes from an interior reflection that takes place when the student thinks about the drills she/he has done. Such process leads the student to the interiorization of her/his actions [7]. In this sense, it is known that appropriate games can be important to motivate the students in an interesting and creative way. Specially in Mathematics the games have been seen as a good resource. This way the computers can be strong allies to avoid the frequent aversion children have about Mathematics, one of the most insistent problems in Math education. The literature has plenty of studies concerning the benefits the games can bring in the teaching/learning process [1][6] .

2. Aim

This paper presents the project **TEGRAM** - a tutoring system about concepts of plane geometry to be used by elementary school students. It consists of practical activities involving concepts of plane geometry. These activities are presented according to the student profile and to a teaching strategy that distinguishes the learning by discovery. This way the student is lead to discover, through activities, the concepts that the system and the teacher intend to teach. The system has a mixed initiative control: it does not restrict the student to a fixed sequence of tasks, neither let her/him too free to hinder a reasonable teaching plan.

To motivate the students for exploring the concepts to be taught, all the activities are based on **Tangram**, a millenary Chinese game [5] [11] . It consists of seven pieces: five triangles, one square and one parallelogram. All of them are originated from the decomposition of the whole square as shown in Figure 1.

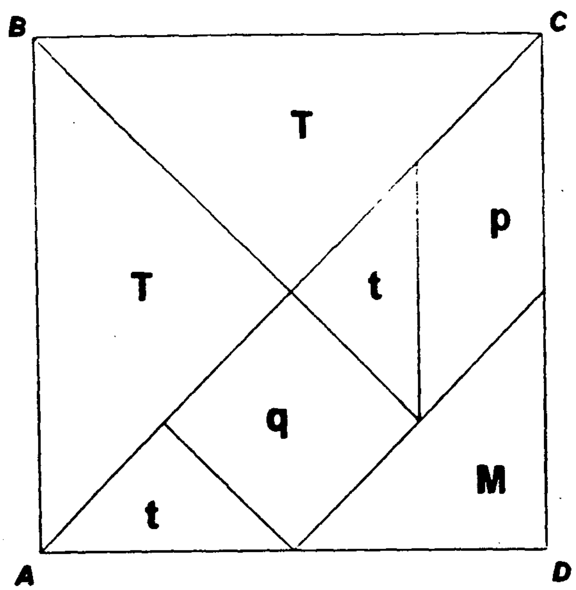

Figure 1. Tangram.

There are countless didactic activities based on Tangram that can be explored for children and students of different ages and school levels [12]. For children at the first grades it is expected to use TEGRAM at first to play both individually and in group. This way they soon recognize the Tangram pieces. Meanwhile they are supposed to create figures like cat, swan, duck, boat, chicken and so on. This stage is too important since it provides a familiarity with the Tangram pieces and the perception of some relations such as identification of the shapes, comparison of number and measure of angles, number of boundary lines of geometrical figures, number of vertexes etc. Moreover, such activities can help to teach to read and write.

Later, according to the student advance, the activities' goal is to emphasize the pieces geometrical characteristics and to lead the student to the achievement of properties and definitions of each shape. This way the next step is the development of classification of other geometric figures made through the composition of the pieces. Some activities explore concepts of length, area and perimeter. Others can explore congruence and similarity among geometric figures or notion of fraction, rate and proportion or even the Pythagorean theorem.

The underlying idea is that the student can assimilate these concepts in an intuitive way, throughout a guided realization of the system activities. The formulation of the concepts would be achieved only *a posteriori*.

3. The System Architecture

The TEGRAM system's architecture is composed of four classical components: the **domain module**, the **student model**, the **tutorial module** and the **communication module** [2] [15] (Figure 2).

The **domain module** has the knowledge of the topics to be taught and is used as a guidance to evaluate the student perfomance related to the activities proposed by the system. The domain module is divided into a **concepts network** and a **definitions network**. A subgraph of concepts to be presented is selected according to the student model. A concept can be considered nuclear (NC) - if it involves major ideas - or satellite (SC) - if it involves complementary ideas. Nuclear concepts consist of all subject topics TEGRAM can teach.

For example, if the nuclear concept is the notion of area , there are some fundamental geometric concepts that may be called **satellite ideas**. Suppose that the student starts the activity by discovering the small square area. So, some satellite ideas are:

- The hole is the sum of the parts.
- The diagonal of a square cuts it in two triangles.
- Those triangles are rectangles, isosceles and congruents.

For the other shapes analogous ideas could be presented. Shall we say for one of the big triangles:

- The hole is the sum of the parts.
- The big triangle is covered by the small square and two small triangles.
- The big triangle is covered by two small triangles and one parallelogram.
- The big triangle is covered by one midle size and two small triangles.
- The parallelogram is covered by two small triangles.
- The midle size triangle is covered by two small triangles.
- The small square is covered by two small triangles.

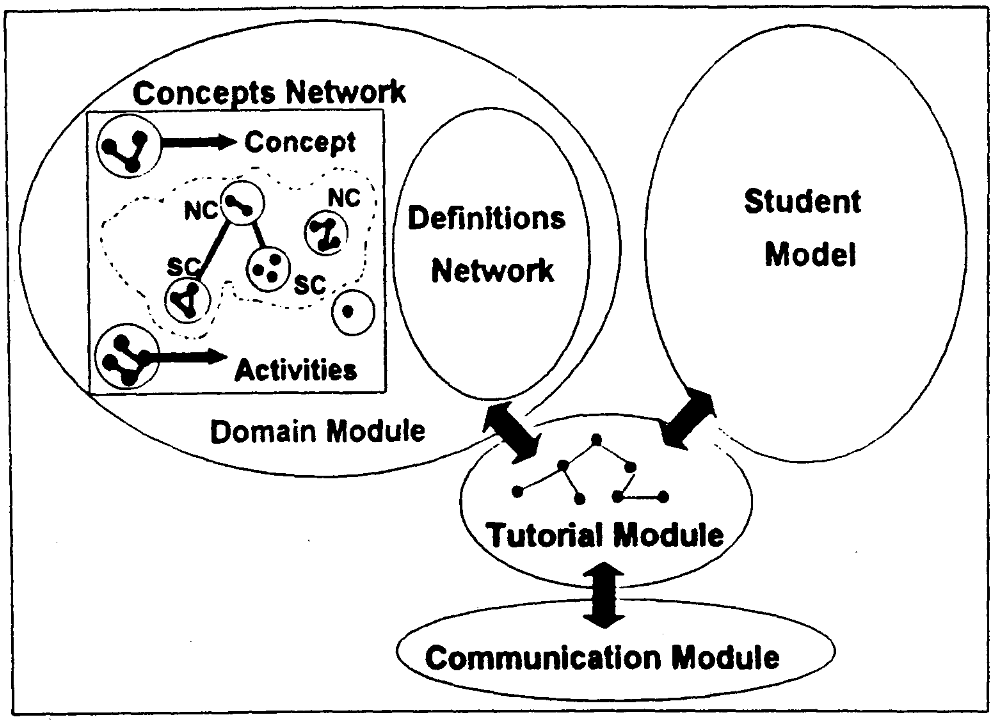

Figure 2. The TEGRAM's architecture.

All the concepts explored by the system are stored in a **concepts base** where there is no difference between nuclear concept and satellite concept. The interaction with a student allows the identification of the concepts as nuclear or satellite in order to explore those concepts that are supposed to fit well to that student level.

Internally the mathematical concepts structure to be used by TEGRAM system contains the following slots:

Slot 1: Identification of the mathematical concept.

Slot 2: Name of the mathematical concept.

Slot 3: A list of the suitable grades to teach this concept.

Slot 4: A list of activity/grade that can be proposed to explore this concept.

Slot 5: A list of following concepts consonant this concept. This list must take into account the relation of precedence of the concepts (prerequisites).

The below example shows how a concept is stored by the system:

Slot 1: C2

Slot 2: Area

Slot 3: [2,3,4]

Slot 4: [[A01,2] [A05,2] [A06,2] [A02,3] [A03,4]]

Slot 5: [C3,C4,C5]

The five slots show that the activities A01, A05 and A06 are suitable to teaching area concept for the second grade's student. The activity A02 for the third grade and A03 for the fourth grade.

Each **nuclear concept** originates an **activities network** which is submitted to the student so that the order is established according to the student model. Depending on the student performance during an activity, the tutor can indicate some revision or a related activity that will better cover the detected problems.

The student is supposed to do only the activities related to the NC that has been taught. Since the system is intended to be used by students with different knowledge levels and different *"modus operandi"*, it is too important to the system to have several different activities to teach each concept. This is a condition of great importance if the main goal is that the learning process takes place in a meaningfull and pleasant way.

The data structure of the activities to be used by TEGRAM system has several slots which are described below:

Slot 1: Identification of the activity.

Slot 2: Name of the activity.

Slot 3: The difficulty level to accomplish the activity. For example, constructing a square using three pieces of Tangram seems to be more difficulty than constructing a square using two of them.

Slot 4: A list of satellite concepts related to this activity. Each activity explores a special nuclear concept (NC) and consequently has a minimal number of prerequisites (SC) to be successfully accomplished.

It is shown below an example of how an activity is stored in the system:

Slot 1: A01

Slot 2: Square area

Slot 3: 2

Slot 4: [C1,C7]

The slots above show that the activity A01 explore the square area concept with difficulty level two and satellites concepts C1 and C7.

The **definitions network** consists of the knowledge about the geometric figures that are the Tangram's parts: square, small triangle, midle size triangle, big triangle, parallelogram and all of the possible combinations of them to generate other geometric shapes. Recognizing the geometric shapes is an essential virtue to enhance the system expertise. In this sense, more than recognize, the system is able to reason about the geometric figures constructed by the student.

The key for the student's individualization in a intelligent tutoring system is its knowledge about its users. The module which contains this knowledge is called **student model** [3] [14]. The student model should contain all the aspects of the user's knowledge and behavior which can reflect on her or his perfomance of learning. The task of selecting and relating this information is quite difficult, or even impossible, since the communication channel used in the user-system dialogue is still very restrictive [10].

In the TEGRAM system, the student model is dynamically updated with information given by her/his choice about the activities. It is also updated by the activities and concepts the student has covered, the student performance and the detected problems in each activity. There is a **students base** with models of students that have used the system.

The date structure of the student model used by the system has the following slots:

Slot 1: Name of the student who is current using or has used the system.

Slot 2: The student password to use the system.

Slot 3: The school grade of the student.

Slot 4: The first session date of this student in the system.

Slot 5: The last session date of this student in the system.

Slot 6: How often the system has been used by this student.

Slot 7: A list of nuclear concepts to be explored by this student.

Slot 8: A list of the concepts that are supposed to be learned by the student. A nuclear concept is considered as learned by the student if she/he has been able to accomplish successfully the activities shown in the slot below.

Slot 9: A structure containing the following information related to the activities the student have done:

- What activity has been done (identification of the activity).
- Date of the activity realization.
- How many times the student have done this activity.
- Student perfomance during the activity realization.
- A list of all the solutions that the student have found in this activity.

The **tutorial module** consists of a set of rules which associate information from the domain module with information from the student model. The result of the application of a tutorial rule includes a set of actions related to system's tasks - the presentation of instructional material. of an exercise, of a help information, etc. - and also the updating of the student model.

The TEGRAM system has a non explicit tutorial module. All of its rule are implicit in the system. The tutorial rules are essentially represented in a production system that combines knowledge from the domain module and the student model to selected the material to be presented to the student. The main underlying strategy is to guide the student through a reasoning path that can lead her/him to formulate an unknown (for her/him) concept.

The interface role in the user-computer interaction has had more and more importance through the last decade. An interactive system will be successful or not depending on the quality of its interface with the user. ITSs are not exceptions. On the contrary, since its main goal is to help the learning, it is fundamental that the communication with the student takes place so that she/he feels comportable, stimulated and challenged. The communication module role is exactly to provide a flexible interaction between the ITS and the student. So, it is essential to have a specific module to control this interaction: the **communication module**.

In this project, the main feature of the communication module is its graphical facilities, since TEGRAM is supposed to be used specially by children. Most of the activities are presented so that the problem is stated in one window and the solution has to be built in a lower one. Both of them are supplied by graphic resources for figures movements. A work window for text editing is also available for the student.

4. Interaction with the system

In order to have a better visualization of TEGRAM system working, in this session it will be shown an example of interaction between the system and a student.

To identify the user, the system presents at first an identification window. The student is asked to fill the slot with her/his name. The system then searches for the name in the **students base**. If it fails, the student is novice, i.e., she/he has never used the system. If the name is there, the system asks the student his password and she/he student model is actived. Henceforward the system constructs and/or updates the student model for this user.

Suppose that the student is a novice user. Thus the system presents a second identification window. This window asks the student's password and her/his school grade. If the student is not a novice, the system asks only the user password to access the system.

Suppose that the student of the example is at first grade. The system then searches in the **concepts base** the concerning mathematical concepts. Now the system construct a *concepts network* with those concepts to be taught to this student. This network is ordered according to slot 5 of the concepts structure (relation of precedence of concepts - prerequisites).

Based on this network, the system creates a list of nuclear concepts to be explored by the student (see slot 7 of the student model). This concepts network is built initially at the first session. Otherwise the system uses directly the slot 7 of the student model. If the student has seen all the nuclear concepts stored in the slot 7 (student model), the system increases the knowledge level of student, builds again a concepts network and updates the slot 7 of the student model.

For the student of the example, the list of nuclear concepts is [C1,C3,C4,C2]. Considering this list, the system teaches at first the concept C1. To do this there are some activities to be done, for instance [A01,A04,A03], ordered according to its difficulty level (see slot 3 of the activities structure). This way an activities menu is dynamically created during the interaction with the student. The selected activities are presented to her/him and she/he is free to choose among them (Figure 3).

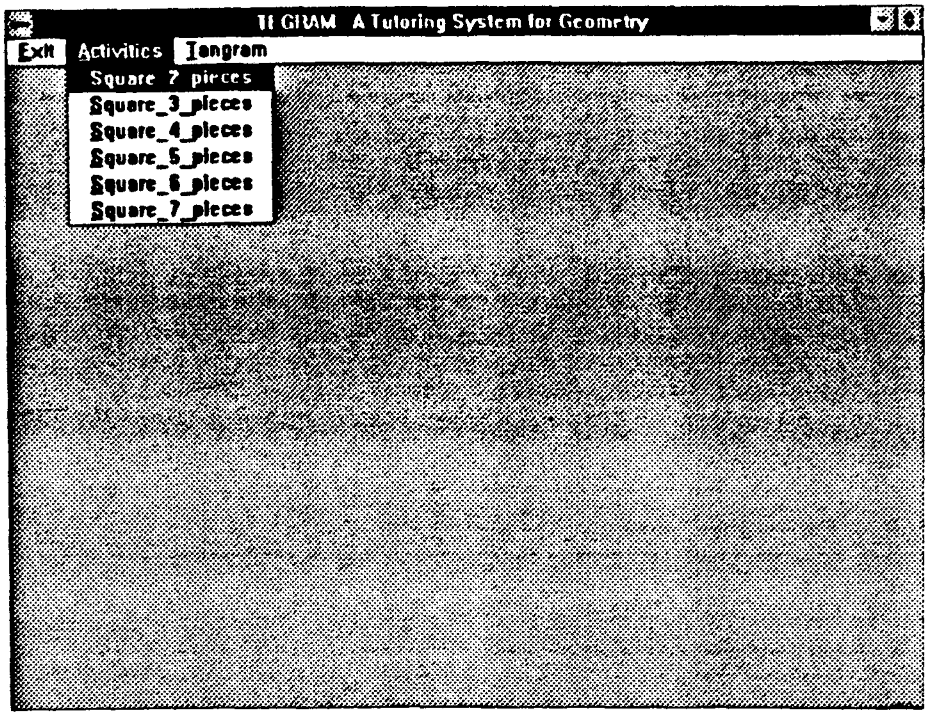

Figure 3: Selected activities to the student of the example.

Suppose the student has chosen the activity **Square-2-pieces**. It will be shown to him a new window with all the relevant information about this activity: enunciation, the Tangram pieces, the buttons to movie the pieces (rotation, translation and reflection), the satellite concepts, the hints to help to reach the solution, etc. (Figure 4).

The student will only experience the next concept if she/he has a satisfactory perfomance in the current activities being done. If she/he has a good perfomance, the system updates the corresponding slot, namely, the slot of supposedly learned concepts (slot 8 of the student model) and creates another activities menu for a new mathematical concept.

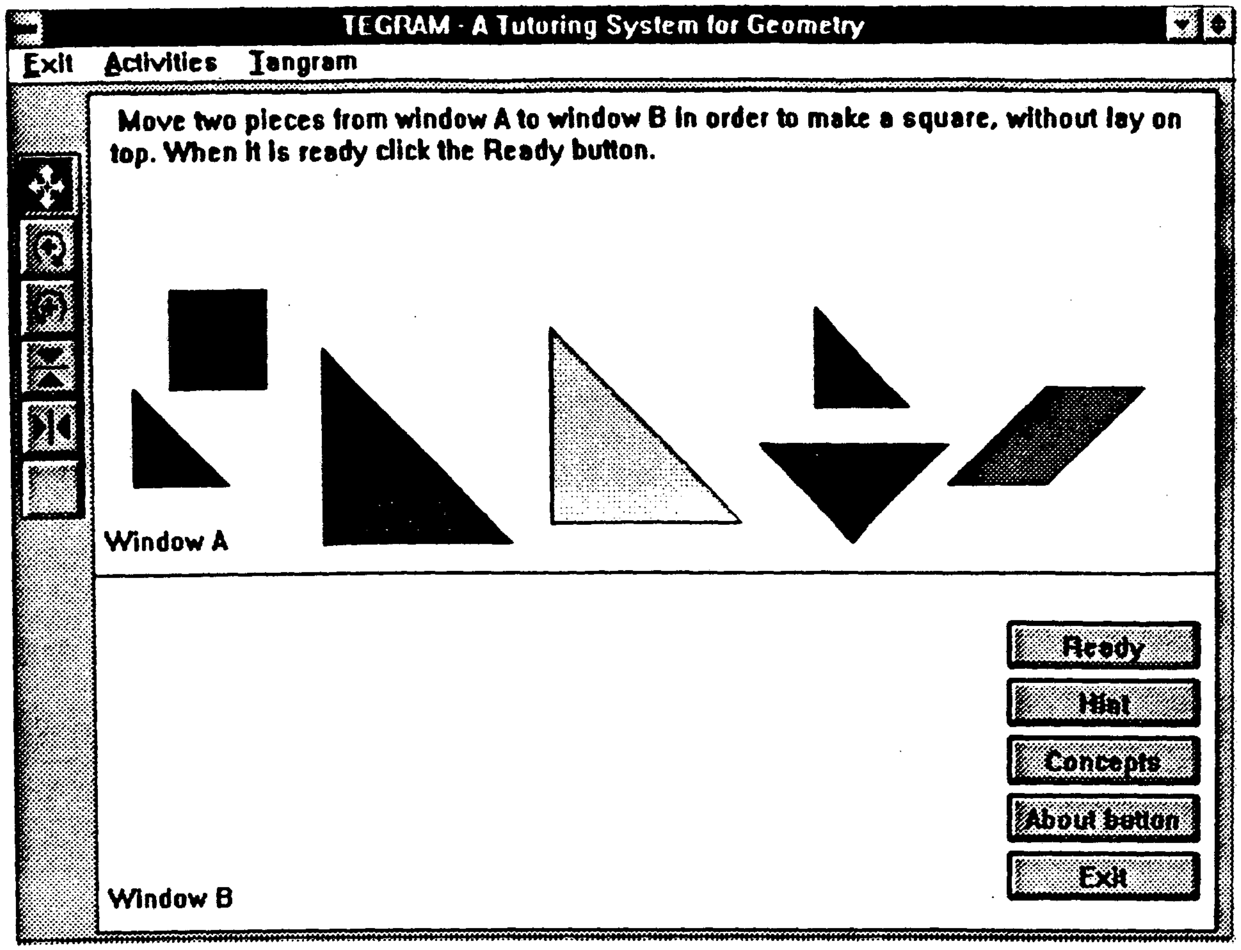

Figure 4: Activity **Square-2-pieces**.

1. Conclusions and future work

The ongoing project partially described above attempts to build a tutoring system for helping teaching concepts of plane geometry.

A first version of TEGRAM system is almost ready. In this version, five mathematical concepts have been explored: geometric shapes, area, congruence, similarity and perimeter.

Ten activities have been implemented up till now:

- Identify the triangles among the seven pieces of Tangram.

- Identify the square among the pieces of Tangram.
- Identify the parallelogram among the pieces of Tangram.
- Construction of a square using two pieces of Tangram.
- Construction of a square using three pieces of Tangram.
- Construction of a triangle using two pieces of Tangram.
- Construction of a triangle using three pieces of Tangram.
- Supposing the measurement of the region of the small triangle is 1, give the measurement of the other pieces of Tangram.
- Supposing the square area is 1, calculate the area of the other pieces.
- Supposing the perimeter of the square is 4, calculate the perimeter of the other pieces of Tangram.

To better explore these mathematical concepts more activities have been thought. The experience with those already implemented activities will bring feedback to the necessary reformulation as well as to new activities to be proposed.

This paper discussed the interaction between the system and the student. However, the student is not the only user; the interaction with the teacher is also very important, although it is often forgotten by the designers. In this project one of the most important features is the possibility of generating explanations to the teacher, so that she/he can reach data about the student's perfomance concerning the possible ways they learn the mathematical concepts.

One of the next phases of this project is constructing a specific interface to the teacher. It will be an interface so that the teacher can modify the way to present the concepts based on the student's perfomance. It will be possible to visualize the concepts network proposed to each grade and to follow the development of the student.

The system is being implemented for PC, using the environment BORLAND C++ for Windows.

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