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Accelerated life tests with one stress variable: a Bayesian analysis of the eyring model

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ACCELERATED LIFE TESTS WITH ONE STRESS VARIABLE:

A BAYESIAN ANALYSIS OF THE EYRING MODEL

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S U M M A R Y

"In this paper, we consider accelerated life tests with an exponential distribution and an Eyring model with one stress variable. Assuming type II censored data and Jeffreys priors for the parameters of the model, we develop a Bayesian analysis using the Laplace's method for approximation of integrals when we cannot find analytical explicitly solutions for the marginal posterior densities of interest. We also find the predictive density for a future observation in a specified stress level, and we use this predictive density to develop quality control tests".

KEY WORDS: accelerated life tests, Eyring model, Bayesian analysis, exponential distribution, predictive densities.

1. INTRODUCTION

Usually, industries of eletronical componentes use accelerated life tests to obtain measures of the reliability of the devices under the usual stress level (see for example, Mann, Schaffer and Singpurwalla, 1974).

In this paper, we assume an exponential distribution for the life times T of the components, with density,

$$f(t;\lambda_i) = \lambda_i \exp\{-\lambda_i t\} \tag{1}$$

where $t > 0$, $\lambda_i > 0$ and the Eyring model with one stress variable V_i, given by,

$$\lambda_i = V_i \exp\{\alpha - \frac{\beta}{V_i}\} \tag{2}$$

where α and β are unknown parameters, $i = 1,2,\ldots,k$ (number of stress levels).

We develop a Bayesian analysis assuming an experiment with type II censored data (see for example, Lawless, 1982) and noninformative Jeffreys prior densities for the parameters (see for example, Box and Tiao, 1973). When we cannot find explicitly analytical solutions for the marginal posterior densities of interest, we use the Laplace's method for approximation of integrals (see for example, Tierney and Kadane, 1986).

We also find the predictive density of a future observation in a stress level V_i and we use this predictive density to develop quality control tests.

2. THE CLASSICAL APPROACH FOR THE PROBLEM

Let T be a nonnegative random variable representing the life time of an unity with exponential density (1) and the Eyring model (2) for the parameter λ_i under the stress variable V_i, $i = 1,2,\ldots,k$.

Let us consider a type II censoring mechanism, that is, the experiment terminates when we observe r_i failures for each stress level i, $i = 1,2,\ldots,k$. Thus, with n_i unities at the beginning of each test with stress V_i, we have the ordered uncensored observations given by t_{1i}, $t_{2i},\ldots,t_{r_i i}$ and $n_i - r_i$ censored observations equal to $t_{r_i i}$, $i=1,2,\ldots,k$.

The likelihood function for α and β considering the data under the stress level V_i is given by,

$$L(\alpha,\beta) = \lambda_i^{r_i} \exp\{-\lambda_i A_i\} \tag{3}$$

where

$$A_i = \sum_{j=1}^{r_i} t_{ji} + (n_i - r_i) t_{r_i i}.$$

Considering the data of k stress levels $V_1, V_2, \ldots, V_k$ taken at random, the logarithm of the likelihood function for α and β is given by,

$$\ell(\alpha,\beta) \propto \alpha r - \beta a_1 - e^{\alpha} \sum_{j=1}^{k} V_j A_j e^{-\beta/V_j} \tag{4}$$

where $a_1 = \sum_{j=1}^{k} r_j/V_j$ and $r = \sum_{j=1}^{k} r_j$ (total number for observed failures).

The maximum likelihood estimators for α and β are given by,

$$\hat{\alpha} = \log\{\frac{r}{\sum_{j=1}^{k} V_j A_j e^{-\hat{\beta}/V_j}}\}$$

$$\frac{\sum_{j=1}^{k} A_j e^{-\hat{\beta}/V_j}}{\sum_{j=1}^{k} V_j A_j e^{\hat{\beta}/V_j}} = \frac{a_1}{r} \qquad (5)$$

For inferences about α and β, or even functions of the parameters, researchers usually use the asymptotical normality of the maximum likelihood estimators,

$$(\hat{\alpha},\hat{\beta}) \overset{a}{\sim} N\ (\alpha,\beta);\ I^{-1}(\hat{\alpha},\hat{\beta})\} \qquad (6)$$

where $I(\alpha,\beta)$ is the Fisher information matrix for α and β, given by,

$$I(\alpha,\beta) = \begin{bmatrix} r & -a_1 \\ -a_1 & a_2 \end{bmatrix}$$

where $r = \sum_{j=1}^{k} r_j$, $a_1 = \sum_{j=1}^{k} r_j/V_j$ and $a_2 = \sum_{j=1}^{k} r_j/V_j^2$. Since the elements of $I(\alpha,\beta)$ are constants, the normal approximation (6) is very accurate (see for example, Sprott, 1973). The asymptotical variances and covariance for $\hat{\alpha}$ and $\hat{\beta}$ are given by, $var(\hat{\alpha}) \cong a_2/(ra_2 - a_1^2)$, $var(\hat{\beta}) \cong r/(ra_2 - a_1^2)$ and $cov(\hat{\alpha},\hat{\beta}) \cong a_1/(ra_2 - a_1^2)$.

Usually, industrial researchers have interest in inferences about $\theta_1 = 1/\lambda_1$, the mean life time under the stress level V_1, and they consider the asymptotical normality of the maximum likelihood estimator for $\theta_1 = e^{\beta/V_1}/(V_1 e^{\alpha})$ obtained from (6) using the "delta method" (see for example, Miller, 1981), given by,

$$\hat{\theta}_1 \overset{a}{\sim} N\{\frac{e^{\beta/V_1}}{V_1 e^{\alpha}};\ \hat{\sigma}^2_{\theta_1}\} \tag{7}$$

where $\hat{\sigma}^2_{\theta_1} = \frac{\hat{\theta}_1^2}{(ra_2 - a_1^2)}\{a_2 + \frac{r}{V_1^2} - \frac{2a_1}{V_1}\}$.

Usually, this normal approximation is not very accurate, that is, we do not find good inferences about θ_1 using (7). In fact, if we consider the likelihood function for θ_1 and β, the Fisher information matrix for θ_1 and β is given by,

$$I(\theta_1,\beta) = \begin{bmatrix} \frac{r}{\theta_1^2} & \frac{a_1 - r/V_1}{\theta_1} \\ \frac{a_1 - r/V_1}{\theta_1} & a_2 - \frac{2a_1}{V_1} + \frac{r}{V_1^2} \end{bmatrix}$$

that is, the elements of $I(\theta_1,\beta)$ are not constants (they depend on θ_1). This implies that the normal approximation for the maximum likelihood estimators $\hat{\theta}_1$ and $\hat{\beta}$, usually is not accurate for small or moderate sample sizes. We also observe that $\hat{\sigma}^2_{\theta_1}$, the asymptotical variance of $\hat{\theta}_1$, is the same given in (7).

It is interesting to observe that, considering other parametrizations, as an orthogonal parametrization θ_1 and $\lambda = \beta + \frac{(a_1 - r/V_1)}{(a_2 - \frac{2a_1}{V_1} + \frac{r}{V_1^2})} \log \theta_1$ (see for example, Cox and Reid, 1987), with Fisher information matrix,

$$I(\theta_1,\lambda) = \begin{bmatrix} \dfrac{\dfrac{[r - \frac{(a_1 - r/V_1)^2}{2a_1}]}{(a_2 - \frac{2a_1}{V_1} + \frac{r}{V_1^2})}}{\theta_1^2} & 0 \\ 0 & a_2 - \dfrac{2a_1}{V_1} + \dfrac{r}{V_1^2} \end{bmatrix}$$

we find the same normal approximation for the maximum likelihood estimator $\hat{\theta}_1$ with $\hat{\sigma}^2_{\theta_1}$ given in (7).

We observe, in the different parametrizations considered above, the same asymptotical normality for $\hat{\theta}_1$, which can not be very accurate for small or moderate sample sizes. This motivates us, to develop a Bayesian analysis of the Eyring model.

3. A BAYESIAN ANALYSIS ASSUMING α AND β UNKNOWN

The Jeffreys prior density for α and β is (see for example, Box and Tiao, 1973) given by,

$$\begin{aligned} \pi(\alpha,\beta) &\propto \{\det I(\alpha,\beta)\}^{1/2} \\ &\propto \text{constant} \end{aligned} \tag{8}$$

where $-\infty < \alpha,\beta < \infty$.

The joint posterior density for α and β considering the noninformative prior (8) is given by,

$$\pi(\alpha,\beta|\text{data}) \propto \exp\{\alpha r - \beta a_1 - e^{\alpha} \sum_{j=1}^{k} V_j A_j e^{-\beta/V_j}\} \tag{9}$$

where $-\infty < \alpha, \beta < \infty$ and $a_1 = \sum_{j=1}^{k} r_j / V_j$.

The marginal posterior density for β is given by,

$$\pi(\beta | \text{data}) \propto e^{-\beta a_1} / \{ \sum_{j=1}^{k} A_j V_j e^{-\beta / V_j} \}^r \qquad (10)$$

where $-\infty < \beta < \infty$, and the marginal posterior density for α approximated by the Laplace's method (see for example, Tierney and Kadane, 1986), is given by,

$$\pi(\alpha | \text{data}) \propto \frac{\exp\{\alpha(r - \frac{1}{2}) - \hat{\beta} a_1 - e^{\alpha} \sum_{j=1}^{k} A_j V_j e^{-\hat{\beta}/V_j}\}}{\{ \sum_{j=1}^{k} \frac{A_j}{V_j} e^{-\hat{\beta}/V_j} \}^{1/2}} \qquad (11)$$

where $-\infty < \alpha < \infty$ and $\hat{\beta}$ is the maximum of $-a_1 + e^{\alpha} \sum_{j=1}^{k} A_j e^{-\beta/V_j}$ for each value of α.

The mean life time under the normal use stress level V_1 is given by $\theta_1 = e^{\beta/V_1}/(V_1 e^{\alpha})$. Considering the transformation of variables $\delta = \log\theta_1$ and $\beta = \beta$, for which the Jeffreys prior density is locally uniform, the marginal posterior density for $\delta = \log\theta_1$ is given by,

$$\pi(\delta | \text{data}) \propto e^{-\delta r} \int_{-\infty}^{\infty} e^{-nh_\delta(\beta)} d\beta \qquad (12)$$

where

$$-nh_\delta(\beta) = -\beta(a_1 - \frac{r}{V_1}) - e^{-\delta} \sum_{j=1}^{k} A_j \frac{V_j}{V_1} e^{-\beta(\frac{1}{V_j} - \frac{1}{V_1})}$$

Using the Laplace's method for approximation of integrals, we have,

$$\pi(\delta|\text{data}) \propto \frac{e^{-\delta(r-1/2)} \exp\{-\hat{\beta}(a_1 - \frac{r}{V_1}) - \frac{e^{-\delta}}{V_1}\sum_{j=1}^{k} A_j V_j e^{-\beta(\frac{1}{V_j}-\frac{1}{V_1})}\}}{\{\sum_{j=1}^{k} A_j V_j (\frac{1}{V_j}-\frac{1}{V_1})^2 \exp[-\hat{\beta}(\frac{1}{V_j}-\frac{1}{V_1})]\}^{1/2}} \quad (13)$$

where $-\infty < \delta < \infty$ and $\hat{\beta}$ maximizes $-nh_\delta(\beta)$.

4. PREDICTIVE DENSITY FOR A FUTURE OBSERVATION

Assuming β known and a Jeffreys prior density $\pi(\alpha) \propto$ constant, $-\infty < \alpha < \infty$, the posterior density for α is given by,

$$\pi(\alpha|\text{data}) = \frac{\{\sum_{j=1}^{k} A_j V_j e^{-\beta/V_j}\}^r}{\Gamma(r)} \exp\{\alpha r - e^\alpha \sum_{j=1}^{k} A_j V_j e^{-\beta/V_j}\} \quad (14)$$

where $-\infty < \alpha < \infty$.

The predictive density for a future observation $T_{(n+1)i}$, where $n = \sum_{j=1}^{k} n_j$ is the number of observations in test, considering a stress level V_i is given by (see for example, Aitchison and Dunsmore, 1975),

$$f^i(t_{(n+1)i}|\text{data}) = \int_{-\infty}^{\infty} f^i(t_{(n+1)i}|\alpha)\pi(\alpha|\text{data})d\alpha = \underset{\alpha|\text{data}}{E}\{f^i(t_{(n+1)i}|\alpha)\} \quad (15)$$

where $f^i(t_{(n+1)i}|\alpha) = V_i e^{\alpha-\beta/V_i} \exp\{-V_i e^{\alpha-\beta/V_i} t_{(n+1)i}\}$, and $\pi(\alpha|\text{data})$ is the posterior density for α given in (14).

That is,

$$f^i(t_{(n+1)i}|\text{data}) = \frac{\Gamma(r+1)V_i e^{-\beta/V_i}\{\sum_{j=1}^{k} A_j V_j e^{-\beta/V_j}\}^r}{\Gamma(r)\{V_i e^{-\beta/V_i} t_{(n+1)i} + \sum_{j=1}^{k} A_j V_j e^{-\beta/V_j}\}^{r+1}}$$

where $t_{(n+1)i} > 0$.

Since $\Gamma(r+1) = r\Gamma(r)$, we have,

$$f^i(t_{(n+1)i}|\text{data}) = \frac{rV_i e^{-\beta/V_i}\{\sum_{j=1}^{k} A_j V_j e^{-\beta/V_j}\}^r}{\{V_i e^{-\beta/V_i} t_{(n+1)i} + \sum_{j=1}^{k} A_j V_j e^{-\beta/V_j}\}^{r+1}} \quad (16)$$

where $t_{(n+1)i} > 0$ (a Pareto density).

5. USE OF THE PREDICTIVE DENSITY $f^i(t_{(n+1)i}|\text{data})$ IN QUALITY CONTROL

We can use the predictive density $f^i(t_{(n+1)i}|\text{data})$ to formulate a quality control procedure in life testing. Usually, the quality engineers select random samples of each batch of manufactured components to verify if the process line is under control. To minimize the cost and time of test, they consider unitis in life tests with a high stress level V_i and a fixed period of time L_i.

Using the predictive density (16) with β known, and considering a fixed probability $1-\gamma$, we can find the required values of V_i and L_i to have,

$$\mathbb{P}^i(T_{(n+1)i} > L_i|\text{data}) = 1-\gamma \quad (17)$$

From (16), we have,

$$\mathbb{P}^{i}(T_{(n+1)i} > L_i \mid \text{data}) = \int_{L_i}^{\infty} f^{i}(t_{(n+1)i} \mid \text{data})\, dt_{(n+1)i} =$$

$$= r\left(\frac{C}{V_i e^{-\beta/V_i}}\right)^{r} \int_{L_i}^{\infty} \frac{dt_{(n+1)i}}{\left\{t_{(n+1)i} + \frac{C}{V_i e^{-\beta/V_i}}\right\}^{r+1}} \tag{18}$$

where $C = \sum_{j=1}^{k} A_j V_j e^{-\beta/V_j}$.

That is,

$$\frac{\left(\frac{C}{V_i e^{-\beta/V_i}}\right)^{r}}{\left\{L_i + \frac{C}{V_i e^{-\beta/V_i}}\right\}^{r}} = 1 - \gamma \tag{19}$$

From (19), we can find the required values of V_i and L_i to be used in quality control tests, and consider the following procedure: put m new unities in test with the stress level V_i and during the period of time L_i. Let X be the number of failures, and assume that $X \sim b(m, p^i)$ (a binomial distribution), where $p^i = \mathbb{P}^i(T_{(n+1)i} \le L_i \mid \text{data})$. The production line is under control if we do not reject the hypothesis H_0: $p^i \le \gamma$.

6. AN EXAMPLE

Consider the data of Table 1 generatd by an Eyring model (2) and exponential density (1) with $\alpha = -10$ and $\beta = 5$. The maximum likelihood estimators for α and β are given by $\hat{\alpha} = -9.99714$ and $\hat{\beta} = 5.20520$ (see (5)). From the asymptotical normal distribution for $\hat{\alpha}$ and $\hat{\beta}$ given in (6), with

$$I^{-1}(\hat{\alpha},\hat{\beta}) = \begin{bmatrix} 0.0351 & 0.7961 \\ 0.7961 & 24.4460 \end{bmatrix}$$

we find 95% confidence intervals for α and β given by $-10.3643 \le \alpha \le -9.6299$ and $-4.4856 \le \beta \le 14.8960$.

TABLE 1 - Generated Data with $\alpha = -10$ and $\beta = 5$ (Lifetime in Hours).

i	V_i	n_i	r_i	θ_i	A_i	NONCENSORED OBSERVATIONS
1	10	20	5	3631.55	18085.00	178, 301, 574, 920, 1007
2	15	20	6	2049.36	13182.00	101, 115, 147, 609, 705, 767
3	20	20	7	1414.13	9907.03	89, 96, 106, 246, 315, 347, 622
4	25	20	9	1076.13	9646.02	95, 98, 131, 221, 236, 283, 377, 561, 637
5	30	20	10	867.37	8676.00	24, 62, 245, 314, 332, 338, 386, 401, 491, 553
6	35	20	12	725.97	8763.96	73, 96, 132, 177, 211, 345, 361, 378, 399, 416, 560, 624
7	40	20	13	623.98	7837.96	13, 16, 52, 52, 148, 229, 255, 288, 317, 359, 560, 581, 621
8	45	20	14	547.00	7466.06	17, 21, 32, 52, 68, 87, 112, 197, 366, 384, 395, 490, 611, 662
9	50	20	15	486.86	7137.90	10, 13, 83, 87, 129, 135, 196, 244, 250, 262, 264, 293, 318, 624, 705
10	55	20	18	438.59	8372.88	11, 39, 53, 90, 96, 115, 153, 228, 259, 323, 356, 388, 411, 597, 761, 763, 823, 969

The maximum likelihood estimator for $\theta_1 = e^{\beta/V_1}/(V_1 e^{\alpha})$, the mean life time under the normal use stress level $V_1 = 10$ is given by $\hat{\theta}_1 = 3696.2453$. Since $a_1 = 3.5496$, $a_2 = 0.15646$ and $r = 109$, $\hat{\theta}_1$ has an asymptotical normal distribution (7) with variance $\hat{\sigma}^2_{\theta_1} = 1645609.091$. Therefore, a 95% approximate confidence interval for θ_1 is given by $1181.932 \le \theta_1 \le 6210.559$.

Usually, we should be careful to get inferences on θ_1 based on the asymptotical normal distribution (7). Clearly, we observe good eliptical form for the contour plots of the likelihood function for α and β, that is, good normality for the likelihood of α and β (see Figure 1), but bad eliptical form for the contour plots of the likelihood function for θ_1 and β (see Figure 2).

In Figures 3, 4 and 5, we have the plots of the marginal posterior densities for α, β and $\delta = \log\theta_1$ given in (10), (11) and (13), respectively. The modes of these posterior densities are given by $\tilde{\alpha} = -9.9998$, $\tilde{\beta} = 5.2040$ and $\tilde{\delta} = 8.2172$ ($\tilde{\theta}_1 = 3704.00$). These values are very close to the maximum likelihood estimators for α, β and θ_1.

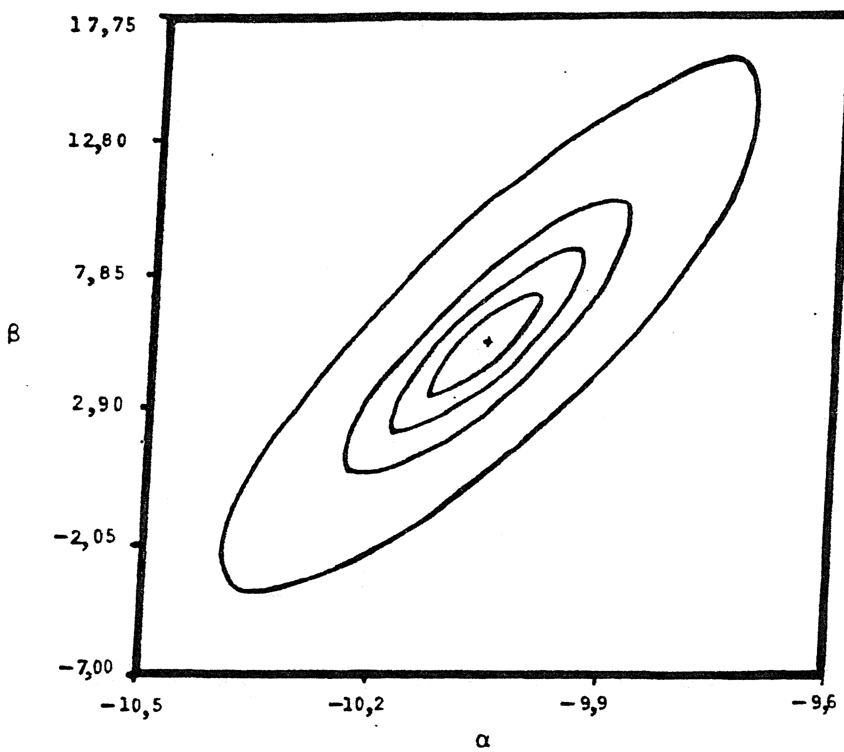

FIGURE 1 - Contours of the Likelihood Function for α and β.

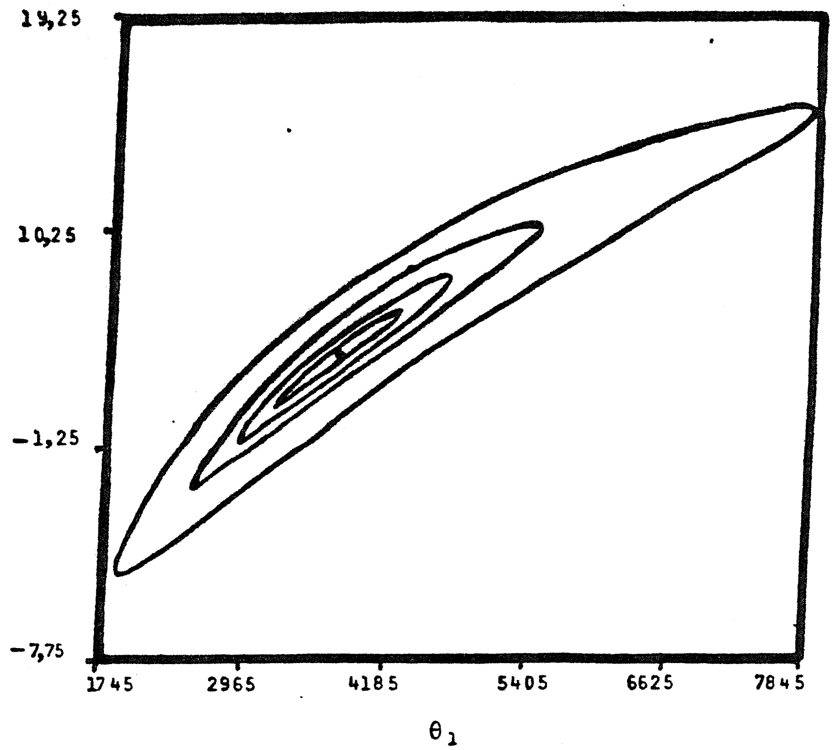

FIGURE 2 - Contours of the Likelihood Function for θ_1 and β.

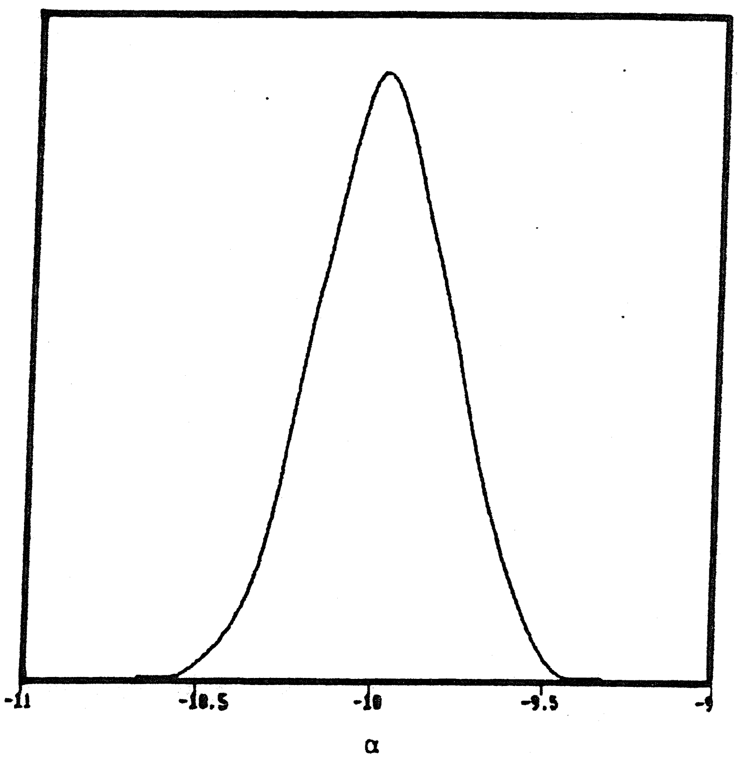

FIGURE 3 - Marginal Posterior Density for α.

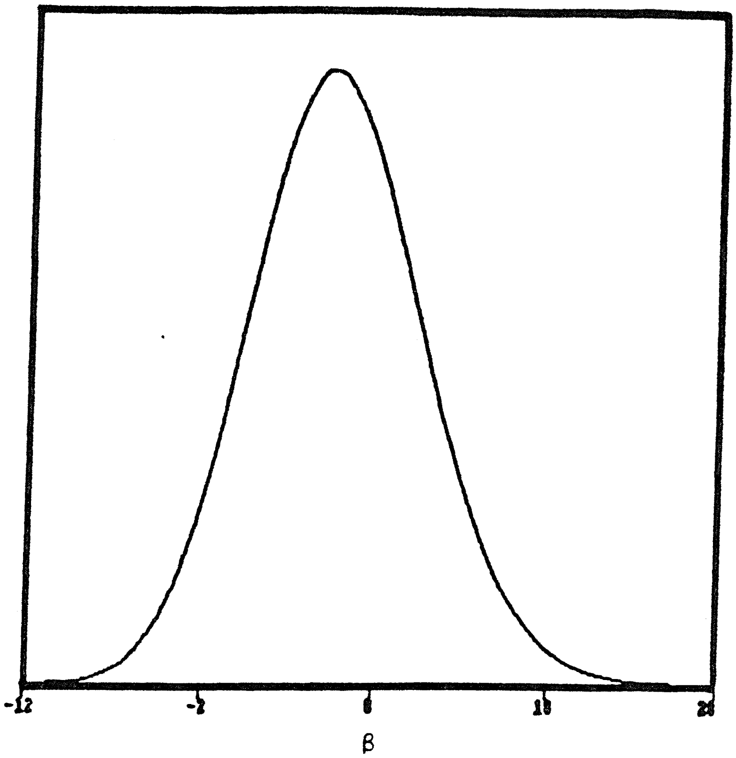

FIGURE 4 - Marginal Posterior Density for β.

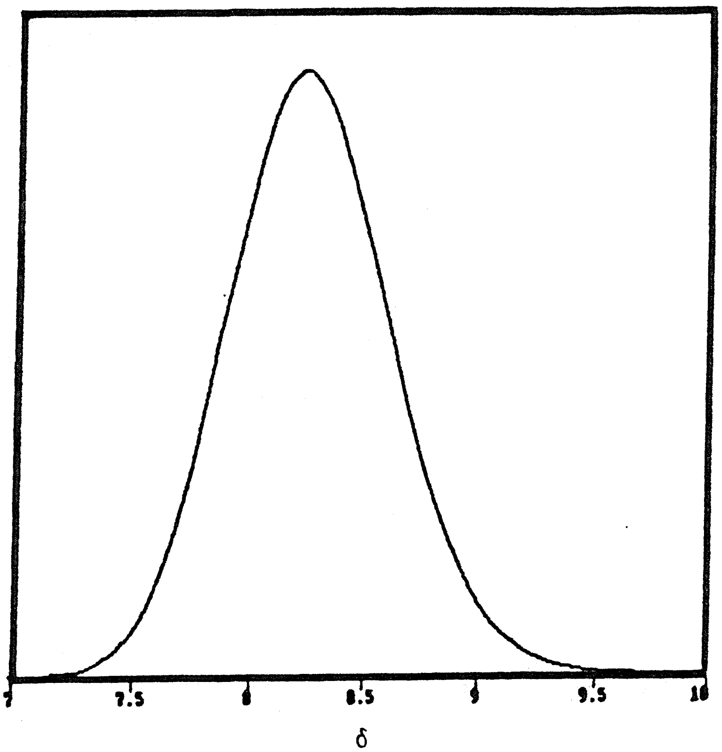

FIGURE 5 - Marginal Posterior Density for $\delta = \log\theta_1$.

In Table 2, we have some confidence and credible intervals for α, β and θ_1. Observe that, we have good agreement for the obtained intervals for α and β, but different results for θ_1. Using the marginal posterior density for $\delta = \log\theta_1$ given in (13) obtained by Laplace's method, we get a 95% HPD interval for θ_1 very close to the 95% confidence interval for θ_1 obtained by the asymptotical likelihood ratio statistics which is invariant to parametrizations.

We observe that in the parametrization $\delta = \log\theta_1$ and β, the approximate 95% confidence interval for δ obtained by the asymptotical normality of the maximum likelihood estimators $\hat{\delta}$ and $\hat{\beta}$ is given by (7.5362; 8.8966). That is, the researcher gets an approximate 95% confidence interval for $\theta_1 = e^{\delta}$ given by (1874.6741; 7307.0433).

TABLE 2 - 95% Confidence or Credible Intervals for α, β and θ_1.

ASYMPTOTICAL NORMALITY OF $\hat{\alpha}$ AND $\hat{\beta}$	ASYMPTOTICAL NORMALITY OF $\hat{\theta}_1$
$(-10.364 \leq \alpha \leq -9.630)$	$(1182.057 \leq \theta_1 \leq 6210.434)$
$(-4.486 \leq \beta \leq 14.896)$	
ASYMPTOTICAL LIKELIHOOD RATIO STATISTICS	
(A) Parametrization α and β	(B) Parametrization θ_1 and β
$(-10.486 \leq \alpha \leq -9.623)$	$(2055.555 \leq \theta_1 \leq 7800.000)$
$(-3.633 \leq \beta \leq 15.676)$	
HPD INTERVALS BASED ON POSTERIOR DENSITIES FOR α, β AND $\delta = \log\theta_1$ GIVEN IN (10), (11) AND (13)	
$(-10.357 \leq \alpha \leq -9.605)$	$(2067.156 \leq \theta_1 \leq 7653.030)$
$(-3.588 \leq \beta \leq 15.984)$	

Considering the data of Table 1, we can use the predictive density (16) to find the required stress level V_i and the required fixed period of time L_i to be used in a quality control test. We have $C = 1806870.275$ (see (18)) and $r = 109$. Assuming $\beta = 5$, $1-\gamma = 0.80$ and a fixed value of V_i, we use (19) to find L_i required for a quality control test (in Table 3, we have some values of L_i considering different values for V_i).

TABLE 3 - Required Values for L_i Considering V_i Fixed ($1-\gamma = 0.80$).

V_i	L_i
10	814.297
20	317.088
25	241.298
30	194.490
35	162.783
40	132.914
50	109.168

In Table 4, we fix some values of L_i and we find the required stress level V_i using (19).

TABLE 4 - Required Values for V_i Considering L_i Fixed ($1-\gamma = 0.80$).

L_i	V_i
800	10.119
700	11.080
600	12.343
500	14.087
400	16.667
350	18.492
300	20.910
250	24.274
200	29.291
150	37.608
100	54.166

7. CONCLUSIONS

The use of Bayesian methods can be of great practical interest. The marginal posterior density for θ_1 given in (13) is a good alternative for the classical asymptotical existing methods for inferences on θ_1 considering the Eyring model with an exponential distribution for the life times of unitis. We also observe that the use of the predictive density (16) can simplify the design of quality control in the accelerated life tests.

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