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I.C.M.S.C.

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ACCELERATED LIFE TESTS WITH AN EXPONENTIAL DISTRIBUTION: A BAYESIAN APPROACH WITH THE POWER RULE MODEL AND TYPE II CENSORED DATA

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SUMMARY

"In this paper, we present a Bayesian analysis of the power rule model $\theta_i = \alpha / V_i^{\beta}$, $i = 1,2,\ldots,k$ considering k levels of a stress V, an exponential distribution with mean θ_i for the life times of the components and assuming a type II censoring mechanism. We assume a Jeffreys prior density for α and β and we use the Laplace's method to find the marginal posterior density for θ_1 considering V_1, the usual level of stress. We also present some considerations for design of experiments and a numerical example with generated data".

Key words: accelerated life tests, power rule model, Bayesian analysis, exponential distribution.

1. INTRODUCTION

Let $T > 0$ be a random variable denoting the life time of a device with an exponential density,

$$f(t;\lambda_i) = \lambda_i \exp\{-\lambda_i t\} \tag{1}$$

where $\lambda_i > 0$ and $t > 0$. The parameter λ_i denotes the constant rate of failures under a stress level V_i, $i = 1,2,...,k$. The mean time of failure under the stress V_i is given by $\theta_i = 1/\lambda_i$.

To minimize the time and cost of the experiment, usually the industries use k levels of a stress to estimate the mean time of failure under the usual stress level.

We assume the power rule model given by,

$$\theta_i = \frac{\alpha}{V_i^{\beta}} \tag{2}$$

where $\alpha > 0$ and $-\infty < \beta < \infty$ are unknown parameters.

An application of this model is given by Levenbach (1957) for dieletric capacitors.

With k stress levels, we assume a type II censoring mechanism, that is, the experiment terminates when we observe r_i failures for each level i of stress. Thus, with n_i unities at the beginning of each test with stress V_i, we have the ordered uncensored observations given by $t_{1i}, t_{2i}, ..., t_{r_i i}$ and $n_i - r_i$ censored observations equal to $t_{r_i i}$, $i = 1,2,...,k$.

The likelihood function for α and β considering the data under stress level V_i is given by,

$$L(\alpha,\beta) = \prod_{j=1}^{r_i} f(t_{ji};\ \lambda_i) S^{n_i - r_i}(t_{r_i i};\lambda_i) \tag{3}$$

where $S(t_{r_i i};\ \lambda_i)$ is the reliability function given by,

$$S(t_{r_i i};\ \lambda_i) = P\{T > t_{r_i i}\} = e^{-\lambda_i t_{r_i i}}$$

and $\lambda_i = V_i^{\beta}/\alpha$.

That is,

$$L(\alpha,\beta) = \frac{V_i^{\beta r_i}}{\alpha^{r_i}} \exp\{-\frac{A_i V_i^{\beta}}{\alpha}\} \tag{4}$$

where $A_i = \sum_{j=1}^{r_i} t_{ji} + (n_i - r_i) t_{r_i i}$.

Considering the data of the k stress levels $V_1, V_2, \ldots, V_k$, the logarithm of the likelihood function for α and β is given by,

$$\ell(\alpha,\beta) = \beta \sum_{i=1}^{k} r_i \log V_i - r \log \alpha - \frac{1}{\alpha} \sum_{i=1}^{k} A_i V_i^{\beta} \tag{5}$$

where $r = \sum_{i=1}^{k} r_i$ (total number of observed failures).

From $\partial\ell/\partial\alpha = 0$ and $\partial\ell/\partial\beta = 0$, we find the maximun likelihood estimators of α and β given by the equations,

$$\hat{\alpha} = \frac{1}{r} \sum_{i=1}^{k} A_i V_i^{\hat{\beta}}$$

$$\sum_{i=1}^{k} r_i \log V_i = \frac{r \sum_{i=1}^{k} A_i V_i^{\hat{\beta}} \log V_i}{\sum_{i=1}^{k} A_i V_i^{\hat{\beta}}} \tag{6}$$

The second partial derivatives of $\ell(\alpha,\beta)$ are given by,

$$\frac{\partial^2 \ell}{\partial \alpha^2} = \frac{r}{\alpha^2} - \frac{2}{\alpha^3} \sum_{i=1}^{k} A_i V_i^{\beta}$$

$$\frac{\partial^2 \ell}{\partial \beta^2} = - \frac{1}{\alpha} \sum_{i=1}^{k} A_i V_i^{\beta} (\log V_i)^2 \tag{7}$$

$$\frac{\partial^2 \ell}{\partial \alpha \partial \beta} = \frac{1}{\alpha^2} \sum_{i=1}^{k} A_i V_i^{\beta} (\log V_i)$$

Since r_i is fixed (type II censoring), we have $E(A_i) = r_i \alpha / V_i^{\beta}$. Thus, the Fisher information matrix is given by,

$$I(\alpha,\beta) = \begin{pmatrix} \frac{r}{\alpha^2} & -\frac{1}{\alpha} \sum_{i=1}^{k} r_i (\log V_i) \\ -\frac{1}{\alpha} \sum_{i=1}^{k} r_i (\log V_i) & \sum_{i=1}^{k} r_i (\log V_i)^2 \end{pmatrix} \tag{8}$$

To construct confidence intervals and hypothesis tests for the parameters α and β, we usually use the asymptotical normality of the maximum likelihood estimators $\hat{\alpha}$ and $\hat{\beta}$ given by,

$$(\hat{\alpha},\hat{\beta}) \overset{a}{\sim} N\{(\alpha,\beta);\ I^{-1}(\alpha,\beta)\} \tag{9}$$

2. A BAYESIAN ANALYSIS ASSUMING α AND β UNKNOWN

The Jeffreys prior density for and (see for example, Box and Tiao, 1973) is given by,

$$\pi(\alpha,\beta) \propto \{\det I(\alpha,\beta)\}^{1/2}$$

$$\propto \frac{1}{\alpha}\{r \sum_{i=1}^{k} A_i(\log V_i)^2 - (\sum_{i=1}^{k} r_i \log V_i)^2\}^{1/2} \tag{10}$$

That is,

$$\pi(\alpha,\beta) \propto \frac{1}{\alpha} \tag{11}$$

where $\alpha > 0$ and $-\infty < \beta < \infty$.

The joint posterior density for α and β is given by,

$$\pi(\alpha,\beta|\text{data}) \propto \alpha^{-(r+1)} (\prod_{i=1}^{k} V_i^{\beta r_i}) \exp\{-\alpha^{-1} \sum_{i=1}^{k} A_i V_i^{\beta}\} \tag{12}$$

The marginal posterior density for β is given by,

$$\pi(\beta|\text{data}) \quad \frac{\prod_{i=1}^{k} V_i^{\beta r_i}}{\{\sum_{i=1}^{k} A_i V_i^{\beta}\}^{r}} \tag{13}$$

2.1. POSTERIOR DENSITY FOR THE MEAN LIFE TIME UNDER THE USUAL STRESS LEVEL V_1

Let us consider the transformation of variables $\beta = \beta$ and $\theta_1 = \alpha/V_1^{\beta}$, where V_1 is the usual stress level. The joint posterior density for θ_1 and β is given by:

$$\pi(\theta_1,\beta|\text{data}) \propto \frac{\left(\prod_{i=1}^{k} V_i^{\beta r_i}\right)}{\theta_1^{r+1} V_1^{\beta r}} \exp\left\{-\frac{1}{\theta_1 V_1^{\beta}} \sum_{i=1}^{k} A_i V_i^{\beta}\right\} \tag{14}$$

The marginal posterior density for θ_1 is given by,

$$\pi(\theta_1|\text{data}) \propto \frac{1}{\theta_1^{r+1}} \int_{-\infty}^{\infty} e^{-nh_{\theta_1}(\beta)} \, d\beta \tag{15}$$

where $nh_{\theta_1}(\beta) = \beta r \log V_1 - \beta \sum_{i=1}^{k} r_i \log V_i + \frac{1}{\theta_1 V_1^{\beta}} \sum_{i=1}^{k} A_i V_i^{\beta}$.

Thus, the marginal posterior density for θ_1 approximated by the Laplace method (see for example, Tierney & Kadane, 1986), is given by,

$$\pi(\theta_1|\text{data}) \propto \frac{1}{\theta_1^{r+1}} \{h''_{\theta_1}(\hat{\beta})\}^{-1/2} \frac{\prod_{i=1}^{k} V_i^{\hat{\beta} r_i}}{V_1^{\hat{\beta} r}} \times$$

$$\exp\left\{-\frac{1}{\theta_1 V_1^{\hat{\beta}}} \sum_{i=1}^{k} A_i V_i^{\hat{\beta}}\right\} \tag{16}$$

where $\theta_1 > 0$, $\hat{\beta}$ maximizes $-nh_{\theta 1}(\beta)$ and $h''_{\theta_1}(\beta)$ is given by,

$$h''_{\theta_1}(\beta) = \frac{1}{n\theta_1 V_1^{\beta}} \Big\{ \sum_{i=1}^{k} A_i V_i^{\beta} (\log V_i)^2 - \log V_1 \Big(\sum_{i=1}^{k} A_i V_i^{\beta} \log V_i\Big) -$$

$$- \log V_1 \Big[\sum_{i=1}^{k} A_i V_i^{\beta} \log V_i - \log V_1 \Big(\sum_{i=1}^{k} A_i V_i^{\beta}\Big)\Big]\Big\}$$

3. A BAYESIAN ANALYSIS ASSUMING β KNOWN

Assuming β known, the likelihood function for α is given by,

$$L(\alpha) \propto \prod_{i=1}^{k} \frac{V_i^{\beta r_i}}{\alpha^{r_i}} \exp\left\{-\frac{A_i V_i^{\beta}}{\alpha}\right\} \tag{17}$$

Considering the Jeffreys prior $\pi(\alpha) \propto \alpha^{-1}$, $\alpha > 0$, the posterior density for α is given by,

$$\pi(\alpha|\text{data}) = \frac{\left\{\sum_{i=1}^{k} A_i V_i^{\beta}\right\}^r}{\Gamma(r)} \alpha^{-(r+1)} \exp\left\{-\frac{1}{\alpha}\sum_{i=1}^{k} A_i V_i^{\beta}\right\} \tag{18}$$

where $\alpha > 0$.

The posterior density for $\theta_1 = \alpha/V_1^{\beta}$ is given by,

$$\pi(\theta_1|\text{data}) = \frac{\left\{\sum_{i=1}^{k} A_i V_i^{\beta}\right\}^r}{V_1^{\beta}\theta_1^{r+1}\Gamma(r)} \exp\left\{-\frac{1}{\theta_1 V_1^{\beta}}\sum_{i=1}^{k} A_i V_i^{\beta}\right\} \tag{19}$$

where $\theta_1 > 0$.

The mode of the posterior density (19) is given by,

$$\hat{\theta}_1^* = \frac{\sum_{i=1}^{k} A_i (V_i/V_1)^{\beta}}{r+1} \tag{20}$$

From (19), we observe that the posterior density for $2\left(\sum_{i=1}^{k} A_i V_i^{\beta}\right)/\theta_1 V_1^{\beta}$ is a chi-square density with 2r degrees of freedom. Thus, a Bayesian interval with probability $1-\alpha^*$ for the mean life time θ_1 of an unity with the usual stress V_1 is given by:

$$\left(\frac{2(\sum_{i=1}^{k} A_i V_i^{\beta})}{V_1^{\beta} X_{2r}^2 (1-\alpha^*/2)} \; ; \; \frac{2(\sum_{i=1}^{k} A_i V_i^{\beta})}{V_1^{\beta} X_{2r}^2 (\alpha^*/2)} \right) \qquad (21)$$

where $X_{2r}^2(\alpha^*/2)$ is the $100(1-\alpha^*)\%$ percentile of a chi-square distribution with 2r degrees of freedom.

The length of the interval (21) is given by,

$$L = \frac{2(\sum_{i=1}^{k} A_i V_i^{\beta})}{V_1^{\beta}} \left\{ \frac{1}{X_{2r}^2(\alpha^*/2)} - \frac{1}{X_{2r}^2(1-\alpha^*/2)} \right\} \qquad (22)$$

4. SOME CONSIDERATIONS ABOUT THE DETERMINATION OF THE REQUIRED NUMBER OF FAILURES IN AN ACCELERATED LIFE TEST

Usually, the researcher wants to estimate the fixed number of unities in each stress to have a fixed length L of the HPD interval for θ_1 with probability $1-\alpha^*$. Assuming K stress levels V_i in a first stage we could estimate the required number r_{K+1} of failures in an additional stress level V_{K+1} to find a fixed length L using the information of the first stage. Thus, considering β known, we have the "data 1" of the first stage with $n = \sum_{i=1}^{k} n_i$ unities, $r = \sum_{i=1}^{k} r_i$ failures, K stress levels $V_1, V_2, \ldots, V_k$ and the statistics $A_1, A_2, \ldots, A_k$ given in (4). Let us assume that, based on the "data 1" of the first stage, we find a $1-\alpha^*$ HPD interval for θ_1 with length $L_1 (L_1 > L)$.

The posterior density (18) for α based on the "data 1" is given by,

$$\pi(\alpha|\text{data }1) = \frac{\{\sum_{i=1}^{k} A_i V_i^{\beta}\}^r}{\Gamma(r)} \alpha^{-(r+1)} \exp\{-\frac{1}{\alpha}\sum_{i=1}^{k} A_i V_i^{\beta}\} \qquad (23)$$

where $\alpha > 0$.

As "data 2", consider n_{K+1} additional unities in a stress level V_{K+1} with r_{K+1} failures at the beginning of the test, and the statistics,

$$A_{K+1} = \sum_{j=1}^{r_{K+1}} t_{j(K+1)} + (n_{K+1} - r_{K+1}) t_{r_{K+1}, K+1} .$$

The likelihood function for α (based on the "data 2") is given by,

$$L(\alpha|\text{data }2) \propto \frac{V_{K+1}^{\beta r_{K+1}}}{\alpha^{r_{K+1}}} \exp\{-\frac{A_{K+1} V_{K+1}^{\beta}}{\alpha}\} \qquad (24)$$

Considering the posterior density (23) based on the "data 1" as the prior density for α, the posterior density for α based on the "data 2" is given by,

$$\pi(\alpha|\text{data }2) = \frac{\{\sum_{i=1}^{K+1} A_i V_i^{\beta}\}^{r+r_{K+1}}}{\Gamma(r + r_{K+1})} \alpha^{-(r + r_{K+1}+1)} \times$$

$$\exp\{-\frac{1}{\alpha}\sum_{i=1}^{k+1} A_i V_i^{\beta}\} \qquad (25)$$

where $\alpha > 0$.

With a transformation of variables $\theta_1 = \alpha/V_1^\beta$ in (25), we find a $1-\alpha^*$ HPD interval for θ_1 given by,

$$\left(\frac{2(\sum_{i=1}^{k+1} A_i V_i^\beta)}{V_1^\beta \chi^2_{2(r+r_{K+1})}(1-\alpha^*/2)} \; ; \; \frac{2(\sum_{i=1}^{k+1} A_i V_i^\beta)}{V_1^\beta \chi^2_{2(r+r_{K+1})}(\alpha^*/2)} \right) \qquad (26)$$

The length of this interval is given by,

$$L = \frac{2(\sum_{i=1}^{K+1} A_i V_i^\beta)}{V_1^\beta} \left\{ \frac{1}{\chi^2_{2(r+r_{K+1})}(\alpha^*/2)} - \frac{1}{\chi^2_{2(r+r_{K+1})}(1-\alpha^*/2)} \right\}$$

Assume now, that we have the "data 1" and we want to estimate the required number of falilures r_{K+1} for the second stage of study to have a specified length L for the $1-\alpha^*$ HPD interval for θ_1. Considering a $100(1-\gamma^*)\%$ confidence coeficient (observe that A_{K+1} is a random variable), we should find r_{K+1}, such that,

$$\mathbb{P}\left\{ \frac{2(\sum_{i=1}^{k} A_i V_i^\beta + A_{K+1} V_{K+1}^\beta)\, a(r_{K+1})}{V_1^\beta} < L \right\} = 1-\gamma^* \qquad (28)$$

where r is fixed in the first stage, β is known, L is fixed

$\sum_{i=1}^{k} \Lambda_i V_i^\beta$ is known from the observed first stage,

$V_1, V_2, \ldots, V_K, V_{K+1}$ are fixed stress levels,

$\Lambda_{K+1} = \sum_{j=1}^{r_{K+1}} t_{j,r_{K+1}} + (n_{K+1} - r_{K+1})\, t_{r_{K+1},\,K+1}$ and

$$a(r_{K+1}) = \frac{1}{X^2_{2(r+r_{K+1})}(\alpha^*/2)} - \frac{1}{X^2_{2(r+r_{K+1})}(1-\alpha^*/2)}$$

Observe that (28) can be given by,

$$P\{\Lambda_{K+1} \leq \frac{1}{V_{K+1}^{\beta}} \left(\frac{LV_1^\beta}{2a(r_{K+1})} - B\right)\} = 1 - \gamma^* \qquad (29)$$

where $B = \sum_{i=1}^{k} A_i V_i^\beta$ (fixed from "data 1").

Since $2\Lambda_{K+1}/\theta_{K+1} \sim X^2_{2r_{K+1}}$ where $\theta_{K+1} = \alpha/V_{K+1}^\beta$ (see for example, Mann, Schaffer, and Singpurwalla, 1974), we have $Y = 2\Lambda_{K+1}V_{K+1}^\beta/\alpha \sim X^2_{2r_{K+1}}$. Therefore, we should find r_{K+1} such that,

$$\mathbb{P}\{Y \leq \frac{2}{\alpha} \left(\frac{LV_1^\beta}{2a(r_{K+1})} - B\right)\} \approx 1 - \gamma^* \qquad (30)$$

where $Y \sim X^2_{2r_{K+1}}$.

Since α is unknown, we could use an estimator of α based on the "data 1". For example, we could consider the mode of the posterior density (23) given by $\hat{\alpha} = B/(r+1)$

5. AN EXAMPLE

Consider the data of table 1 (generated using the power rule model for 5 stress levels with $\alpha = 500$ and $\beta = 0.8$).

i	V_i	θ_i	n_i	r_i	UNCENSORED OBSERVATIONS
1	10	79.24	30	5	6, 8, 10, 12, 14
2	20	45.52	30	8	4, 5, 5, 6, 8, 8, 9, 14
3	30	32.90	30	12	2, 3, 3, 5, 6, 7, 7, 8, 8, 9, 10, 17
4	40	26.14	30	18	3, 3, 4, 5, 6, 6, 8, 9, 10, 10, 12, 12, 13, 14, 14, 14, 15, 24
5	50	21.87	30	22	2, 3, 4, 5, 5, 8, 8, 8, 9, 10, 12, 13, 14, 14, 15, 18, 18, 18, 19, 20, 20, 27

Table 1. GENERATED DATA WITH $\alpha = 500$ AND $\beta = 0.8$

The logarithm of the likelihood function (5) for α and β is given by,

$$\ell(\alpha,\beta) = 228.7575\beta - 65 \log\beta - \frac{1}{\alpha} \sum_{i=1}^{5} A_i V_i^{\beta} \tag{31}$$

The maximum likelihood estimators for α and β are given by $\hat{\alpha} = 502.0524$ and $\hat{\beta} = 0.8003$. Thus, the maximum likelihood estimator for θ_1 is given by $\hat{\theta}_1 = \hat{\alpha}/V_1^{\hat{\beta}} = 79.5150$.

The Fisher information matrix (8) is given by,

$$I(\hat{\alpha},\hat{\beta}) = \begin{pmatrix} 0.000258 & -0.455645 \\ -0.455645 & 818.749806 \end{pmatrix}$$

Thus, the maximum likelihood estimators for α and β have an asymptotical normal distribution given by,

$$(\hat{\alpha},\hat{\beta}) \overset{a}{\sim} N\{(\alpha,\beta);\ I^{-1}(\hat{\alpha},\hat{\beta})\} \tag{32}$$

where $I^{-1}(\hat{\alpha},\hat{\beta}) = \begin{pmatrix} 232204.60 & 129.22 \\ 129.22 & 0.07314 \end{pmatrix}$

From (32), we can construct confidence intervals for α and β. A 90% confidence interval for α is given by (-288.2244; 1292.3292) and a 90% confidence interval for β is given by (0.3568; 1.2438). Using the "delta method" (see for example, Miller, 1981), we can find a confidence interval for θ_1 using the asymptotical normal distribution (32). A 90% confidence interval for $\theta_1 = \alpha/V_1^{\beta}$ is given by (33.6654; 125.4033).

We can check the adequability of the power rule model (2) observing that we have a linear relation given by $\log \lambda_i = -\log \alpha + \beta \log V_i$. Considering the maximum likelihood estimators $\hat{\lambda}_i = r_i/A_i$, $i = 1,2,\ldots 5$ for each stress level, we observe an approximate linear relation between $\log \hat{\lambda}_i$ and $\log V_i$ (see figure 1), indicating the adequability of the power rule model (2) for the data of table 1.

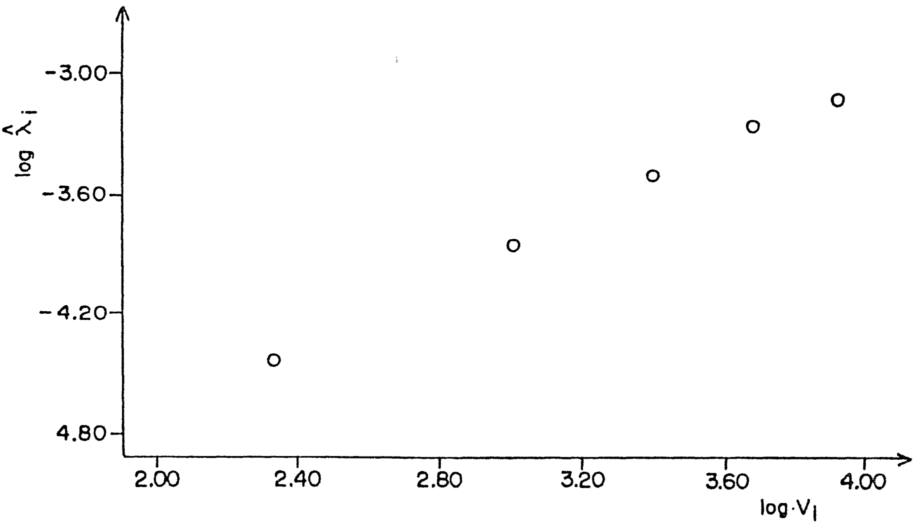

Figure 1. LOG($\hat{\lambda}_i$) VERSUS LOG(V_i)

Assuming α and β unknown, and the Jeffreys prior (11), the marginal posterior density for β (13) is given by,

$$\pi(\beta|\text{data}) \propto \frac{\{2.23 \times 10^{99}\}^{\beta}}{\{400(10^{\beta})+367(20^{\beta})+391(30^{\beta})+470(40^{\beta})+486(50^{\beta})\}^{65}}$$

where $-\infty < \beta < \infty$.

In figure 2, we have the plot of $\pi(\beta|\text{data})$. The mode of this posterior density is given by $\hat{\beta} \simeq 0.800$.

In figure 3, we have the plot of the marginal posterior density for $\theta_1 = \alpha / V_1^{\beta}$ (given in (16)), using the Laplace's method for approximation of integrals.

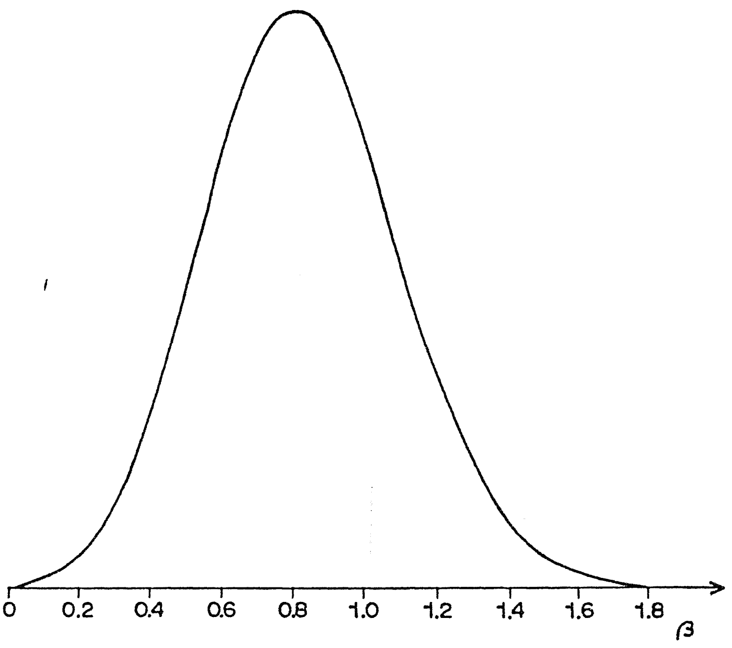

Figure 2. MARGINAL POSTERIOR DENSITY FOR β

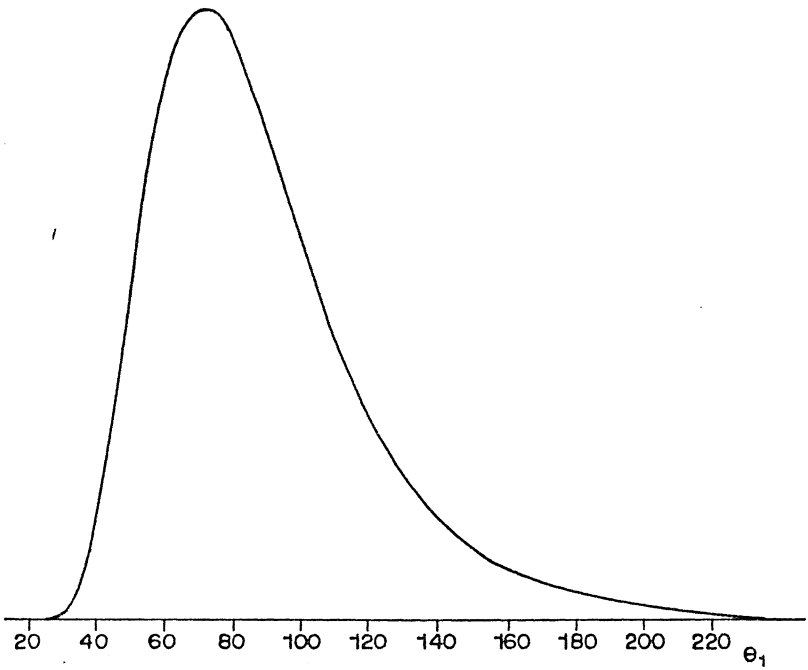

Figure 3. MARGINAL POSTERIOR DENSITY FOR θ_1

From figure 3, we can find a 90% HPD interval for θ_1. This Bayesian interval is given by approximately (50; 140) which is different of the 90% confidence interval for θ_1 given by (33.6654; 125.4033) using the asymptotical normality of the maximum likelihood estimators for α and β. In fact, we observe in figure 4 that a contour plot for the likelihood function of α and β does not present a good eliptical form, that is, we do not have good normal approximation for the maximum likelihood estimators $\hat{\alpha}$ and $\hat{\beta}$.

In figure 5, we have the plot of the posterior density for θ_1(19) assuming $\beta = 0.8$ known. In this case, a 90% HPD interval for θ_1 (see (21)) is given by (65.6824; 98.9963) with length $L = 33.3139$. The mode of the posterior density for θ_1 is given (from (20) by $\hat{\theta}_1^* = 78.28$.

Assume now, that the resercher wants a length interval $L = 32$. Thus, he would like to find the required number of additional failures in the level stress $V_6 = 60$ to obtain a 90% HPD interval for θ_1 with length $L = 32$. Considering a confidence coeficient $1 - \gamma^* = 0.90$ (see (30)), observe from table 2 that the required number of failures in the level stress $V_6 = 60$ is given by $r_6 = 13$.

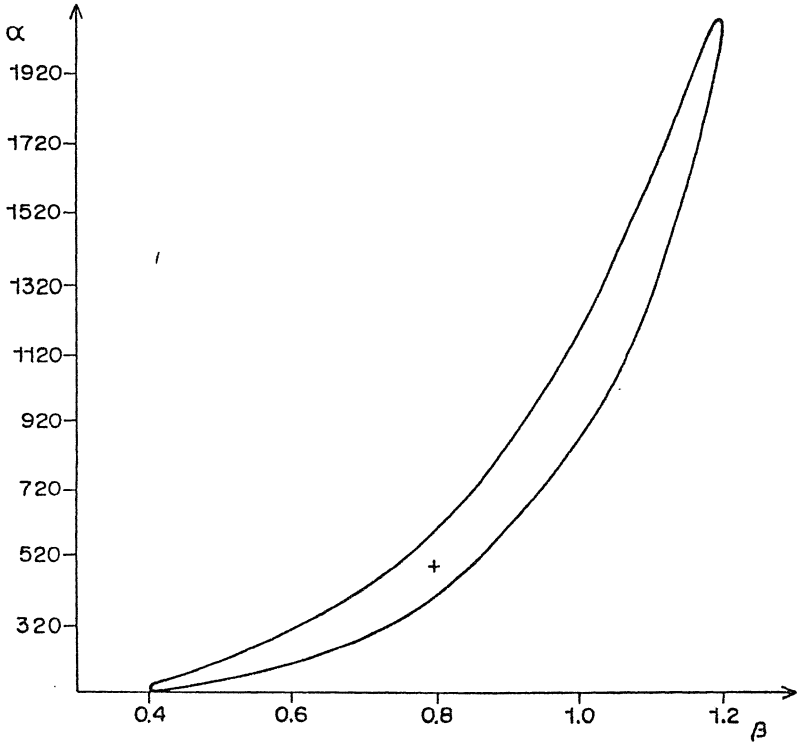

Figure 4. A CONTOUR OF THE LIKELIHOOD FUNCTION FOR α AND β

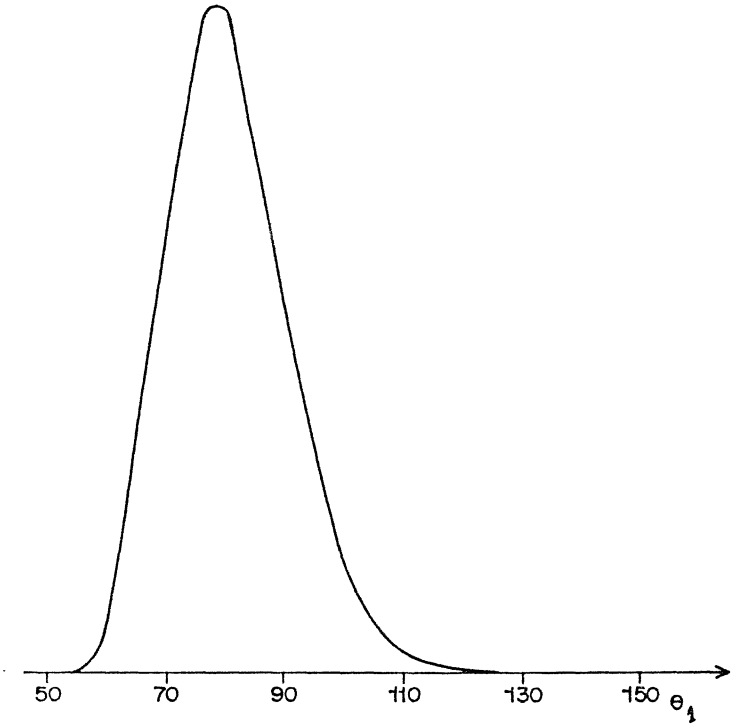

Figure 5. POSTERIOR DENSITY FOR θ_1 WITH $\beta = 0.8$ KNOWN

r_6	$1 - \gamma^*$
2	0.06
3	0.30
4	0.45
5	0.54
6	0.62
7	0.65
8	0.75
9	0.78
10	0.81
11	0.84
12	0.87
13	0.90
14	0.91
15	0.92
16	0.94
17	0.95

Table 2. OBSERVED CONFIDENCE $1 - \gamma^*$ FOR DIFFERENT VALUES OF r_6 ($r=65$; $L=32$; $1-\alpha^*=0.90$)

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