Oficina de sistemas diferenciais polinomiais

Resumo das palestras

1. **Palestrante:** Wilker Thiago Resende Fernandes (UFSJ).

   Titulo: a ser anunciado

   Resumo: a ser informado

2. **Palestrante:** Francisco Braun (UFSCAR)

   Título: Sobre o número de folhas inseparáveis que um sistema polinomial cordal pode ter.

   Resumo: Um sistema polinomial no plano $\dot{x} = P(x, y)$, $\dot{y} = Q(x, y)$ é chamado *cordal* quando P e Q não tem zeros (reais) em comum. O *grau* deste sistema é o máximo entre os graus de P e Q. Duas órbitas γ_1 e γ_2 deste sistema são ditas *inseparáveis* se para quaisquer seções transversais S_1 e S_2 a γ_1 e γ_2, respectivamente, existe uma órbita do sistema que intersecta S_1 e S_2. Claro está que as órbitas inseparáveis são as separatrizes do nosso sistema polinomial. Assim, se procuramos saber quantos sistemas polinomiais cordais de grau n (a menos de equivalência topológica) existem, segue de um conhecido resultado de Markus-Neumann-Peixoto que um primeiro passo é saber quantas folhas inseparáveis um campo polinomial cordal de grau n pode ter. Denotamos este importante número por $s(n)$, isto é, s(n) é o número máximo de folhas inseparáveis que um sistema polinomial cordal de grau n pode ter. Nesta palestra, faremos um breve histórico dos resultados conhecidos sobre cotas inferiores e superiores de s(n), bem como apresentaremos nossa contribuição ao problema.

   Este é um trabalho em desenvolvimento com Filipe Balduíno Pires Fernandes.

3. **Palestrante:** Camila Rodrigues (UFSC)

   Title: Quadratic systems with an invariant algebraic curve of degree 3 and a Darboux invariant

   Abstract: Denote by $\mathbb{R}[x, y]$ the ring of the real polynomials in the variables x and y. Consider the differential system in $\mathbb{R}^2$ given by

$$\dot{x} = P(x, y), \qquad \dot{y} = Q(x, y), \tag{1}$$

   where $P, Q \in \mathbb{R}[x, y]$. Here the dot denotes derivative with respect to the *time* t and the degree of system (1) is $m = \max\{\deg P, \deg Q\}$. When $m = 2$ we say that system (1) is a *quadratic polynomial differential system* or simply a *quadratic system*. The quadratic systems appear in the modelling of many natural phenomena described in different branches of science, in biological and physical applications. Besides the applications the quadratic systems became a matter of interest for the mathematicians.

   In this talk we present our investigation on quadratic systems having an invariant cubic. We present their normal forms and we classify the phase portraits in the Poincaré disc of all planar quadratic polynomial differential systems with invariant cubic curve and having a Darboux invariant defined by it. Quadratic system with conics have been studied by Oliveira-Llibre (2015 and 2018).

   This is a joint work with Jaume Llibre (UAB) and Regilene Oliveira (ICMC).

4. **Palestrante:** Leonardo P. C. da Cruz (UFSCar).

   Título: O Teorema de Bendixson–Dulac para alguns sistemas por partes

   Resumo: O Teorema de Bendixson–Dulac fornece um critério para encontrar limites superiores para o número de ciclos limite em sistemas diferenciais analíticos. Estendemos este resultado clássico para algumas classes de sistemas diferenciais por partes. Podemos aplicá-lo a três diferentes sistemas diferenciais de Liénard $\ddot{x}+f^{\pm}(x)\dot{x}+x=0$. O primeiro é linear, o segundo é racional e o último corresponde a uma particular extensão do oscilador cubico de van der Pol. Em todos os casos, os sistemas apresentam regiões no espaço de parâmetros sem ciclos limite e outros tendo no máximo um.

5. **Palestrante:** Ingrid Sofia Mesa Sarmiento (UFSCar).

   Title: Quadratic slow-fast systems on the plane.

   Abstract: In this work singularly perturbed quadratic polynomial differential systems

$$\varepsilon\dot{x} = P_\varepsilon(x,y) = P(x,y,\varepsilon), \quad \dot{y} = Q_\varepsilon(x,y) = Q(x,y,\varepsilon)$$

   with $x, y \in \mathbb{R}, \varepsilon \geqslant 0$ and $(P_\varepsilon, Q_\varepsilon) = 1$ for $\varepsilon > 0$, are considered. We prove that there are 10 classes of affine equivalence for these systems. We describe the dynamics of these 10 classes on the Poincaré disc when $\varepsilon = 0$. For $\varepsilon > 0$, we classify the possible finite and infinite singularities. We proved a kind of "Fenichel theorem". More specifically, by combining the slow and the fast dynamics with the dynamics at infinity, we describe the possible elliptic, hyperbolic, and parabolic sectors for the singularities at $\mathbb{S}^1$ (the equator of the Poincaré sphere). Moreover, when the critical manifold is normally hyperbolic we obtain algebraic invariants that allow us to describe globally the dynamics for $\varepsilon > 0$ small. In fact, when the critical manifold is a hyperbola we obtain 33 possible (and realizable) distinct phase portraits and when it is formed by two straight lines we get 11 possible (and realizable) distinct phase portraits on the Poincaré disc.

   This is a joint work with Paulo Ricardo da Silva (IBILCE-UNESP) and Regilene Oliveira (ICMC-USP).

6. **Palestrante:** Miriam Manoel (ICMC/USP).

   Title: Symmetric pairs of planar foliations

   Abstract: In this talk we explain how to recognize symmetries in binary differential equations (BDEs). These are implicit differential equations given by the zeros of a quadratic 1-form, $a(x, y)dy^2 + b(x, y)dxdy + c(x, y)dx^2 = 0$, for a, b, c smooth real functions defined on an open set of $\mathbb{R}^2$. Generically, solutions of a BDE are given as leaves of a pair of foliations, and the action of a symmetry must depend not only whether it preserves or inverts the plane orientation, but also whether it preserves or interchanges the foliations. We first present this dependence, which reveals to be given algebraically by a simple formula. We then present the general expressions of the symmetric quadratic 1-forms under all representations of the orthogonal group $\mathbf{O}(2)$.

   This is a joint work with Patricia Tempesta, UFSJ - São João del Rei, MG.

7. **Palestrante:** Alex Carlucci Rezende (UFSCar).

   Title: New results on the structurally unstable quadratic differential systems of codimensions one and two.

   Abstract: In this talk we present the new results concerning the structurally unstable quadratic differential systems of codimension one and two. In 1998, Artés, Kooij and Llibre proved that there exist 44 structurally stable topologically distinct phase portraits in the Poincaré disc of quadratic vector fields modulo limit cycles, and, in 2018, Artés, Llibre and Rezende showed the existence of at least 204 (at most 211) structurally unstable topologically distinct phase portraits of codimension-one quadratic systems, modulo limit cycles. Now, we begin to study the codimension-two quadratic systems. Combining the groups of codimension-one quadratic vector fields one to each other, we obtain ten new groups. We first consider group obtained by the coalescence of two finite singular points, yielding either a triple saddle, or a triple node, or a cusp point, or two saddle-nodes. We obtain all the possible topological phase portraits of group AA and prove their realization. We got 34 new topologically distinct phase portraits in the Poincaré disc modulo limit cycles.

8. **Palestrante:** Jackson Itikawa (UFIR)

   Título: Exemplos simples de sistemas diferenciais hamiltonianos planares com centros isócronos.

   Abstract: Apresentamos algumas famílias simples de sistemas diferenciais hamiltonianos em $\mathbb{R}^2$ com centros globais isócronos. Tais famílias constituem uma pequena contribuição proposta de caracterização de sistemas diferenciais planares que possuem centros globais, proposto por Conti em 1998.

9. **Palestrante:** Marcos Coutinho Mota (ICMC-USP)

   Title: A study of a new three-dimensional autonomous chaotic system

   Abstract: We present the study of dynamic aspects of the autonomous system

   $$\dot{x} = yz, \quad \dot{y} = x - y, \quad \dot{z} = 1 - x(\alpha y + \beta x),$$

   where $(x, y, z) \in \mathbb{R}^3$ and $\alpha, \beta \in [0, 1]$ are two parameters. It contains the Sprott B and the Sprott C systems at the two extremes of its parameter spectrum and we called it Sprott BC system. Here we present the complete description of its equilibrium points and we show that this system passes through a Hopf bifurcation at $\alpha = 0$ and we compute the respective first Lyapunov coefficient. Using the Poincaré compactification of a polynomial vector field in $\mathbb{R}^3$ we give a complete description of its dynamic on the Poincaré sphere at infinity.

   This work is part of my PhD Thesis under supervision of Regilene Oliveira, Alex C. Rezende (UFSCar) e Joan C. Artes (UAB)

10. **Palestrante:** Ana Maria Travaglini (ICMC-USP).

    Title: Darboux integrability for quadratic systems with invariant hyperbola.

    Abstract: In this talk, I will present the basic definitions and properties of the Darboux integrability theory in order to guarantee or not the existence of a first integral for a family of planar systems through invariant algebraic curves and exponential factors.

    This work is part of my PhD Thesis under supervision of Regilene Oliveira and Dana Schlomiuk (University of Montreal, Canada).